Preliminaries to Automatic Generation of Mizar Documentation for Circuits

Grzegorz Bancerek¹ Białystok Technical University

Adam Naumowicz² University of Białystok

Summary. In this paper we introduce technical notions used by a system which automatically generates Mizar documentation for specified circuits. They provide a ready for use elements needed to justify correctness of circuits' construction. We concentrate on the concept of stabilization and analyze one-gate circuits and their combinations.

MML Identifier: CIRCCMB3.

The articles [21], [26], [20], [11], [10], [27], [7], [12], [2], [3], [8], [1], [9], [14], [4], [6], [22], [25], [23], [5], [17], [16], [15], [18], [19], [13], and [24] provide the notation and terminology for this paper.

1. Stabilizing Circuits

The following proposition is true

(1) Let S be a non-void circuit-like non empty many sorted signature, A be a non-empty circuit of S, s be a state of A, and x be a set. If $x \in \text{InputVertices}(S)$, then for every natural number n holds (Following(s,n))(x) = s(x).

Let S be a non-void circuit-like non empty many sorted signature, let A be a non-empty circuit of S, and let s be a state of A. We say that s is stabilizing if and only if:

¹This paper was written when the first author visited Shinshu University as a two-year JSPS Fellow.

²The paper was prepared during the author's cooperative research at Shinshu University 'Verification of circuit designs with the aid of the Mizar system'.

(Def. 1) There exists a natural number n such that Following(s, n) is stable.

Let S be a non-void circuit-like non empty many sorted signature and let A be a non-empty circuit of S. We say that A is stabilizing if and only if:

(Def. 2) Every state of A is stabilizing.

We say that A has a stabilization limit if and only if:

(Def. 3) There exists a natural number n such that for every state s of A holds Following(s, n) is stable.

Let S be a non void circuit-like non empty many sorted signature. Note that every non-empty circuit of S which has a stabilization limit is also stabilizing.

Let S be a non-void circuit-like non empty many sorted signature, let A be a non-empty circuit of S, and let s be a state of A. Let us assume that s is stabilizing. The functor Result(s) yields a state of A and is defined as follows:

(Def. 4) Result(s) is stable and there exists a natural number n such that Result(s) = Following(s, n).

Let S be a non-void circuit-like non empty many sorted signature, let A be a non-empty circuit of S, and let s be a state of A. Let us assume that s is stabilizing. The stabilization time of s is a natural number and is defined by the conditions (Def. 5).

- (Def. 5)(i) Following (s, the stabilization time of s) is stable, and
 - (ii) for every natural number n such that n < the stabilization time of s holds Following(s, n) is not stable.

The following propositions are true:

- (2) Let S be a non void circuit-like non empty many sorted signature, A be a non-empty circuit of S, and s be a state of A. If s is stabilizing, then $\operatorname{Result}(s) = \operatorname{Following}(s, \text{the stabilization time of } s)$.
- (3) Let S be a non-void circuit-like non-empty many sorted signature, A be a non-empty circuit of S, s be a state of A, and n be a natural number. If Following(s, n) is stable, then the stabilization time of $s \leq n$.
- (4) Let S be a non-void circuit-like non-empty many sorted signature, A be a non-empty circuit of S, s be a state of A, and n be a natural number. If Following(s, n) is stable, then Result(s) = Following(s, n).
- (5) Let S be a non void circuit-like non empty many sorted signature, A be a non-empty circuit of S, s be a state of A, and n be a natural number. Suppose s is stabilizing and $n \ge$ the stabilization time of s. Then Result(s) = Following(s, n).
- (6) Let S be a non-void circuit-like non-empty many sorted signature, A be a non-empty circuit of S, and s be a state of A. If s is stabilizing, then for every set x such that $x \in \text{InputVertices}(S)$ holds (Result(s))(x) = s(x).
- (7) Let S_1 , S be non void circuit-like non empty many sorted signatures, A_1 be a non-empty circuit of S_1 , A be a non-empty circuit of S, s be a state

- of A, and s_1 be a state of A_1 . If $s_1 = s$ the carrier of S_1 , then for every vertex v_1 of S_1 holds $s_1(v_1) = s(v_1)$.
- (8) Let S_1 , S_2 be non void circuit-like non empty many sorted signatures. Suppose InputVertices (S_1) misses InnerVertices (S_2) and InputVertices (S_2) misses InnerVertices (S_1) . Let S be a non void circuit-like non empty many sorted signature. Suppose $S = S_1 + \cdot S_2$. Let A_1 be a non-empty circuit of S_1 and A_2 be a non-empty circuit of S_2 . Suppose $A_1 \approx A_2$. Let A be a non-empty circuit of S. Suppose $A = A_1 + \cdot A_2$. Let S be a state of S_2 be a state of S_3 suppose $S_3 = S$ the carrier of S_3 and S_4 is stabilizing and S_4 is stabilizing. Then S_3 is stabilizing.
- (9) Let S_1 , S_2 be non void circuit-like non empty many sorted signatures. Suppose InputVertices (S_1) misses InnerVertices (S_2) and InputVertices (S_2) misses InnerVertices (S_1) . Let S be a non void circuit-like non empty many sorted signature. Suppose $S = S_1 + \cdot S_2$. Let A_1 be a non-empty circuit of S_1 and S_2 be a non-empty circuit of S_2 . Suppose $S_1 = S_1 + \cdot S_2 = S_2 + \cdot S_2 = S_1 + \cdot S_2 = S_2 + \cdot S_2 = S_2 + \cdot S_2 = S_2 = S_2 + \cdot S_2 = S_2$
- (10) Let S_1 , S_2 be non void circuit-like non empty many sorted signatures. Suppose InputVertices (S_1) misses InnerVertices (S_2) . Let S be a non void circuit-like non empty many sorted signature. Suppose $S = S_1 + \cdot S_2$. Let A_1 be a non-empty circuit of S_1 and A_2 be a non-empty circuit of S_2 . Suppose $A_1 \approx A_2$. Let A be a non-empty circuit of S. Suppose $A = A_1 + \cdot A_2$. Let S be a state of S_1 and S_2 be a state of S_3 suppose S_4 and S_4 is stabilizing. Let S_4 be a state of S_4 . Suppose S_4 are Following (S_4) , the stabilization time of S_4) the carrier of S_4 and S_4 is stabilizing. Then S_4 is stabilizing.
- (11) Let S_1 , S_2 be non void circuit-like non empty many sorted signatures. Suppose InputVertices (S_1) misses InnerVertices (S_2) . Let S be a non void circuit-like non empty many sorted signature. Suppose $S = S_1 + \cdot S_2$. Let A_1 be a non-empty circuit of S_1 and A_2 be a non-empty circuit of S_2 . Suppose $A_1 \approx A_2$. Let A be a non-empty circuit of S. Suppose $A = A_1 + \cdot A_2$. Let S be a state of S_1 and S_2 be a state of S_3 . Suppose S_4 is stabilizing. Let S_2 be a state of S_3 suppose S_4 is stabilizing. Then the stabilization time of S_3 (the stabilization time of S_3).
- (12) Let S_1 , S_2 , S be non void circuit-like non empty many sorted signatures.

