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Preliminaries to Automatic Generation of Mizar Documentation for Circuits

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Summary. In this paper we introduce technical notions used by a system which automatically generates Mizar documentation for specified circuits. They provide a ready for use elements needed to justify correctness of circuits' construction. We concentrate on the concept of stabilization and analyze one-gate circuits and their combinations.

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The articles [21], [26], [20], [11], [10], [27], [7], [12], [2], [3], [8], [1], [9], [14], [4], [6], [22], [25], [23], [5], [17], [16], [15], [18], [19], [13], and [24] provide the notation and terminology for this paper.

1. Stabilizing Circuits

The following proposition is true

(1) Let S be a non void circuit-like non empty many sorted signature, A be a non-empty circuit of S, s be a state of A, and x be a set. If $x \in \mathrm{InputVertices}(S)$, then for every natural number n holds $(\mathrm{Following}(s,n))(x) = s(x)$.

Let S be a non void circuit-like non empty many sorted signature, let A be a non-empty circuit of S, and let s be a state of A. We say that s is stabilizing if and only if:

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(Def. 1) There exists a natural number n such that Following(s, n) is stable.

Let S be a non void circuit-like non empty many sorted signature and let A be a non-empty circuit of S. We say that A is stabilizing if and only if:

(Def. 2) Every state of A is stabilizing.

We say that A has a stabilization limit if and only if:

(Def. 3) There exists a natural number n such that for every state s of A holds Following(s, n) is stable.

Let S be a non void circuit-like non empty many sorted signature. Note that every non-empty circuit of S which has a stabilization limit is also stabilizing.

Let S be a non void circuit-like non empty many sorted signature, let A be a non-empty circuit of S, and let s be a state of A. Let us assume that s is stabilizing. The functor Result(s) yields a state of A and is defined as follows:

(Def. 4) Result(s) is stable and there exists a natural number n such that Result$(s) =$ Following(s, n).

Let S be a non void circuit-like non empty many sorted signature, let A be a non-empty circuit of S, and let s be a state of A. Let us assume that s is stabilizing. The stabilization time of s is a natural number and is defined by the conditions (Def. 5).

(Def. 5)(i) Following$(s,$ the stabilization time of $s)$ is stable, and

(ii) for every natural number n such that $n <$ the stabilization time of s holds Following(s, n) is not stable.

The following propositions are true:

(2) Let S be a non void circuit-like non empty many sorted signature, A be a non-empty circuit of S, and s be a state of A. If s is stabilizing, then Result$(s) =$ Following$(s,$ the stabilization time of $s)$.

(3) Let S be a non void circuit-like non empty many sorted signature, A be a non-empty circuit of S, s be a state of A, and n be a natural number. If Following(s, n) is stable, then the stabilization time of $s \leqslant n$.

(4) Let S be a non void circuit-like non empty many sorted signature, A be a non-empty circuit of S, s be a state of A, and n be a natural number. If Following(s, n) is stable, then Result$(s) =$ Following(s, n).

(5) Let S be a non void circuit-like non empty many sorted signature, A be a non-empty circuit of S, s be a state of A, and n be a natural number. Suppose s is stabilizing and $n \geqslant$ the stabilization time of s. Then Result$(s) =$ Following(s, n).

(6) Let S be a non void circuit-like non empty many sorted signature, A be a non-empty circuit of S, and s be a state of A. If s is stabilizing, then for every set x such that $x \in$ InputVertices(S) holds $(\text{Result}(s))(x) = s(x)$.

(7) Let S_1, S be non void circuit-like non empty many sorted signatures, A_1 be a non-empty circuit of S_1, A be a non-empty circuit of S, s be a state

of A, and s_1 be a state of A_1. If $s_1 = s \restriction$the carrier of S_1, then for every vertex v_1 of S_1 holds $s_1(v_1) = s(v_1)$.

(8) Let S_1, S_2 be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2) and InputVertices(S_2) misses InnerVertices(S_1). Let S be a non void circuit-like non empty many sorted signature. Suppose $S = S_1 +\cdot S_2$. Let A_1 be a non-empty circuit of S_1 and A_2 be a non-empty circuit of S_2. Suppose $A_1 \approx A_2$. Let A be a non-empty circuit of S. Suppose $A = A_1 +\cdot A_2$. Let s be a state of A, s_1 be a state of A_1, and s_2 be a state of A_2. Suppose $s_1 = s \restriction$the carrier of S_1 and $s_2 = s \restriction$the carrier of S_2 and s_1 is stabilizing and s_2 is stabilizing. Then s is stabilizing.

(9) Let S_1, S_2 be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2) and InputVertices(S_2) misses InnerVertices(S_1). Let S be a non void circuit-like non empty many sorted signature. Suppose $S = S_1 +\cdot S_2$. Let A_1 be a non-empty circuit of S_1 and A_2 be a non-empty circuit of S_2. Suppose $A_1 \approx A_2$. Let A be a non-empty circuit of S. Suppose $A = A_1 +\cdot A_2$. Let s be a state of A and s_1 be a state of A_1. Suppose $s_1 = s \restriction$the carrier of S_1 and s_1 is stabilizing. Let s_2 be a state of A_2. Suppose $s_2 = s \restriction$the carrier of S_2 and s_2 is stabilizing. Then the stabilization time of s = max(the stabilization time of s_1, the stabilization time of s_2).

(10) Let S_1, S_2 be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2). Let S be a non void circuit-like non empty many sorted signature. Suppose $S = S_1 +\cdot S_2$. Let A_1 be a non-empty circuit of S_1 and A_2 be a non-empty circuit of S_2. Suppose $A_1 \approx A_2$. Let A be a non-empty circuit of S. Suppose $A = A_1 +\cdot A_2$. Let s be a state of A and s_1 be a state of A_1. Suppose $s_1 = s \restriction$the carrier of S_1 and s_1 is stabilizing. Let s_2 be a state of A_2. Suppose s_2 = Following(s, the stabilization time of s_1)$\restriction$the carrier of S_2 and s_2 is stabilizing. Then s is stabilizing.