Suppose InputVertices(S_1) misses InnerVertices(S_2) and $S = S_1 + \cdot S_2$. Let A_1 be a non-empty circuit of S_1 , A_2 be a non-empty circuit of S_2 , and A be a non-empty circuit of S. Suppose $A_1 \approx A_2$ and $A = A_1 + \cdot A_2$. Let s be a state of A and s_1 be a state of A_1 . Suppose $s_1 = s \upharpoonright$ the carrier of S_1 and s_1 is stabilizing. Let s_2 be a state of A_2 . Suppose $s_2 = \text{Following}(s, \text{the stabilization time of } s_1) \upharpoonright$ the carrier of S_2 and s_2 is stabilizing. Then Result(s) \upharpoonright the carrier of $S_1 = \text{Result}(s_1)$.

2. One-gate Circuits

We now state three propositions:

- (13) Let x be a set, X be a non empty finite set, n be a natural number, p be a finite sequence with length n, g be a function from X^n into X, and s be a state of 1GateCircuit(p,g). Then $s \cdot p$ is an element of X^n .
- (14) For all sets x_1, x_2, x_3, x_4 holds $\operatorname{rng}\langle x_1, x_2, x_3, x_4 \rangle = \{x_1, x_2, x_3, x_4\}.$
- (15) For all sets x_1 , x_2 , x_3 , x_4 , x_5 holds $rng\langle x_1, x_2, x_3, x_4, x_5 \rangle = \{x_1, x_2, x_3, x_4, x_5\}.$

Let x_1, x_2, x_3, x_4 be sets. Then $\langle x_1, x_2, x_3, x_4 \rangle$ is a finite sequence with length 4. Let x_5 be a set. Then $\langle x_1, x_2, x_3, x_4, x_5 \rangle$ is a finite sequence with length 5.

Let S be a many sorted signature. We say that S is one-gate if and only if the condition (Def. 6) is satisfied.

(Def. 6) There exists a non empty finite set X and there exists a natural number n and there exists a finite sequence p with length n and there exists a function f from X^n into X such that S = 1GateCircStr(p, f).

Let S be a non empty many sorted signature and let A be an algebra over S. We say that A is one-gate if and only if the condition (Def. 7) is satisfied.

(Def. 7) There exists a non empty finite set X and there exists a natural number n and there exists a finite sequence p with length n and there exists a function f from X^n into X such that S = 1GateCircStr(p, f) and A = 1GateCircuit(p, f).

Let p be a finite sequence and let x be a set. Observe that 1GateCircStr(p, x) is finite.

Let us note that every many sorted signature which is one-gate is also strict, non void, non empty, unsplit, and finite and has arity held in gates.

One can check that every non empty many sorted signature which is one-gate has also denotation held in gates.

Let X be a non empty finite set, let n be a natural number, let p be a finite sequence with length n, and let f be a function from X^n into X. Note that 1GateCircStr(p,f) is one-gate.

One can check that there exists a many sorted signature which is one-gate.

Let S be an one-gate many sorted signature. Observe that every circuit of S which is one-gate is also strict and non-empty.

Let X be a non empty finite set, let n be a natural number, let p be a finite sequence with length n, and let f be a function from X^n into X. One can check that 1GateCircuit(p, f) is one-gate.

Let S be an one-gate many sorted signature. Observe that there exists a circuit of S which is one-gate and non-empty.

Let S be an one-gate many sorted signature. The functor Output S yields a vertex of S and is defined as follows:

(Def. 8) Output $S = \bigcup$ (the operation symbols of S).

Let S be an one-gate many sorted signature. Observe that Output S is pair. Next we state several propositions:

- (16) Let S be an one-gate many sorted signature, p be a finite sequence, and x be a set. If S = 1GateCircStr(p, x), then Output $S = \langle p, x \rangle$.
- (17) For every one-gate many sorted signature S holds InnerVertices $(S) = \{\text{Output } S\}.$
- (18) Let S be an one-gate many sorted signature, A be an one-gate circuit of S, n be a natural number, X be a finite non empty set, f be a function from X^n into X, and p be a finite sequence with length n. If A = 1GateCircuit(p, f), then S = 1GateCircStr(p, f).
- (19) Let n be a natural number, X be a finite non empty set, f be a function from X^n into X, p be a finite sequence with length n, and s be a state of 1GateCircuit(p, f). Then (Following(s))(Output 1GateCircStr(p, f)) = $f(s \cdot p)$.
- (20) Let S be an one-gate many sorted signature, A be an one-gate circuit of S, and s be a state of A. Then Following(s) is stable.

Let S be a non void circuit-like non empty many sorted signature. Observe that every non-empty circuit of S which is one-gate has also a stabilization limit.

We now state two propositions:

- (21) Let S be an one-gate many sorted signature, A be an one-gate circuit of S, and s be a state of A. Then Result(s) = Following(s).
- (22) Let S be an one-gate many sorted signature, A be an one-gate circuit of S, and s be a state of A. Then the stabilization time of $s \leq 1$.

In this article we present several logical schemes. The scheme OneGate1Ex deals with a set \mathcal{A} , a non empty finite set \mathcal{B} , and a unary functor \mathcal{F} yielding an element of \mathcal{B} , and states that:

There exists an one-gate many sorted signature S and there exists an one-gate circuit A of S such that InputVertices $(S) = \{A\}$ and for every state s of A holds $(Result(s))(Output <math>S) = \mathcal{F}(s(A))$ for all values of the parameters.

The scheme OneGate2Ex deals with sets \mathcal{A} , \mathcal{B} , a non empty finite set \mathcal{C} , and a binary functor \mathcal{F} yielding an element of \mathcal{C} , and states that:

There exists an one-gate many sorted signature S and there exists an one-gate circuit A of S such that InputVertices $(S) = \{A, B\}$ and for every state s of A holds $(Result(s))(Output <math>S) = \mathcal{F}(s(A), s(B))$ for all values of the parameters.

The scheme OneGate3Ex deals with sets \mathcal{A} , \mathcal{B} , \mathcal{C} , a non empty finite set \mathcal{D} , and a ternary functor \mathcal{F} yielding an element of \mathcal{D} , and states that:

There exists an one-gate many sorted signature S and there exists an one-gate circuit A of S such that InputVertices $(S) = \{A, B, C\}$ and for every state s of A holds (Result(s))(Output S)

$$= \mathcal{F}(s(\mathcal{A}), s(\mathcal{B}), s(\mathcal{C}))$$

for all values of the parameters.