(11) Let S_1, S_2 be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2). Let S be a non void circuit-like non empty many sorted signature. Suppose $S = S_1 +\cdot S_2$. Let A_1 be a non-empty circuit of S_1 and A_2 be a non-empty circuit of S_2. Suppose $A_1 \approx A_2$. Let A be a non-empty circuit of S. Suppose $A = A_1 +\cdot A_2$. Let s be a state of A and s_1 be a state of A_1. Suppose $s_1 = s \restriction$the carrier of S_1 and s_1 is stabilizing. Let s_2 be a state of A_2. Suppose s_2 = Following(s, the stabilization time of s_1)$\restriction$the carrier of S_2 and s_2 is stabilizing. Then the stabilization time of s = (the stabilization time of s_1) + (the stabilization time of s_2).

(12) Let S_1, S_2, S be non void circuit-like non empty many sorted signatures.

Suppose InputVertices(S_1) misses InnerVertices(S_2) and $S = S_1 {+\cdot} S_2$. Let A_1 be a non-empty circuit of S_1, A_2 be a non-empty circuit of S_2, and A be a non-empty circuit of S. Suppose $A_1 \approx A_2$ and $A = A_1 {+\cdot} A_2$. Let s be a state of A and s_1 be a state of A_1. Suppose $s_1 = s \upharpoonright$the carrier of S_1 and s_1 is stabilizing. Let s_2 be a state of A_2. Suppose $s_2 =$ Following(s, the stabilization time of s_1)$\upharpoonright$the carrier of S_2 and s_2 is stabilizing. Then Result(s)$\upharpoonright$the carrier of $S_1 =$ Result(s_1).

2. One-gate Circuits

We now state three propositions:

(13) Let x be a set, X be a non empty finite set, n be a natural number, p be a finite sequence with length n, g be a function from X^n into X, and s be a state of 1GateCircuit(p, g). Then $s \cdot p$ is an element of X^n.

(14) For all sets x_1, x_2, x_3, x_4 holds $\mathrm{rng}\langle x_1, x_2, x_3, x_4\rangle = \{x_1, x_2, x_3, x_4\}$.

(15) For all sets x_1, x_2, x_3, x_4, x_5 holds $\mathrm{rng}\langle x_1, x_2, x_3, x_4, x_5\rangle = \{x_1, x_2, x_3, x_4, x_5\}$.

Let x_1, x_2, x_3, x_4 be sets. Then $\langle x_1, x_2, x_3, x_4\rangle$ is a finite sequence with length 4. Let x_5 be a set. Then $\langle x_1, x_2, x_3, x_4, x_5\rangle$ is a finite sequence with length 5.

Let S be a many sorted signature. We say that S is one-gate if and only if the condition (Def. 6) is satisfied.

(Def. 6) There exists a non empty finite set X and there exists a natural number n and there exists a finite sequence p with length n and there exists a function f from X^n into X such that $S =$ 1GateCircStr(p, f).

Let S be a non empty many sorted signature and let A be an algebra over S. We say that A is one-gate if and only if the condition (Def. 7) is satisfied.

(Def. 7) There exists a non empty finite set X and there exists a natural number n and there exists a finite sequence p with length n and there exists a function f from X^n into X such that $S =$ 1GateCircStr(p, f) and $A =$ 1GateCircuit(p, f).

Let p be a finite sequence and let x be a set. Observe that 1GateCircStr(p, x) is finite.

Let us note that every many sorted signature which is one-gate is also strict, non void, non empty, unsplit, and finite and has arity held in gates.

One can check that every non empty many sorted signature which is one-gate has also denotation held in gates.

Let X be a non empty finite set, let n be a natural number, let p be a finite sequence with length n, and let f be a function from X^n into X. Note that 1GateCircStr(p, f) is one-gate.

One can check that there exists a many sorted signature which is one-gate.

Let S be an one-gate many sorted signature. Observe that every circuit of S which is one-gate is also strict and non-empty.

Let X be a non empty finite set, let n be a natural number, let p be a finite sequence with length n, and let f be a function from X^n into X. One can check that 1GateCircuit(p, f) is one-gate.

Let S be an one-gate many sorted signature. Observe that there exists a circuit of S which is one-gate and non-empty.

Let S be an one-gate many sorted signature. The functor Output S yields a vertex of S and is defined as follows:

(Def. 8) Output $S = \bigcup$(the operation symbols of S).

Let S be an one-gate many sorted signature. Observe that Output S is pair.

Next we state several propositions:

(16) Let S be an one-gate many sorted signature, p be a finite sequence, and x be a set. If $S = $ 1GateCircStr(p, x), then Output $S = \langle p, x\rangle$.

(17) For every one-gate many sorted signature S holds InnerVertices$(S) = \{$Output $S\}$.

(18) Let S be an one-gate many sorted signature, A be an one-gate circuit of S, n be a natural number, X be a finite non empty set, f be a function from X^n into X, and p be a finite sequence with length n. If $A = $ 1GateCircuit(p, f), then $S = $ 1GateCircStr(p, f).

(19) Let n be a natural number, X be a finite non empty set, f be a function from X^n into X, p be a finite sequence with length n, and s be a state of 1GateCircuit(p, f). Then (Following(s))(Output 1GateCircStr(p, f)) $= f(s \cdot p)$.

(20) Let S be an one-gate many sorted signature, A be an one-gate circuit of S, and s be a state of A. Then Following(s) is stable.

Let S be a non void circuit-like non empty many sorted signature. Observe that every non-empty circuit of S which is one-gate has also a stabilization limit.

We now state two propositions:

(21) Let S be an one-gate many sorted signature, A be an one-gate circuit of S, and s be a state of A. Then Result$(s) = $ Following(s).

(22) Let S be an one-gate many sorted signature, A be an one-gate circuit of S, and s be a state of A. Then the stabilization time of $s \leqslant 1$.

In this article we present several logical schemes. The scheme *OneGate1Ex* deals with a set $\mathcal{A}$, a non empty finite set $\mathcal{B}$, and a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, and states that:

> There exists an one-gate many sorted signature S and there exists an one-gate circuit A of S such that InputVertices$(S) = \{\mathcal{A}\}$ and for every state s of A holds (Result(s))(Output S) $= \mathcal{F}(s(\mathcal{A}))$

for all values of the parameters.

The scheme *OneGate2Ex* deals with sets $\mathcal{A}$, $\mathcal{B}$, a non empty finite set $\mathcal{C}$, and a binary functor $\mathcal{F}$ yielding an element of $\mathcal{C}$, and states that:

> There exists an one-gate many sorted signature S and there exists an one-gate circuit A of S such that $\mathrm{InputVertices}(S) = \{\mathcal{A}, \mathcal{B}\}$ and for every state s of A holds $(\mathrm{Result}(s))(\mathrm{Output}\, S) = \mathcal{F}(s(\mathcal{A}), s(\mathcal{B}))$

for all values of the parameters.

The scheme *OneGate3Ex* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, a non empty finite set $\mathcal{D}$, and a ternary functor $\mathcal{F}$ yielding an element of $\mathcal{D}$, and states that:

> There exists an one-gate many sorted signature S and there exists an one-gate circuit A of S such that $\mathrm{InputVertices}(S) = \{\mathcal{A}, \mathcal{B}, \mathcal{C}\}$ and for every state s of A holds $(\mathrm{Result}(s))(\mathrm{Output}\, S)$ $= \mathcal{F}(s(\mathcal{A}), s(\mathcal{B}), s(\mathcal{C}))$

for all values of the parameters.

The scheme *OneGate4Ex* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, a non empty finite set $\mathcal{E}$, and a 4-ary functor $\mathcal{F}$ yielding an element of $\mathcal{E}$, and states that:

> There exists an one-gate many sorted signature S and there exists an one-gate circuit A of S such that $\mathrm{InputVertices}(S) = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}\}$ and for every state s of A holds $(\mathrm{Result}(s))(\mathrm{Output}\, S)$ $= \mathcal{F}(s(\mathcal{A}), s(\mathcal{B}), s(\mathcal{C}), s(\mathcal{D}))$

for all values of the parameters.

The scheme *OneGate5Ex* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, a non empty finite set $\mathcal{F}$, and a 5-ary functor $\mathcal{F}$ yielding an element of $\mathcal{F}$, and states that:

> There exists an one-gate many sorted signature S and there exists an one-gate circuit A of S such that $\mathrm{InputVertices}(S) = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}\}$ and for every state s of A holds $(\mathrm{Result}(s))(\mathrm{Output}\, S)$ $= \mathcal{F}(s(\mathcal{A}), s(\mathcal{B}), s(\mathcal{C}), s(\mathcal{D}), s(\mathcal{E}))$

for all values of the parameters.

3. Mono-sorted Circuits

One can prove the following propositions:

(23) For every constant function f holds $f = \mathrm{dom}\, f \longmapsto$ the value of f.

(24) For all non empty sets X, Y and for all natural numbers n, m such that $n \neq 0$ and $X^n = Y^m$ holds $X = Y$ and $n = m$.

(25) For all non empty many sorted signatures S_1, S_2 holds every vertex of S_1 is a vertex of $S_1 {+}{\cdot} S_2$.

(26) For all non empty many sorted signatures S_1, S_2 holds every vertex of S_2 is a vertex of $S_1 {+}{\cdot} S_2$.

Let X be a non empty finite set. A non void non empty unsplit many sorted signature with arity held in gates with denotation held in gates is said to be a signature over X if it satisfies the condition (Def. 9).

(Def. 9) There exists a circuit A of it such that the sorts of A are constant and the value of the sorts of $A = X$ and A has denotation held in gates.

Next we state the proposition

(27) Let n be a natural number, X be a non empty finite set, f be a function from X^n into X, and p be a finite sequence with length n. Then 1GateCircStr(p, f) is a signature over X.

Let X be a non empty finite set. Observe that there exists a signature over X which is strict and one-gate.

Let n be a natural number, let X be a non empty finite set, let f be a function from X^n into X, and let p be a finite sequence with length n. Then 1GateCircStr(p, f) is a strict signature over X.

Let X be a non empty finite set and let S be a signature over X. A circuit of S is called a circuit over X and S if:

(Def. 10) It has denotation held in gates and the sorts of it are constant and the value of the sorts of it $= X$.

Let X be a non empty finite set and let S be a signature over X. One can check that every circuit over X and S is non-empty and has denotation held in gates.

Next we state the proposition

(28) Let n be a natural number, X be a non empty finite set, f be a function from X^n into X, and p be a finite sequence with length n. Then 1GateCircuit(p, f) is a circuit over X and 1GateCircStr(p, f).

Let X be a non empty finite set and let S be an one-gate signature over X. One can check that there exists a circuit over X and S which is strict and one-gate.

Let X be a non empty finite set and let S be a signature over X. One can check that there exists a circuit over X and S which is strict.

Let n be a natural number, let X be a non empty finite set, let f be a function from X^n into X, and let p be a finite sequence with length n. Then 1GateCircuit(p, f) is a strict circuit over X and 1GateCircStr(p, f).

One can prove the following propositions:

(29) For every non empty finite set X and for all signatures S_1, S_2 over X holds $S_1 \approx S_2$.

(30) Let X be a non empty finite set, S_1, S_2 be signatures over X, A_1 be a circuit over X and S_1, and A_2 be a circuit over X and S_2. Then $A_1 \approx A_2$.

(31) Let X be a non empty finite set, S_1, S_2 be signatures over X, A_1 be a circuit over X and S_1, and A_2 be a circuit over X and S_2. Then $A_1 {+\cdot} A_2$ is a circuit of $S_1 {+\cdot} S_2$.

(32) Let X be a non empty finite set, S_1, S_2 be signatures over X, A_1 be a circuit over X and S_1, and A_2 be a circuit over X and S_2. Then $A_1 {+\cdot} A_2$

has denotation held in gates.

(33) Let X be a non empty finite set, S_1, S_2 be signatures over X, A_1 be a circuit over X and S_1, and A_2 be a circuit over X and S_2. Then the sorts of $A_1 +\cdot A_2$ are constant and the value of the sorts of $A_1 +\cdot A_2 = X$.

Let S_1, S_2 be finite non empty many sorted signatures. Note that $S_1 +\cdot S_2$ is finite.

Let X be a non empty finite set and let S_1, S_2 be signatures over X. One can verify that $S_1 +\cdot S_2$ has denotation held in gates.

Let X be a non empty finite set and let S_1, S_2 be signatures over X. Then $S_1 +\cdot S_2$ is a strict signature over X.

Let X be a non empty finite set, let S_1, S_2 be signatures over X, let A_1 be a circuit over X and S_1, and let A_2 be a circuit over X and S_2. Then $A_1 +\cdot A_2$ is a strict circuit over X and $S_1 +\cdot S_2$.