The scheme OneGate4Ex deals with sets \mathcal{A} , \mathcal{B} , \mathcal{C} , \mathcal{D} , a non empty finite set \mathcal{E} , and a 4-ary functor \mathcal{F} yielding an element of \mathcal{E} , and states that:

There exists an one-gate many sorted signature S and there exists an one-gate circuit A of S such that InputVertices $(S) = \{A, B, C, D\}$ and for every state s of A holds (Result(s))(Output S)

$$= \mathcal{F}(s(\mathcal{A}), s(\mathcal{B}), s(\mathcal{C}), s(\mathcal{D}))$$

for all values of the parameters.

The scheme OneGate5Ex deals with sets \mathcal{A} , \mathcal{B} , \mathcal{C} , \mathcal{D} , \mathcal{E} , a non empty finite set \mathcal{F} , and a 5-ary functor \mathcal{F} yielding an element of \mathcal{F} , and states that:

There exists an one-gate many sorted signature S and there exists an one-gate circuit A of S such that InputVertices $(S) = \{A, B, C, D, \mathcal{E}\}$ and for every state s of A holds (Result(s))(Output S)

$$= \mathcal{F}(s(\mathcal{A}), s(\mathcal{B}), s(\mathcal{C}), s(\mathcal{D}), s(\mathcal{E}))$$

for all values of the parameters.

3. Mono-sorted Circuits

One can prove the following propositions:

- (23) For every constant function f holds $f = \text{dom } f \mapsto \text{the value of } f$.
- (24) For all non empty sets X, Y and for all natural numbers n, m such that $n \neq 0$ and $X^n = Y^m$ holds X = Y and n = m.
- (25) For all non empty many sorted signatures S_1 , S_2 holds every vertex of S_1 is a vertex of $S_1 + S_2$.
- (26) For all non empty many sorted signatures S_1 , S_2 holds every vertex of S_2 is a vertex of $S_1 + S_2$.

Let X be a non empty finite set. A non void non empty unsplit many sorted signature with arity held in gates with denotation held in gates is said to be a signature over X if it satisfies the condition (Def. 9).

(Def. 9) There exists a circuit A of it such that the sorts of A are constant and the value of the sorts of A = X and A has denotation held in gates.

Next we state the proposition

(27) Let n be a natural number, X be a non empty finite set, f be a function from X^n into X, and p be a finite sequence with length n. Then 1GateCircStr(p, f) is a signature over X.

Let X be a non empty finite set. Observe that there exists a signature over X which is strict and one-gate.

Let n be a natural number, let X be a non empty finite set, let f be a function from X^n into X, and let p be a finite sequence with length n. Then 1GateCircStr(p, f) is a strict signature over X.

Let X be a non empty finite set and let S be a signature over X. A circuit of S is called a circuit over X and S if:

(Def. 10) It has denotation held in gates and the sorts of it are constant and the value of the sorts of it = X.

Let X be a non empty finite set and let S be a signature over X. One can check that every circuit over X and S is non-empty and has denotation held in gates.

Next we state the proposition

(28) Let n be a natural number, X be a non empty finite set, f be a function from X^n into X, and p be a finite sequence with length n. Then 1GateCircuit(p, f) is a circuit over X and 1GateCircStr(p, f).

Let X be a non empty finite set and let S be an one-gate signature over X. One can check that there exists a circuit over X and S which is strict and one-gate.

Let X be a non empty finite set and let S be a signature over X. One can check that there exists a circuit over X and S which is strict.

Let n be a natural number, let X be a non empty finite set, let f be a function from X^n into X, and let p be a finite sequence with length n. Then 1GateCircuit(p, f) is a strict circuit over X and 1GateCircStr(p, f).

One can prove the following propositions:

- (29) For every non empty finite set X and for all signatures S_1 , S_2 over X holds $S_1 \approx S_2$.
- (30) Let X be a non empty finite set, S_1 , S_2 be signatures over X, A_1 be a circuit over X and S_1 , and A_2 be a circuit over X and S_2 . Then $A_1 \approx A_2$.
- (31) Let X be a non empty finite set, S_1 , S_2 be signatures over X, A_1 be a circuit over X and S_1 , and A_2 be a circuit over X and S_2 . Then $A_1 + A_2$ is a circuit of $S_1 + S_2$.
- (32) Let X be a non empty finite set, S_1 , S_2 be signatures over X, A_1 be a circuit over X and S_1 , and A_2 be a circuit over X and S_2 . Then $A_1 + A_2$

has denotation held in gates.

(33) Let X be a non empty finite set, S_1 , S_2 be signatures over X, A_1 be a circuit over X and S_1 , and A_2 be a circuit over X and S_2 . Then the sorts of $A_1 + A_2$ are constant and the value of the sorts of $A_1 + A_2 = X$.

Let S_1 , S_2 be finite non empty many sorted signatures. Note that $S_1 + \cdot S_2$ is finite.

Let X be a non empty finite set and let S_1 , S_2 be signatures over X. One can verify that $S_1 + S_2$ has denotation held in gates.

Let X be a non empty finite set and let S_1 , S_2 be signatures over X. Then $S_1 + \cdot S_2$ is a strict signature over X.

Let X be a non empty finite set, let S_1 , S_2 be signatures over X, let A_1 be a circuit over X and S_1 , and let A_2 be a circuit over X and S_2 . Then $A_1 + A_2$ is a strict circuit over X and $S_1 + S_2$.

One can prove the following two propositions:

- (34) For all sets x, y holds $\operatorname{rk}(x) \in \operatorname{rk}(\langle x, y \rangle)$ and $\operatorname{rk}(y) \in \operatorname{rk}(\langle x, y \rangle)$.
- (35) Let S be a finite non void non empty unsplit many sorted signature with arity held in gates with denotation held in gates and A be a non-empty circuit of S such that A has denotation held in gates. Then A has a stabilization limit.

Let X be a non empty finite set and let S be a finite signature over X. One can verify that every circuit over X and S has a stabilization limit.

Now we present three schemes. The scheme 1AryDef deals with a non empty set \mathcal{A} and a unary functor \mathcal{F} yielding an element of \mathcal{A} , and states that:

- (i) There exists a function f from \mathcal{A}^1 into \mathcal{A} such that for every element x of \mathcal{A} holds $f(\langle x \rangle) = \mathcal{F}(x)$, and
- (ii) for all functions f_1 , f_2 from \mathcal{A}^1 into \mathcal{A} such that for every element x of \mathcal{A} holds $f_1(\langle x \rangle) = \mathcal{F}(x)$ and for every element x of \mathcal{A} holds $f_2(\langle x \rangle) = \mathcal{F}(x)$ holds $f_1 = f_2$

for all values of the parameters.

The scheme 2AryDef deals with a non empty set \mathcal{A} and a binary functor \mathcal{F} yielding an element of \mathcal{A} , and states that:

- (i) There exists a function f from \mathcal{A}^2 into \mathcal{A} such that for all elements x, y of \mathcal{A} holds $f(\langle x, y \rangle) = \mathcal{F}(x, y)$, and
- (ii) for all functions f_1 , f_2 from \mathcal{A}^2 into \mathcal{A} such that for all elements x, y of \mathcal{A} holds $f_1(\langle x, y \rangle) = \mathcal{F}(x, y)$ and for all elements x, y of \mathcal{A} holds $f_2(\langle x, y \rangle) = \mathcal{F}(x, y)$ holds $f_1 = f_2$

for all values of the parameters.

The scheme 3AryDef deals with a non empty set \mathcal{A} and a ternary functor \mathcal{F} yielding an element of \mathcal{A} , and states that:

(i) There exists a function f from \mathcal{A}^3 into \mathcal{A} such that for all elements x, y, z of \mathcal{A} holds $f(\langle x, y, z \rangle) = \mathcal{F}(x, y, z)$, and

(ii) for all functions f_1 , f_2 from \mathcal{A}^3 into \mathcal{A} such that for all elements x, y, z of \mathcal{A} holds $f_1(\langle x, y, z \rangle) = \mathcal{F}(x, y, z)$ and for all elements x, y, z of \mathcal{A} holds $f_2(\langle x, y, z \rangle) = \mathcal{F}(x, y, z)$ holds $f_1 = f_2$ for all values of the parameters.

We now state three propositions:

- (36) For every function f and for every set x such that $x \in \text{dom } f$ holds $f \cdot \langle x \rangle = \langle f(x) \rangle$.
- (37) Let f be a function and x_1, x_2, x_3, x_4 be sets. If $x_1 \in \text{dom } f$ and $x_2 \in \text{dom } f$ and $x_3 \in \text{dom } f$ and $x_4 \in \text{dom } f$, then $f \cdot \langle x_1, x_2, x_3, x_4 \rangle = \langle f(x_1), f(x_2), f(x_3), f(x_4) \rangle$.
- (38) Let f be a function and x_1, x_2, x_3, x_4, x_5 be sets. Suppose $x_1 \in \text{dom } f$ and $x_2 \in \text{dom } f$ and $x_3 \in \text{dom } f$ and $x_4 \in \text{dom } f$ and $x_5 \in \text{dom } f$. Then $f \cdot \langle x_1, x_2, x_3, x_4, x_5 \rangle = \langle f(x_1), f(x_2), f(x_3), f(x_4), f(x_5) \rangle$.

Now we present several schemes. The scheme OneGate1Result deals with a set \mathcal{A} , a non empty finite set \mathcal{B} , a unary functor \mathcal{F} yielding an element of \mathcal{B} , and a function \mathcal{C} from \mathcal{B}^1 into \mathcal{B} , and states that:

For every state s of 1GateCircuit($\langle \mathcal{A} \rangle, \mathcal{C}$) and for every element a_1 of \mathcal{B} such that $a_1 = s(\mathcal{A})$ holds (Result(s))(Output 1GateCircStr($\langle \mathcal{A} \rangle, \mathcal{C}$)) = $\mathcal{F}(a_1)$

provided the following requirement is met:

• For every function g from \mathcal{B}^1 into \mathcal{B} holds $g = \mathcal{C}$ iff for every element a_1 of \mathcal{B} holds $g(\langle a_1 \rangle) = \mathcal{F}(a_1)$.

The scheme OneGate2Result deals with sets \mathcal{A} , \mathcal{B} , a non empty finite set \mathcal{C} , a binary functor \mathcal{F} yielding an element of \mathcal{C} , and a function \mathcal{D} from \mathcal{C}^2 into \mathcal{C} , and states that:

For every state s of 1GateCircuit($\langle \mathcal{A}, \mathcal{B} \rangle, \mathcal{D}$) and for all elements a_1 , a_2 of \mathcal{C} such that $a_1 = s(\mathcal{A})$ and $a_2 = s(\mathcal{B})$ holds (Result(s))(Output 1GateCircStr($\langle \mathcal{A}, \mathcal{B} \rangle, \mathcal{D}$)) = $\mathcal{F}(a_1, a_2)$

provided the parameters satisfy the following condition:

• For every function g from C^2 into C holds g = D iff for all elements a_1, a_2 of C holds $g(\langle a_1, a_2 \rangle) = \mathcal{F}(a_1, a_2)$.

The scheme OneGate3Result deals with sets \mathcal{A} , \mathcal{B} , \mathcal{C} , a non empty finite set \mathcal{D} , a ternary functor \mathcal{F} yielding an element of \mathcal{D} , and a function \mathcal{E} from \mathcal{D}^3 into \mathcal{D} , and states that:

Let s be a state of 1GateCircuit($\langle \mathcal{A}, \mathcal{B}, \mathcal{C} \rangle, \mathcal{E}$) and a_1, a_2, a_3 be elements of \mathcal{D} . If $a_1 = s(\mathcal{A})$ and $a_2 = s(\mathcal{B})$ and $a_3 = s(\mathcal{C})$, then $(\text{Result}(s))(\text{Output 1}\text{GateCircStr}(\langle \mathcal{A}, \mathcal{B}, \mathcal{C} \rangle, \mathcal{E})) = \mathcal{F}(a_1, a_2, a_3)$ provided the following requirement is met:

• For every function g from \mathcal{D}^3 into \mathcal{D} holds $g = \mathcal{E}$ iff for all elements a_1, a_2, a_3 of \mathcal{D} holds $g(\langle a_1, a_2, a_3 \rangle) = \mathcal{F}(a_1, a_2, a_3)$.

The scheme OneGate4Result deals with sets $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$, a non empty finite

set \mathcal{E} , a 4-ary functor \mathcal{F} yielding an element of \mathcal{E} , and a function \mathcal{F} from \mathcal{E}^4 into \mathcal{E} , and states that:

Let s be a state of 1GateCircuit($\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D} \rangle, \mathcal{F}$) and a_1, a_2, a_3, a_4 be elements of \mathcal{E} . If $a_1 = s(\mathcal{A})$ and $a_2 = s(\mathcal{B})$ and $a_3 = s(\mathcal{C})$ and $a_4 = s(\mathcal{D})$, then (Result(s))(Output 1GateCircStr($\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D} \rangle, \mathcal{F}$)) = $\mathcal{F}(a_1, a_2, a_3, a_4)$

provided the following condition is met:

• Let g be a function from \mathcal{E}^4 into \mathcal{E} . Then $g = \mathcal{F}$ if and only if for all elements a_1 , a_2 , a_3 , a_4 of \mathcal{E} holds $g(\langle a_1, a_2, a_3, a_4 \rangle) = \mathcal{F}(a_1, a_2, a_3, a_4)$.