One can prove the following two propositions:

(34) For all sets x, y holds $\mathrm{rk}(x) \in \mathrm{rk}(\langle x, y \rangle)$ and $\mathrm{rk}(y) \in \mathrm{rk}(\langle x, y \rangle)$.

(35) Let S be a finite non void non empty unsplit many sorted signature with arity held in gates with denotation held in gates and A be a non-empty circuit of S such that A has denotation held in gates. Then A has a stabilization limit.

Let X be a non empty finite set and let S be a finite signature over X. One can verify that every circuit over X and S has a stabilization limit.

Now we present three schemes. The scheme *1AryDef* deals with a non empty set $\mathcal{A}$ and a unary functor $\mathcal{F}$ yielding an element of $\mathcal{A}$, and states that:

(i) There exists a function f from $\mathcal{A}^1$ into $\mathcal{A}$ such that for every element x of $\mathcal{A}$ holds $f(\langle x \rangle) = \mathcal{F}(x)$, and

(ii) for all functions f_1, f_2 from $\mathcal{A}^1$ into $\mathcal{A}$ such that for every element x of $\mathcal{A}$ holds $f_1(\langle x \rangle) = \mathcal{F}(x)$ and for every element x of $\mathcal{A}$ holds $f_2(\langle x \rangle) = \mathcal{F}(x)$ holds $f_1 = f_2$

for all values of the parameters.

The scheme *2AryDef* deals with a non empty set $\mathcal{A}$ and a binary functor $\mathcal{F}$ yielding an element of $\mathcal{A}$, and states that:

(i) There exists a function f from $\mathcal{A}^2$ into $\mathcal{A}$ such that for all elements x, y of $\mathcal{A}$ holds $f(\langle x, y \rangle) = \mathcal{F}(x, y)$, and

(ii) for all functions f_1, f_2 from $\mathcal{A}^2$ into $\mathcal{A}$ such that for all elements x, y of $\mathcal{A}$ holds $f_1(\langle x, y \rangle) = \mathcal{F}(x, y)$ and for all elements x, y of $\mathcal{A}$ holds $f_2(\langle x, y \rangle) = \mathcal{F}(x, y)$ holds $f_1 = f_2$

for all values of the parameters.

The scheme *3AryDef* deals with a non empty set $\mathcal{A}$ and a ternary functor $\mathcal{F}$ yielding an element of $\mathcal{A}$, and states that:

(i) There exists a function f from $\mathcal{A}^3$ into $\mathcal{A}$ such that for all elements x, y, z of $\mathcal{A}$ holds $f(\langle x, y, z \rangle) = \mathcal{F}(x, y, z)$, and

(ii) for all functions f_1, f_2 from $\mathcal{A}^3$ into $\mathcal{A}$ such that for all elements x, y, z of $\mathcal{A}$ holds $f_1(\langle x,y,z\rangle) = \mathcal{F}(x,y,z)$ and for all elements x, y, z of $\mathcal{A}$ holds $f_2(\langle x,y,z\rangle) = \mathcal{F}(x,y,z)$ holds $f_1 = f_2$

for all values of the parameters.

We now state three propositions:

(36) For every function f and for every set x such that $x \in \mathrm{dom}\, f$ holds $f \cdot \langle x\rangle = \langle f(x)\rangle$.

(37) Let f be a function and x_1, x_2, x_3, x_4 be sets. If $x_1 \in \mathrm{dom}\, f$ and $x_2 \in \mathrm{dom}\, f$ and $x_3 \in \mathrm{dom}\, f$ and $x_4 \in \mathrm{dom}\, f$, then $f \cdot \langle x_1,x_2,x_3,x_4\rangle = \langle f(x_1), f(x_2), f(x_3), f(x_4)\rangle$.

(38) Let f be a function and x_1, x_2, x_3, x_4, x_5 be sets. Suppose $x_1 \in \mathrm{dom}\, f$ and $x_2 \in \mathrm{dom}\, f$ and $x_3 \in \mathrm{dom}\, f$ and $x_4 \in \mathrm{dom}\, f$ and $x_5 \in \mathrm{dom}\, f$. Then $f \cdot \langle x_1,x_2,x_3,x_4,x_5\rangle = \langle f(x_1), f(x_2), f(x_3), f(x_4), f(x_5)\rangle$.

Now we present several schemes. The scheme *OneGate1Result* deals with a set $\mathcal{A}$, a non empty finite set $\mathcal{B}$, a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, and a function $\mathcal{C}$ from $\mathcal{B}^1$ into $\mathcal{B}$, and states that:

For every state s of 1GateCircuit$(\langle\mathcal{A}\rangle,\mathcal{C})$ and for every element a_1 of $\mathcal{B}$ such that $a_1 = s(\mathcal{A})$ holds $(\mathrm{Result}(s))(\mathrm{Output}\, \mathrm{1GateCircStr}(\langle\mathcal{A}\rangle,\mathcal{C})) = \mathcal{F}(a_1)$

provided the following requirement is met:

- For every function g from $\mathcal{B}^1$ into $\mathcal{B}$ holds $g = \mathcal{C}$ iff for every element a_1 of $\mathcal{B}$ holds $g(\langle a_1\rangle) = \mathcal{F}(a_1)$.

The scheme *OneGate2Result* deals with sets $\mathcal{A}$, $\mathcal{B}$, a non empty finite set $\mathcal{C}$, a binary functor $\mathcal{F}$ yielding an element of $\mathcal{C}$, and a function $\mathcal{D}$ from $\mathcal{C}^2$ into $\mathcal{C}$, and states that:

For every state s of 1GateCircuit$(\langle\mathcal{A},\mathcal{B}\rangle,\mathcal{D})$ and for all elements a_1, a_2 of $\mathcal{C}$ such that $a_1 = s(\mathcal{A})$ and $a_2 = s(\mathcal{B})$ holds $(\mathrm{Result}(s))(\mathrm{Output}\, \mathrm{1GateCircStr}(\langle\mathcal{A},\mathcal{B}\rangle,\mathcal{D})) = \mathcal{F}(a_1,a_2)$

provided the parameters satisfy the following condition:

- For every function g from $\mathcal{C}^2$ into $\mathcal{C}$ holds $g = \mathcal{D}$ iff for all elements a_1, a_2 of $\mathcal{C}$ holds $g(\langle a_1,a_2\rangle) = \mathcal{F}(a_1,a_2)$.