The scheme OneGate5Result deals with sets \mathcal{A} , \mathcal{B} , \mathcal{C} , \mathcal{D} , \mathcal{E} , a non empty finite set \mathcal{F} , a 5-ary functor \mathcal{F} yielding an element of \mathcal{F} , and a function \mathcal{G} from \mathcal{F}^5 into \mathcal{F} , and states that:

Let s be a state of 1GateCircuit($\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E} \rangle, \mathcal{G}$) and a_1 , a_2 , a_3 , a_4 , a_5 be elements of \mathcal{F} . Suppose $a_1 = s(\mathcal{A})$ and $a_2 = s(\mathcal{B})$ and $a_3 = s(\mathcal{C})$ and $a_4 = s(\mathcal{D})$ and $a_5 = s(\mathcal{E})$. Then (Result(s))(Output 1GateCircStr($\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E} \rangle, \mathcal{G}$)) = $\mathcal{F}(a_1, a_2, a_3, a_4, a_5)$

provided the following requirement is met:

• Let g be a function from \mathcal{F}^5 into \mathcal{F} . Then $g = \mathcal{G}$ if and only if for all elements a_1 , a_2 , a_3 , a_4 , a_5 of \mathcal{F} holds $g(\langle a_1, a_2, a_3, a_4, a_5 \rangle) = \mathcal{F}(a_1, a_2, a_3, a_4, a_5)$.

4. Input of a Compound Circuit

We now state a number of propositions:

- (39) Let n be a natural number, X be a non empty finite set, f be a function from X^n into X, p be a finite sequence with length n, and S be a signature over X. If $\operatorname{rng} p \subseteq \operatorname{the} \operatorname{carrier}$ of S and $\operatorname{Output} 1\operatorname{GateCircStr}(p,f) \notin \operatorname{InputVertices}(S)$, then $\operatorname{InputVertices}(S+1\operatorname{GateCircStr}(p,f)) = \operatorname{InputVertices}(S)$.
- (40) Let X_1 , X_2 be sets, X be a non empty finite set, n be a natural number, f be a function from X^n into X, p be a finite sequence with length n, and S be a signature over X. Suppose $\operatorname{rng} p = X_1 \cup X_2$ and $X_1 \subseteq \operatorname{the carrier of } S$ and X_2 misses $\operatorname{InnerVertices}(S)$ and $\operatorname{Output} \operatorname{1GateCircStr}(p,f) \notin \operatorname{InputVertices}(S)$. Then $\operatorname{InputVertices}(S+\operatorname{1GateCircStr}(p,f)) = \operatorname{InputVertices}(S) \cup X_2$.
- (41) Let x_1 be a set, X be a non empty finite set, f be a function from X^1 into X, and S be a signature over X. If $x_1 \in$ the carrier of S and Output 1GateCircStr $(\langle x_1 \rangle, f) \notin$ InputVertices(S), then InputVertices $(S+\cdot 1$ GateCircStr $(\langle x_1 \rangle, f)) =$ InputVertices(S).

- (42) Let x_1 , x_2 be sets, X be a non empty finite set, f be a function from X^2 into X, and S be a signature over X. Suppose $x_1 \in$ the carrier of S and $x_2 \notin \text{InnerVertices}(S)$ and $\text{Output 1GateCircStr}(\langle x_1, x_2 \rangle, f) \notin \text{InputVertices}(S)$. Then $\text{InputVertices}(S+\cdot 1 \text{GateCircStr}(\langle x_1, x_2 \rangle, f)) = \text{InputVertices}(S) \cup \{x_2\}$.
- (43) Let x_1, x_2 be sets, X be a non empty finite set, f be a function from X^2 into X, and S be a signature over X. Suppose $x_2 \in$ the carrier of S and $x_1 \notin \text{InnerVertices}(S)$ and $\text{Output 1GateCircStr}(\langle x_1, x_2 \rangle, f) \notin \text{InputVertices}(S)$. Then $\text{InputVertices}(S+\cdot 1\text{GateCircStr}(\langle x_1, x_2 \rangle, f)) = \text{InputVertices}(S) \cup \{x_1\}$.
- (44) Let x_1 , x_2 be sets, X be a non empty finite set, f be a function from X^2 into X, and S be a signature over X. Suppose $x_1 \in$ the carrier of S and $x_2 \in$ the carrier of S and Output 1GateCircStr($\langle x_1, x_2 \rangle, f$) \notin InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2 \rangle, f$)) = InputVertices(S).
- (45) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_1 \in$ the carrier of S and $x_2 \notin \text{InnerVertices}(S)$ and $x_3 \notin \text{InnerVertices}(S)$ and Output $1\text{GateCircStr}(\langle x_1, x_2, x_3 \rangle, f) \notin \text{InputVertices}(S)$. Then InputVertices $(S + 1\text{GateCircStr}(\langle x_1, x_2, x_3 \rangle, f)) = \text{InputVertices}(S) \cup \{x_2, x_3\}.$
- (46) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_2 \in$ the carrier of S and $x_1 \notin \text{InnerVertices}(S)$ and $x_3 \notin \text{InnerVertices}(S)$ and Output $1\text{GateCircStr}(\langle x_1, x_2, x_3 \rangle, f) \notin \text{InputVertices}(S)$. Then InputVertices $(S + 1\text{GateCircStr}(\langle x_1, x_2, x_3 \rangle, f)) = \text{InputVertices}(S) \cup \{x_1, x_3\}.$
- (47) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_3 \in$ the carrier of S and $x_1 \notin \text{InnerVertices}(S)$ and $x_2 \notin \text{InnerVertices}(S)$ and Output $1\text{GateCircStr}(\langle x_1, x_2, x_3 \rangle, f) \notin \text{InputVertices}(S)$. Then InputVertices $(S + 1\text{GateCircStr}(\langle x_1, x_2, x_3 \rangle, f)) = \text{InputVertices}(S) \cup \{x_1, x_2\}.$
- (48) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_1 \in$ the carrier of S and $x_2 \in$ the carrier of S and $x_3 \notin$ InnerVertices(S) and Output 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$) \notin InputVertices(S). Then InputVertices(S+·1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$)) = InputVertices(S) $\cup \{x_3\}$.
- (49) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_1 \in$ the

carrier of S and $x_3 \in$ the carrier of S and $x_2 \notin$ InnerVertices(S) and Output 1GateCircStr $(\langle x_1, x_2, x_3 \rangle, f) \notin$ InputVertices(S). Then InputVertices $(S+\cdot 1 \text{GateCircStr}(\langle x_1, x_2, x_3 \rangle, f)) = \text{InputVertices}(S) \cup \{x_2\}.$

- (50) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_2 \in$ the carrier of S and $x_3 \in$ the carrier of S and $x_1 \notin \text{InnerVertices}(S)$ and Output $1\text{GateCircStr}(\langle x_1, x_2, x_3 \rangle, f) \notin \text{InputVertices}(S)$. Then InputVertices $(S + 1\text{GateCircStr}(\langle x_1, x_2, x_3 \rangle, f)) = \text{InputVertices}(S) \cup \{x_1\}.$
- (51) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_1 \in$ the carrier of S and $x_2 \in$ the carrier of S and $x_3 \in$ the carrier of S and Output 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$) \notin InputVertices(S). Then InputVertices(S+·1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$)) = InputVertices(S).

5. Result of a Compound Circuit

Next we state the proposition

(52) Let X be a non empty finite set, S be a finite signature over X, A be a circuit over X and S, n be a natural number, f be a function from X^n into X, and p be a finite sequence with length n. Suppose Output 1GateCircStr $(p, f) \notin \text{InputVertices}(S)$. Let s be a state of $A+\cdot 1\text{GateCircuit}(p, f)$ and s' be a state of A. Suppose s'=s the carrier of S. Then the stabilization time of $s \leqslant 1$ + the stabilization time of s'.

Now we present several schemes. The scheme Comb1CircResult deals with a set \mathcal{A} , a non empty finite set \mathcal{B} , a unary functor \mathcal{F} yielding an element of \mathcal{B} , a finite signature \mathcal{C} over \mathcal{B} , a circuit \mathcal{D} over \mathcal{B} and \mathcal{C} , and a function \mathcal{E} from \mathcal{B}^1 into \mathcal{B} , and states that:

Let s be a state of $\mathcal{D}+\operatorname{1GateCircuit}(\langle \mathcal{A} \rangle, \mathcal{E})$ and s' be a state of \mathcal{D} . Suppose s'=s the carrier of \mathcal{C} . Let a_1 be an element of \mathcal{B} . Suppose if $\mathcal{A} \in \operatorname{InnerVertices}(\mathcal{C})$, then $a_1=(\operatorname{Result}(s'))(\mathcal{A})$ and if $\mathcal{A} \notin \operatorname{InnerVertices}(\mathcal{C})$, then $a_1=s(\mathcal{A})$. Then $(\operatorname{Result}(s))(\operatorname{Output} \operatorname{1GateCircStr}(\langle \mathcal{A} \rangle, \mathcal{E}))=\mathcal{F}(a_1)$

provided the parameters meet the following conditions:

- For every function g from \mathcal{B}^1 into \mathcal{B} holds $g = \mathcal{E}$ iff for every element a_1 of \mathcal{B} holds $g(\langle a_1 \rangle) = \mathcal{F}(a_1)$, and
- Output 1GateCircStr($\langle A \rangle, \mathcal{E}$) \notin InputVertices(\mathcal{C}).

The scheme Comb2CircResult deals with sets \mathcal{A} , \mathcal{B} , a non empty finite set \mathcal{C} , a binary functor \mathcal{F} yielding an element of \mathcal{C} , a finite signature \mathcal{D} over \mathcal{C} , a circuit \mathcal{E} over \mathcal{C} and \mathcal{D} , and a function \mathcal{F} from \mathcal{C}^2 into \mathcal{C} , and states that:

Let s be a state of $\mathcal{E}+1$ GateCircuit($\langle \mathcal{A}, \mathcal{B} \rangle, \mathcal{F}$) and s' be a state of \mathcal{E} . Suppose s' = s the carrier of \mathcal{D} . Let a_1 , a_2 be elements of \mathcal{C} . Suppose if $\mathcal{A} \in \text{InnerVertices}(\mathcal{D})$, then $a_1 = (\text{Result}(s'))(\mathcal{A})$ and if $\mathcal{A} \notin \text{InnerVertices}(\mathcal{D})$, then $a_1 = s(\mathcal{A})$ and if $\mathcal{B} \in \text{InnerVertices}(\mathcal{D})$, then $a_2 =$ $(\text{Result}(s'))(\mathcal{B})$ and if $\mathcal{B} \notin \text{InnerVertices}(\mathcal{D})$, then $a_2 = s(\mathcal{B})$. Then (Result(s))(Output 1GateCircStr($\langle \mathcal{A}, \mathcal{B} \rangle, \mathcal{F}$)) = $\mathcal{F}(a_1, a_2)$

provided the parameters meet the following requirements:

- For every function g from C^2 into C holds $g = \mathcal{F}$ iff for all elements $a_1, a_2 \text{ of } \mathcal{C} \text{ holds } g(\langle a_1, a_2 \rangle) = \mathcal{F}(a_1, a_2), \text{ and }$
- Output 1GateCircStr($\langle \mathcal{A}, \mathcal{B} \rangle, \mathcal{F}$) \notin InputVertices(\mathcal{D}).

The scheme Comb3CircResult deals with sets $\mathcal{A}, \mathcal{B}, \mathcal{C}$, a non empty finite set \mathcal{D} , a ternary functor \mathcal{F} yielding an element of \mathcal{D} , a finite signature \mathcal{E} over \mathcal{D} , a circuit \mathcal{F} over \mathcal{D} and \mathcal{E} , and a function \mathcal{G} from \mathcal{D}^3 into \mathcal{D} , and states that:

Let s be a state of $\mathcal{F}+\cdot 1$ GateCircuit($\langle \mathcal{A}, \mathcal{B}, \mathcal{C} \rangle, \mathcal{G}$) and s' be a state of \mathcal{F} . Suppose s'=s the carrier of \mathcal{E} . Let a_1, a_2, a_3 be elements of \mathcal{D} . Suppose that

- if $A \in \text{InnerVertices}(\mathcal{E})$, then $a_1 = (\text{Result}(s'))(A)$, (i)
- (ii) if $\mathcal{A} \notin \text{InnerVertices}(\mathcal{E})$, then $a_1 = s(\mathcal{A})$,
- if $\mathcal{B} \in \text{InnerVertices}(\mathcal{E})$, then $a_2 = (\text{Result}(s'))(\mathcal{B})$, (iii)
- (iv)if $\mathcal{B} \notin \text{InnerVertices}(\mathcal{E})$, then $a_2 = s(\mathcal{B})$,
- if $C \in \text{InnerVertices}(\mathcal{E})$, then $a_3 = (\text{Result}(s'))(C)$, and (\mathbf{v})
- if $\mathcal{C} \notin \text{InnerVertices}(\mathcal{E})$, then $a_3 = s(\mathcal{C})$.

Then (Result(s))(Output 1GateCircStr($\langle \mathcal{A}, \mathcal{B}, \mathcal{C} \rangle, \mathcal{G}$)) = $\mathcal{F}(a_1, a_2, a_3)$ provided the parameters meet the following requirements:

- For every function g from \mathcal{D}^3 into \mathcal{D} holds $g = \mathcal{G}$ iff for all elements $a_1, a_2, a_3 \text{ of } \mathcal{D} \text{ holds } g(\langle a_1, a_2, a_3 \rangle) = \mathcal{F}(a_1, a_2, a_3), \text{ and }$
- Output 1GateCircStr($\langle \mathcal{A}, \mathcal{B}, \mathcal{C} \rangle, \mathcal{G}$) \notin InputVertices(\mathcal{E}).