The scheme *OneGate3Result* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, a non empty finite set $\mathcal{D}$, a ternary functor $\mathcal{F}$ yielding an element of $\mathcal{D}$, and a function $\mathcal{E}$ from $\mathcal{D}^3$ into $\mathcal{D}$, and states that:

Let s be a state of 1GateCircuit$(\langle\mathcal{A},\mathcal{B},\mathcal{C}\rangle,\mathcal{E})$ and a_1, a_2, a_3 be elements of $\mathcal{D}$. If $a_1 = s(\mathcal{A})$ and $a_2 = s(\mathcal{B})$ and $a_3 = s(\mathcal{C})$, then $(\mathrm{Result}(s))(\mathrm{Output}\, \mathrm{1GateCircStr}(\langle\mathcal{A},\mathcal{B},\mathcal{C}\rangle,\mathcal{E})) = \mathcal{F}(a_1,a_2,a_3)$

provided the following requirement is met:

- For every function g from $\mathcal{D}^3$ into $\mathcal{D}$ holds $g = \mathcal{E}$ iff for all elements a_1, a_2, a_3 of $\mathcal{D}$ holds $g(\langle a_1,a_2,a_3\rangle) = \mathcal{F}(a_1,a_2,a_3)$.

The scheme *OneGate4Result* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, a non empty finite set $\mathcal{E}$, a 4-ary functor $\mathcal{F}$ yielding an element of $\mathcal{E}$, and a function $\mathcal{F}$ from $\mathcal{E}^4$ into $\mathcal{E}$, and states that:

Let s be a state of 1GateCircuit($\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D} \rangle, \mathcal{F}$) and a_1, a_2, a_3, a_4 be elements of $\mathcal{E}$. If $a_1 = s(\mathcal{A})$ and $a_2 = s(\mathcal{B})$ and $a_3 = s(\mathcal{C})$ and $a_4 = s(\mathcal{D})$, then (Result(s))(Output 1GateCircStr($\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D} \rangle, \mathcal{F}$)) $= \mathcal{F}(a_1, a_2, a_3, a_4)$

provided the following condition is met:

- Let g be a function from $\mathcal{E}^4$ into $\mathcal{E}$. Then $g = \mathcal{F}$ if and only if for all elements a_1, a_2, a_3, a_4 of $\mathcal{E}$ holds $g(\langle a_1, a_2, a_3, a_4 \rangle) = \mathcal{F}(a_1, a_2, a_3, a_4)$.

The scheme *OneGate5Result* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, a non empty finite set $\mathcal{F}$, a 5-ary functor $\mathcal{F}$ yielding an element of $\mathcal{F}$, and a function $\mathcal{G}$ from $\mathcal{F}^5$ into $\mathcal{F}$, and states that:

Let s be a state of 1GateCircuit($\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E} \rangle, \mathcal{G}$) and a_1, a_2, a_3, a_4, a_5 be elements of $\mathcal{F}$. Suppose $a_1 = s(\mathcal{A})$ and $a_2 = s(\mathcal{B})$ and $a_3 = s(\mathcal{C})$ and $a_4 = s(\mathcal{D})$ and $a_5 = s(\mathcal{E})$. Then (Result(s))(Output 1GateCircStr($\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E} \rangle, \mathcal{G}$)) $= \mathcal{F}(a_1, a_2, a_3, a_4, a_5)$

provided the following requirement is met:

- Let g be a function from $\mathcal{F}^5$ into $\mathcal{F}$. Then $g = \mathcal{G}$ if and only if for all elements a_1, a_2, a_3, a_4, a_5 of $\mathcal{F}$ holds $g(\langle a_1, a_2, a_3, a_4, a_5 \rangle) = \mathcal{F}(a_1, a_2, a_3, a_4, a_5)$.

4. Input of a Compound Circuit

We now state a number of propositions:

(39) Let n be a natural number, X be a non empty finite set, f be a function from X^n into X, p be a finite sequence with length n, and S be a signature over X. If $\operatorname{rng} p \subseteq$ the carrier of S and Output 1GateCircStr(p, f) $\notin$ InputVertices(S), then InputVertices(S+· 1GateCircStr(p, f)) = InputVertices(S).

(40) Let X_1, X_2 be sets, X be a non empty finite set, n be a natural number, f be a function from X^n into X, p be a finite sequence with length n, and S be a signature over X. Suppose $\operatorname{rng} p = X_1 \cup X_2$ and $X_1 \subseteq$ the carrier of S and X_2 misses InnerVertices(S) and Output 1GateCircStr(p, f) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr(p, f)) = InputVertices(S) $\cup X_2$.

(41) Let x_1 be a set, X be a non empty finite set, f be a function from X^1 into X, and S be a signature over X. If $x_1 \in$ the carrier of S and Output 1GateCircStr($\langle x_1 \rangle, f$) $\notin$ InputVertices(S), then InputVertices(S+· 1GateCircStr($\langle x_1 \rangle, f$)) = InputVertices(S).

(42) Let x_1, x_2 be sets, X be a non empty finite set, f be a function from X^2 into X, and S be a signature over X. Suppose $x_1 \in$ the carrier of S and $x_2 \notin$ InnerVertices(S) and Output 1GateCircStr($\langle x_1, x_2\rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2\rangle, f$)) = InputVertices($S$) $\cup \{x_2\}$.

(43) Let x_1, x_2 be sets, X be a non empty finite set, f be a function from X^2 into X, and S be a signature over X. Suppose $x_2 \in$ the carrier of S and $x_1 \notin$ InnerVertices(S) and Output 1GateCircStr($\langle x_1, x_2\rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2\rangle, f$)) = InputVertices($S$) $\cup \{x_1\}$.

(44) Let x_1, x_2 be sets, X be a non empty finite set, f be a function from X^2 into X, and S be a signature over X. Suppose $x_1 \in$ the carrier of S and $x_2 \in$ the carrier of S and Output 1GateCircStr($\langle x_1, x_2\rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2\rangle, f$)) = InputVertices($S$).