The scheme $Comb \not\leftarrow CircResult$ deals with sets $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$, a non empty finite set \mathcal{E} , a 4-ary functor \mathcal{F} yielding an element of \mathcal{E} , a finite signature \mathcal{F} over \mathcal{E} , a circuit \mathcal{G} over \mathcal{E} and \mathcal{F} , and a function \mathcal{H} from \mathcal{E}^4 into \mathcal{E} , and states that:

Let s be a state of $\mathcal{G}+\cdot 1$ GateCircuit($\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D} \rangle, \mathcal{H}$) and s' be a state of \mathcal{G} . Suppose s' = s the carrier of \mathcal{F} . Let a_1, a_2, a_3, a_4 a_4 be elements of \mathcal{E} . Suppose that if $\mathcal{A} \in \text{InnerVertices}(\mathcal{F})$, then $a_1 = (\text{Result}(s'))(\mathcal{A})$ and if $\mathcal{A} \notin \text{InnerVertices}(\mathcal{F})$, then $a_1 = s(\mathcal{A})$ and if $\mathcal{B} \in \text{InnerVertices}(\mathcal{F})$, then $a_2 =$ $(\text{Result}(s'))(\mathcal{B})$ and if $\mathcal{B} \notin \text{InnerVertices}(\mathcal{F})$, then $a_2 = s(\mathcal{B})$ and if $\mathcal{C} \in \text{InnerVertices}(\mathcal{F})$, then $a_3 = (\text{Result}(s'))(\mathcal{C})$ and if $\mathcal{C} \notin$ InnerVertices(\mathcal{F}), then $a_3 = s(\mathcal{C})$ and if $\mathcal{D} \in \text{InnerVertices}(\mathcal{F})$, then $a_4 = (\text{Result}(s'))(\mathcal{D})$ and if $\mathcal{D} \notin \text{InnerVertices}(\mathcal{F})$, then $a_4 =$ $s(\mathcal{D})$. Then (Result(s))(Output 1GateCircStr($\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D} \rangle, \mathcal{H}$)) = $\mathcal{F}(a_1, a_2, a_3, a_4)$

provided the parameters satisfy the following conditions:

- Let g be a function from \mathcal{E}^4 into \mathcal{E} . Then $g = \mathcal{H}$ if and only if for all elements a_1 , a_2 , a_3 , a_4 of \mathcal{E} holds $g(\langle a_1, a_2, a_3, a_4 \rangle) = \mathcal{F}(a_1, a_2, a_3, a_4)$, and
- Output 1GateCircStr($\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D} \rangle, \mathcal{H}$) \notin InputVertices(\mathcal{F}).

The scheme Comb5CircResult deals with sets \mathcal{A} , \mathcal{B} , \mathcal{C} , \mathcal{D} , \mathcal{E} , a non empty finite set \mathcal{F} , a 5-ary functor \mathcal{F} yielding an element of \mathcal{F} , a finite signature \mathcal{G} over \mathcal{F} , a circuit \mathcal{H} over \mathcal{F} and \mathcal{G} , and a function \mathcal{I} from \mathcal{F}^5 into \mathcal{F} , and states that:

Let s be a state of $\mathcal{H}+\cdot 1$ GateCircuit($\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E} \rangle, \mathcal{I}$) and s' be a state of \mathcal{H} . Suppose $s'=s \upharpoonright$ the carrier of \mathcal{G} . Let a_1, a_2, a_3, a_4, a_5 be elements of \mathcal{F} . Suppose that if $\mathcal{A} \in InnerVertices(\mathcal{G})$, then $a_1=(Result(s'))(\mathcal{A})$ and if $\mathcal{A} \notin InnerVertices(\mathcal{G})$, then $a_1=s(\mathcal{A})$ and if $\mathcal{B} \in InnerVertices(\mathcal{G})$, then $a_2=(Result(s'))(\mathcal{B})$ and if $\mathcal{B} \notin InnerVertices(\mathcal{G})$, then $a_2=s(\mathcal{B})$ and if $\mathcal{C} \in InnerVertices(\mathcal{G})$, then $a_3=(Result(s'))(\mathcal{C})$ and if $\mathcal{C} \notin InnerVertices(\mathcal{G})$, then $a_3=s(\mathcal{C})$ and if $\mathcal{D} \notin InnerVertices(\mathcal{G})$, then $a_4=(Result(s'))(\mathcal{D})$ and if $\mathcal{D} \notin InnerVertices(\mathcal{G})$, then $a_4=s(\mathcal{D})$ and if $\mathcal{E} \in InnerVertices(\mathcal{G})$, then $a_5=(Result(s'))(\mathcal{E})$ and if $\mathcal{E} \notin InnerVertices(\mathcal{G})$, then $a_5=s(\mathcal{E})$. Then $(Result(s))(Output 1GateCircStr(<math>\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E} \rangle, \mathcal{I}))=\mathcal{F}(a_1,a_2,a_3,a_4,a_5)$

provided the parameters meet the following conditions:

- Let g be a function from \mathcal{F}^5 into \mathcal{F} . Then $g = \mathcal{I}$ if and only if for all elements a_1 , a_2 , a_3 , a_4 , a_5 of \mathcal{F} holds $g(\langle a_1, a_2, a_3, a_4, a_5 \rangle) = \mathcal{F}(a_1, a_2, a_3, a_4, a_5)$, and
- Output 1GateCircStr($\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E} \rangle, \mathcal{I}$) \notin InputVertices(\mathcal{G}).

6. Inputs Without Pairs

Let S be a non empty many sorted signature. We say that S has nonpair inputs if and only if:

(Def. 11) InputVertices(S) has no pairs.

Note that \mathbb{N} has no pairs. Let X be a set with no pairs. Note that every subset of X has no pairs.

Let us observe that every function which is natural-yielding is also nonpair yielding.

Let us note that every finite sequence of elements of \mathbb{N} is natural-yielding.

Let us observe that there exists a finite sequence which is one-to-one and natural-yielding.

Let n be a natural number. Observe that there exists a finite sequence with length n which is one-to-one and natural-yielding.

Let p be a nonpair yielding finite sequence and let f be a set. Observe that 1GateCircStr(p, f) has nonpair inputs.

One can verify that there exists an one-gate many sorted signature which has nonpair inputs. Let X be a non empty finite set. One can verify that there exists an one-gate signature over X which has nonpair inputs.

Let S be a non empty many sorted signature with nonpair inputs. One can check that InputVertices(S) has no pairs.

The following proposition is true

(53) Let S be a non empty many sorted signature with nonpair inputs and x be a vertex of S. If x is pair, then $x \in \text{InnerVertices}(S)$.

Let S be an unsplit non empty many sorted signature with arity held in gates. One can verify that InnerVertices(S) is relation-like.

Let S be an unsplit non empty non void many sorted signature with denotation held in gates. Note that InnerVertices(S) is relation-like.

Let S_1 , S_2 be unsplit non empty many sorted signatures with arity held in gates with nonpair inputs. One can verify that $S_1 + \cdot S_2$ has nonpair inputs.

One can prove the following propositions:

- (54) For every non pair set x and for every binary relation R holds $x \notin R$.
- (55) Let x_1 be a set, X be a non empty finite set, f be a function from X^1 into X, and S be a signature over X with nonpair inputs. If $x_1 \in$ the carrier of S or x_1 is non pair, then $S+\cdot 1$ GateCircStr($\langle x_1 \rangle, f$) has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x_1 be a vertex of S, and let f be a function from X^1 into X. One can verify that $S+\cdot 1$ GateCircStr($\langle x_1 \rangle, f$) has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x_1 be a non pair set, and let f be a function from X^1 into X. One can verify that $S+\cdot 1$ GateCircStr($\langle x_1 \rangle, f$) has nonpair inputs.

We now state the proposition

(56) Let x_1, x_2 be sets, X be a non empty finite set, f be a function from X^2 into X, and S be a signature over X with nonpair inputs. Suppose $x_1 \in$ the carrier of S or x_1 is non pair but $x_2 \in$ the carrier of S or x_2 is non pair. Then $S+\cdot 1$ GateCircStr($\langle x_1, x_2 \rangle, f$) has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x_1 be a vertex of S, let n_2 be a non pair set, and let f be a function from X^2 into X. Observe that $S+\cdot 1$ GateCircStr($\langle x_1, n_2 \rangle, f$) has nonpair inputs and $S+\cdot 1$ GateCircStr($\langle n_2, x_1 \rangle, f$) has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x_1 , x_2 be vertices of S, and let f be a function from X^2 into X. One can verify that $S+\cdot 1$ GateCircStr($\langle x_1, x_2 \rangle, f$) has nonpair inputs.

One can prove the following proposition

- (57) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X with nonpair inputs. Suppose that
 - (i) $x_1 \in \text{the carrier of } S \text{ or } x_1 \text{ is non pair,}$
 - (ii) $x_2 \in \text{the carrier of } S \text{ or } x_2 \text{ is non pair, and}$
 - (iii) $x_3 \in \text{the carrier of } S \text{ or } x_3 \text{ is non pair.}$

Then $S+\cdot 1$ GateCircStr($\langle x_1, x_2, x_3 \rangle, f$) has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x_1, x_2 be vertices of S, let n be a non pair set, and let f be a function from X^3 into X. One can verify the following observations:

- * $S+\cdot 1$ GateCircStr($\langle x_1, x_2, n \rangle, f$) has nonpair inputs,
- * $S+\cdot 1$ GateCircStr($\langle x_1, n, x_2 \rangle, f$) has nonpair inputs, and
- * $S+\cdot 1$ GateCircStr($\langle n, x_1, x_2 \rangle, f$) has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x be a vertex of S, let n_1 , n_2 be non pair sets, and let f be a function from X^3 into X. One can check the following observations:

- * $S+\cdot 1$ GateCircStr($\langle x, n_1, n_2 \rangle, f$) has nonpair inputs,
- * $S+\cdot 1$ GateCircStr($\langle n_1, x, n_2 \rangle, f$) has nonpair inputs, and
- * $S+\cdot 1$ GateCircStr($\langle n_1, n_2, x \rangle, f$) has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x_1, x_2, x_3 be vertices of S, and let f be a function from X^3 into X. Observe that $S+\cdot 1$ GateCircStr($\langle x_1, x_2, x_3 \rangle, f$) has nonpair inputs.

References

- [1] Grzegorz Bancerek. The fundamental properties of natural numbers. Formalized Mathematics, 1(1):41–46, 1990.
- [2] Grzegorz Bancerek. Sequences of ordinal numbers. Formalized Mathematics, 1(2):281–290, 1990.
- [3] Grzegorz Bancerek. Tarski's classes and ranks. Formalized Mathematics, 1(3):563–567, 1990.
- [4] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. Formalized Mathematics, 1(1):107–114, 1990.
- [5] Grzegorz Bancerek and Yatsuka Nakamura. Full adder circuit. Part I. Formalized Mathematics, 5(3):367–380, 1996.
- [6] Czesław Byliński. Finite sequences and tuples of elements of a non-empty sets. Formalized Mathematics, 1(3):529–536, 1990.
- [7] Czesław Byliński. Functions and their basic properties. Formalized Mathematics, 1(1):55–65, 1990.
- [8] Czesław Byliński. Functions from a set to a set. Formalized Mathematics, 1(1):153–164, 1990.
- 1990. [9] Czesław Byliński. Partial functions. Formalized Mathematics, 1(2):357–367, 1990.
- [10] Czesław Byliński. Some basic properties of sets. Formalized Mathematics, 1(1):47–53, 1990.
- [11] Jing-Chao Chen. A small computer model with push-down stack. Formalized Mathematics, 8(1):175–182, 1999.
- [12] Agata Darmochwał. Finite sets. Formalized Mathematics, 1(1):165–167, 1990.
- [13] Jarosław Kotowicz. Monotone real sequences. Subsequences. Formalized Mathematics, 1(3):471–475, 1990.

- [14] Jarosław Kotowicz. The limit of a real function at infinity. Formalized Mathematics, 2(1):17–28, 1991.
- [15] Yatsuka Nakamura and Grzegorz Bancerek. Combining of circuits. Formalized Mathematics, 5(2):283–295, 1996.
- [16] Yatsuka Nakamura, Piotr Rudnicki, Andrzej Trybulec, and Pauline N. Kawamoto. Introduction to circuits, II. Formalized Mathematics, 5(2):273–278, 1996.
- [17] Yatsuka Nakamura, Piotr Rudnicki, Andrzej Trybulec, and Pauline N. Kawamoto. Preliminaries to circuits, II. Formalized Mathematics, 5(2):215–220, 1996.
- [18] Yatsuka Nakamura, Piotr Rudnicki, Andrzej Trybulec, and Pauline N. Kawamoto. Preliminaries to circuits, I. Formalized Mathematics, 5(2):167–172, 1996.
- [19] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. Formalized Mathematics, 4(1):83–86, 1993.
- [20] Andrzej Trybulec. Enumerated sets. Formalized Mathematics, 1(1):25-34, 1990.
- [21] Andrzej Trybulec. Tarski Grothendieck set theory. Formalized Mathematics, 1(1):9–11,
- [22] Andrzej Trybulec. Many-sorted sets. Formalized Mathematics, 4(1):15–22, 1993.
- [23] Andrzej Trybulec. Many sorted algebras. Formalized Mathematics, 5(1):37-42, 1996.
- [24] Andrzej Trybulec. Moore-Smith convergence. Formalized Mathematics, 6(2):213–225, 1997.
- 1997. [25] Wojciech A. Trybulec. Groups. Formalized Mathematics, 1(5):821–827, 1990.
- Zinaida Trybulec. Properties of subsets. Formalized Mathematics, 1(1):67–71, 1990.
- [27] Edmund Woronowicz. Relations and their basic properties. Formalized Mathematics, 1(1):73–83, 1990.

Received July 26, 2002