(45) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_1 \in$ the carrier of S and $x_2 \notin$ InnerVertices(S) and $x_3 \notin$ InnerVertices(S) and Output 1GateCircStr($\langle x_1, x_2, x_3\rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2, x_3\rangle, f$)) = InputVertices($S$) $\cup$ $\{x_2, x_3\}$.

(46) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_2 \in$ the carrier of S and $x_1 \notin$ InnerVertices(S) and $x_3 \notin$ InnerVertices(S) and Output 1GateCircStr($\langle x_1, x_2, x_3\rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2, x_3\rangle, f$)) = InputVertices($S$) $\cup$ $\{x_1, x_3\}$.

(47) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_3 \in$ the carrier of S and $x_1 \notin$ InnerVertices(S) and $x_2 \notin$ InnerVertices(S) and Output 1GateCircStr($\langle x_1, x_2, x_3\rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2, x_3\rangle, f$)) = InputVertices($S$) $\cup$ $\{x_1, x_2\}$.

(48) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_1 \in$ the carrier of S and $x_2 \in$ the carrier of S and $x_3 \notin$ InnerVertices(S) and Output 1GateCircStr($\langle x_1, x_2, x_3\rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2, x_3\rangle, f$)) = InputVertices($S$) $\cup$ $\{x_3\}$.

(49) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_1 \in$ the

carrier of S and $x_3 \in$ the carrier of S and $x_2 \notin$ InnerVertices(S) and Output 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$)) = InputVertices($S$) $\cup$ $\{x_2\}$.

(50) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_2 \in$ the carrier of S and $x_3 \in$ the carrier of S and $x_1 \notin$ InnerVertices(S) and Output 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$)) = InputVertices($S$) $\cup$ $\{x_1\}$.

(51) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_1 \in$ the carrier of S and $x_2 \in$ the carrier of S and $x_3 \in$ the carrier of S and Output 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$)) = InputVertices($S$).

5. Result of a Compound Circuit

Next we state the proposition

(52) Let X be a non empty finite set, S be a finite signature over X, A be a circuit over X and S, n be a natural number, f be a function from X^n into X, and p be a finite sequence with length n. Suppose Output 1GateCircStr(p, f) $\notin$ InputVertices(S). Let s be a state of A+· 1GateCircuit(p, f) and s' be a state of A. Suppose $s' = s{\restriction}$the carrier of S. Then the stabilization time of $s \leqslant 1 +$ the stabilization time of s'.

Now we present several schemes. The scheme *Comb1CircResult* deals with a set $\mathcal{A}$, a non empty finite set $\mathcal{B}$, a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, a finite signature $\mathcal{C}$ over $\mathcal{B}$, a circuit $\mathcal{D}$ over $\mathcal{B}$ and $\mathcal{C}$, and a function $\mathcal{E}$ from $\mathcal{B}^1$ into $\mathcal{B}$, and states that:

> Let s be a state of $\mathcal{D}$+· 1GateCircuit($\langle \mathcal{A} \rangle, \mathcal{E}$) and s' be a state of $\mathcal{D}$. Suppose $s' = s{\restriction}$the carrier of $\mathcal{C}$. Let a_1 be an element of $\mathcal{B}$. Suppose if $\mathcal{A} \in$ InnerVertices($\mathcal{C}$), then $a_1 =$ (Result(s'))($\mathcal{A}$) and if $\mathcal{A} \notin$ InnerVertices($\mathcal{C}$), then $a_1 = s(\mathcal{A})$. Then (Result(s))(Output 1GateCircStr($\langle \mathcal{A} \rangle, \mathcal{E}$)) $= \mathcal{F}(a_1)$

provided the parameters meet the following conditions:

- For every function g from $\mathcal{B}^1$ into $\mathcal{B}$ holds $g = \mathcal{E}$ iff for every element a_1 of $\mathcal{B}$ holds $g(\langle a_1 \rangle) = \mathcal{F}(a_1)$, and
- Output 1GateCircStr($\langle \mathcal{A} \rangle, \mathcal{E}$) $\notin$ InputVertices($\mathcal{C}$).

The scheme *Comb2CircResult* deals with sets $\mathcal{A}$, $\mathcal{B}$, a non empty finite set $\mathcal{C}$, a binary functor $\mathcal{F}$ yielding an element of $\mathcal{C}$, a finite signature $\mathcal{D}$ over $\mathcal{C}$, a circuit $\mathcal{E}$ over $\mathcal{C}$ and $\mathcal{D}$, and a function $\mathcal{F}$ from $\mathcal{C}^2$ into $\mathcal{C}$, and states that:

Let s be a state of $\mathcal{E}+\cdot$ 1GateCircuit($\langle\mathcal{A},\mathcal{B}\rangle,\mathcal{F}$) and s' be a state of $\mathcal{E}$. Suppose $s' = s\restriction$the carrier of $\mathcal{D}$. Let a_1, a_2 be elements of $\mathcal{C}$. Suppose if $\mathcal{A} \in$ InnerVertices($\mathcal{D}$), then $a_1 =$ (Result(s'))($\mathcal{A}$) and if $\mathcal{A} \notin$ InnerVertices($\mathcal{D}$), then $a_1 = s(\mathcal{A})$ and if $\mathcal{B} \in$ InnerVertices($\mathcal{D}$), then $a_2 =$ (Result(s'))($\mathcal{B}$) and if $\mathcal{B} \notin$ InnerVertices($\mathcal{D}$), then $a_2 = s(\mathcal{B})$. Then (Result(s))(Output 1GateCircStr($\langle\mathcal{A},\mathcal{B}\rangle,\mathcal{F}$)) $= \mathcal{F}(a_1, a_2)$

provided the parameters meet the following requirements:

- For every function g from $\mathcal{C}^2$ into $\mathcal{C}$ holds $g = \mathcal{F}$ iff for all elements a_1, a_2 of $\mathcal{C}$ holds $g(\langle a_1, a_2\rangle) = \mathcal{F}(a_1, a_2)$, and
- Output 1GateCircStr($\langle\mathcal{A},\mathcal{B}\rangle,\mathcal{F}$) $\notin$ InputVertices($\mathcal{D}$).

The scheme *Comb3CircResult* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, a non empty finite set $\mathcal{D}$, a ternary functor $\mathcal{F}$ yielding an element of $\mathcal{D}$, a finite signature $\mathcal{E}$ over $\mathcal{D}$, a circuit $\mathcal{F}$ over $\mathcal{D}$ and $\mathcal{E}$, and a function $\mathcal{G}$ from $\mathcal{D}^3$ into $\mathcal{D}$, and states that:

Let s be a state of $\mathcal{F}+\cdot$ 1GateCircuit($\langle\mathcal{A},\mathcal{B},\mathcal{C}\rangle,\mathcal{G}$) and s' be a state of $\mathcal{F}$. Suppose $s' = s\restriction$the carrier of $\mathcal{E}$. Let a_1, a_2, a_3 be elements of $\mathcal{D}$. Suppose that

(i) if $\mathcal{A} \in$ InnerVertices($\mathcal{E}$), then $a_1 =$ (Result(s'))($\mathcal{A}$),
(ii) if $\mathcal{A} \notin$ InnerVertices($\mathcal{E}$), then $a_1 = s(\mathcal{A})$,
(iii) if $\mathcal{B} \in$ InnerVertices($\mathcal{E}$), then $a_2 =$ (Result(s'))($\mathcal{B}$),
(iv) if $\mathcal{B} \notin$ InnerVertices($\mathcal{E}$), then $a_2 = s(\mathcal{B})$,
(v) if $\mathcal{C} \in$ InnerVertices($\mathcal{E}$), then $a_3 =$ (Result(s'))($\mathcal{C}$), and
(vi) if $\mathcal{C} \notin$ InnerVertices($\mathcal{E}$), then $a_3 = s(\mathcal{C})$.

Then (Result(s))(Output 1GateCircStr($\langle\mathcal{A},\mathcal{B},\mathcal{C}\rangle,\mathcal{G}$)) $= \mathcal{F}(a_1, a_2, a_3)$

provided the parameters meet the following requirements:

- For every function g from $\mathcal{D}^3$ into $\mathcal{D}$ holds $g = \mathcal{G}$ iff for all elements a_1, a_2, a_3 of $\mathcal{D}$ holds $g(\langle a_1, a_2, a_3\rangle) = \mathcal{F}(a_1, a_2, a_3)$, and
- Output 1GateCircStr($\langle\mathcal{A},\mathcal{B},\mathcal{C}\rangle,\mathcal{G}$) $\notin$ InputVertices($\mathcal{E}$).

The scheme *Comb4CircResult* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, a non empty finite set $\mathcal{E}$, a 4-ary functor $\mathcal{F}$ yielding an element of $\mathcal{E}$, a finite signature $\mathcal{F}$ over $\mathcal{E}$, a circuit $\mathcal{G}$ over $\mathcal{E}$ and $\mathcal{F}$, and a function $\mathcal{H}$ from $\mathcal{E}^4$ into $\mathcal{E}$, and states that:

Let s be a state of $\mathcal{G}+\cdot$ 1GateCircuit($\langle\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D}\rangle,\mathcal{H}$) and s' be a state of $\mathcal{G}$. Suppose $s' = s\restriction$the carrier of $\mathcal{F}$. Let a_1, a_2, a_3, a_4 be elements of $\mathcal{E}$. Suppose that if $\mathcal{A} \in$ InnerVertices($\mathcal{F}$), then $a_1 =$ (Result(s'))($\mathcal{A}$) and if $\mathcal{A} \notin$ InnerVertices($\mathcal{F}$), then $a_1 = s(\mathcal{A})$ and if $\mathcal{B} \in$ InnerVertices($\mathcal{F}$), then $a_2 =$ (Result(s'))($\mathcal{B}$) and if $\mathcal{B} \notin$ InnerVertices($\mathcal{F}$), then $a_2 = s(\mathcal{B})$ and if $\mathcal{C} \in$ InnerVertices($\mathcal{F}$), then $a_3 =$ (Result(s'))($\mathcal{C}$) and if $\mathcal{C} \notin$ InnerVertices($\mathcal{F}$), then $a_3 = s(\mathcal{C})$ and if $\mathcal{D} \in$ InnerVertices($\mathcal{F}$), then $a_4 =$ (Result(s'))($\mathcal{D}$) and if $\mathcal{D} \notin$ InnerVertices($\mathcal{F}$), then $a_4 = s(\mathcal{D})$. Then (Result(s))(Output 1GateCircStr($\langle\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D}\rangle,\mathcal{H}$)) $= \mathcal{F}(a_1, a_2, a_3, a_4)$

provided the parameters satisfy the following conditions:

- Let g be a function from $\mathcal{E}^4$ into $\mathcal{E}$. Then $g = \mathcal{H}$ if and only if for all elements a_1, a_2, a_3, a_4 of $\mathcal{E}$ holds $g(\langle a_1, a_2, a_3, a_4\rangle) = \mathcal{F}(a_1, a_2, a_3, a_4)$, and
- Output 1GateCircStr$(\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}\rangle, \mathcal{H}) \notin$ InputVertices$(\mathcal{F})$.

The scheme *Comb5CircResult* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, a non empty finite set $\mathcal{F}$, a 5-ary functor $\mathcal{F}$ yielding an element of $\mathcal{F}$, a finite signature $\mathcal{G}$ over $\mathcal{F}$, a circuit $\mathcal{H}$ over $\mathcal{F}$ and $\mathcal{G}$, and a function $\mathcal{I}$ from $\mathcal{F}^5$ into $\mathcal{F}$, and states that:

> Let s be a state of $\mathcal{H} +\cdot$ 1GateCircuit$(\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}\rangle, \mathcal{I})$ and s' be a state of $\mathcal{H}$. Suppose $s' = s \restriction$the carrier of $\mathcal{G}$. Let a_1, a_2, a_3, a_4, a_5 be elements of $\mathcal{F}$. Suppose that if $\mathcal{A} \in$ InnerVertices$(\mathcal{G})$, then $a_1 =$ (Result(s'))$(\mathcal{A})$ and if $\mathcal{A} \notin$ InnerVertices$(\mathcal{G})$, then $a_1 = s(\mathcal{A})$ and if $\mathcal{B} \in$ InnerVertices$(\mathcal{G})$, then $a_2 =$ (Result(s'))$(\mathcal{B})$ and if $\mathcal{B} \notin$ InnerVertices$(\mathcal{G})$, then $a_2 = s(\mathcal{B})$ and if $\mathcal{C} \in$ InnerVertices$(\mathcal{G})$, then $a_3 =$ (Result(s'))$(\mathcal{C})$ and if $\mathcal{C} \notin$ InnerVertices$(\mathcal{G})$, then $a_3 = s(\mathcal{C})$ and if $\mathcal{D} \in$ InnerVertices$(\mathcal{G})$, then $a_4 =$ (Result(s'))$(\mathcal{D})$ and if $\mathcal{D} \notin$ InnerVertices$(\mathcal{G})$, then $a_4 = s(\mathcal{D})$ and if $\mathcal{E} \in$ InnerVertices$(\mathcal{G})$, then $a_5 =$ (Result(s'))$(\mathcal{E})$ and if $\mathcal{E} \notin$ InnerVertices$(\mathcal{G})$, then $a_5 = s(\mathcal{E})$. Then (Result(s))(Output 1GateCircStr$(\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}\rangle, \mathcal{I})) = \mathcal{F}(a_1, a_2, a_3, a_4, a_5)$

provided the parameters meet the following conditions:

- Let g be a function from $\mathcal{F}^5$ into $\mathcal{F}$. Then $g = \mathcal{I}$ if and only if for all elements a_1, a_2, a_3, a_4, a_5 of $\mathcal{F}$ holds $g(\langle a_1, a_2, a_3, a_4, a_5\rangle) = \mathcal{F}(a_1, a_2, a_3, a_4, a_5)$, and
- Output 1GateCircStr$(\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}\rangle, \mathcal{I}) \notin$ InputVertices$(\mathcal{G})$.

6. Inputs Without Pairs

Let S be a non empty many sorted signature. We say that S has nonpair inputs if and only if:

(Def. 11) InputVertices(S) has no pairs.

Note that $\mathbb{N}$ has no pairs. Let X be a set with no pairs. Note that every subset of X has no pairs.

Let us observe that every function which is natural-yielding is also nonpair yielding.

Let us note that every finite sequence of elements of $\mathbb{N}$ is natural-yielding.

Let us observe that there exists a finite sequence which is one-to-one and natural-yielding.

Let n be a natural number. Observe that there exists a finite sequence with length n which is one-to-one and natural-yielding.

Let p be a nonpair yielding finite sequence and let f be a set. Observe that 1GateCircStr(p, f) has nonpair inputs.

One can verify that there exists an one-gate many sorted signature which has nonpair inputs. Let X be a non empty finite set. One can verify that there exists an one-gate signature over X which has nonpair inputs.

Let S be a non empty many sorted signature with nonpair inputs. One can check that InputVertices(S) has no pairs.

The following proposition is true

(53) Let S be a non empty many sorted signature with nonpair inputs and x be a vertex of S. If x is pair, then $x \in$ InnerVertices(S).

Let S be an unsplit non empty many sorted signature with arity held in gates. One can verify that InnerVertices(S) is relation-like.

Let S be an unsplit non empty non void many sorted signature with denotation held in gates. Note that InnerVertices(S) is relation-like.

Let S_1, S_2 be unsplit non empty many sorted signatures with arity held in gates with nonpair inputs. One can verify that $S_1 {+\cdot} S_2$ has nonpair inputs.

One can prove the following propositions:

(54) For every non pair set x and for every binary relation R holds $x \notin R$.

(55) Let x_1 be a set, X be a non empty finite set, f be a function from X^1 into X, and S be a signature over X with nonpair inputs. If $x_1 \in$ the carrier of S or x_1 is non pair, then $S {+\cdot}$ 1GateCircStr$(\langle x_1 \rangle, f)$ has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x_1 be a vertex of S, and let f be a function from X^1 into X. One can verify that $S {+\cdot}$ 1GateCircStr$(\langle x_1 \rangle, f)$ has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x_1 be a non pair set, and let f be a function from X^1 into X. One can verify that $S {+\cdot}$ 1GateCircStr$(\langle x_1 \rangle, f)$ has nonpair inputs.

We now state the proposition

(56) Let x_1, x_2 be sets, X be a non empty finite set, f be a function from X^2 into X, and S be a signature over X with nonpair inputs. Suppose $x_1 \in$ the carrier of S or x_1 is non pair but $x_2 \in$ the carrier of S or x_2 is non pair. Then $S {+\cdot}$ 1GateCircStr$(\langle x_1, x_2 \rangle, f)$ has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x_1 be a vertex of S, let n_2 be a non pair set, and let f be a function from X^2 into X. Observe that $S {+\cdot}$ 1GateCircStr$(\langle x_1, n_2 \rangle, f)$ has nonpair inputs and $S {+\cdot}$ 1GateCircStr$(\langle n_2, x_1 \rangle, f)$ has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x_1, x_2 be vertices of S, and let f be a function from X^2 into X. One can verify that $S {+\cdot}$ 1GateCircStr$(\langle x_1, x_2 \rangle, f)$ has nonpair inputs.

One can prove the following proposition

(57) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X with nonpair inputs. Suppose that

(i) $x_1 \in$ the carrier of S or x_1 is non pair,

(ii) $x_2 \in$ the carrier of S or x_2 is non pair, and

(iii) $x_3 \in$ the carrier of S or x_3 is non pair.

Then $S +\cdot$ 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$) has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x_1, x_2 be vertices of S, let n be a non pair set, and let f be a function from X^3 into X. One can verify the following observations:

* $S +\cdot$ 1GateCircStr($\langle x_1, x_2, n \rangle, f$) has nonpair inputs,
* $S +\cdot$ 1GateCircStr($\langle x_1, n, x_2 \rangle, f$) has nonpair inputs, and
* $S +\cdot$ 1GateCircStr($\langle n, x_1, x_2 \rangle, f$) has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x be a vertex of S, let n_1, n_2 be non pair sets, and let f be a function from X^3 into X. One can check the following observations:

* $S +\cdot$ 1GateCircStr($\langle x, n_1, n_2 \rangle, f$) has nonpair inputs,
* $S +\cdot$ 1GateCircStr($\langle n_1, x, n_2 \rangle, f$) has nonpair inputs, and
* $S +\cdot$ 1GateCircStr($\langle n_1, n_2, x \rangle, f$) has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x_1, x_2, x_3 be vertices of S, and let f be a function from X^3 into X. Observe that $S +\cdot$ 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$) has nonpair inputs.

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