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# The Ordering of Points on a Curve. Part III[1]

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The articles [12], [13], [1], [6], [7], [10], [4], [3], [11], [5], [8], [2], and [9] provide the notation and terminology for this paper.

We follow the rules: $C$, $P$ denote simple closed curves and $a$, $b$, $c$, $d$, $e$ denote points of $\mathcal{E}^2_{\rm T}$.

We now state several propositions:

(1) Let $n$ be a natural number, $a$, $p_1$, $p_2$ be points of $\mathcal{E}^n_{\rm T}$, and $P$ be a subset of the carrier of $\mathcal{E}^n_{\rm T}$. Suppose $a \in P$ and $P$ is an arc from $p_1$ to $p_2$. Then there exists a map $f$ from $\mathbb{I}$ into $(\mathcal{E}^n_{\rm T})\restriction P$ and there exists a real number $r$ such that $f$ is a homeomorphism and $f(0) = p_1$ and $f(1) = p_2$ and $0 \leqslant r$ and $r \leqslant 1$ and $f(r) = a$.

(2) $\mathrm{LE}(\text{W-min}\,P, \text{E-max}\,P, P)$.

(3) If $\mathrm{LE}(a, \text{E-max}\,P, P)$, then $a \in \mathrm{UpperArc}\,P$.

(4) If $\mathrm{LE}(\text{E-max}\,P, a, P)$, then $a \in \mathrm{LowerArc}\,P$.

(5) If $\mathrm{LE}(a, \text{W-min}\,P, P)$, then $a \in \mathrm{LowerArc}\,P$.

(6) Let $P$ be a subset of the carrier of $\mathcal{E}^2_{\rm T}$. Suppose $a \neq b$ and $P$ is an arc from $c$ to $d$ and LE $a$, $b$, $P$, $c$, $d$. Then there exists $e$ such that $a \neq e$ and $b \neq e$ and LE $a$, $e$, $P$, $c$, $d$ and LE $e$, $b$, $P$, $c$, $d$.

(7) If $a \in P$, then there exists $e$ such that $a \neq e$ and $\mathrm{LE}(a, e, P)$.

(8) If $a \neq b$ and $\mathrm{LE}(a, b, P)$, then there exists $c$ such that $c \neq a$ and $c \neq b$ and $\mathrm{LE}(a, c, P)$ and $\mathrm{LE}(c, b, P)$.

Let $P$ be a compact non empty subset of $\mathcal{E}^2_{\rm T}$ and let $a$, $b$, $c$, $d$ be points of $\mathcal{E}^2_{\rm T}$. We say that $a$, $b$, $c$, $d$ are in this order on $P$ if and only if:

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(Def. 1) $\mathrm{LE}(a, b, P)$ and $\mathrm{LE}(b, c, P)$ and $\mathrm{LE}(c, d, P)$ or $\mathrm{LE}(b, c, P)$ and $\mathrm{LE}(c, d, P)$ and $\mathrm{LE}(d, a, P)$ or $\mathrm{LE}(c, d, P)$ and $\mathrm{LE}(d, a, P)$ and $\mathrm{LE}(a, b, P)$ or $\mathrm{LE}(d, a, P)$ and $\mathrm{LE}(a, b, P)$ and $\mathrm{LE}(b, c, P)$.

The following propositions are true:

(9) If $a \in P$, then $a$, $a$, $a$, $a$ are in this order on $P$.

(10) If $a$, $b$, $c$, $d$ are in this order on $P$, then $b$, $c$, $d$, $a$ are in this order on $P$.

(11) If $a$, $b$, $c$, $d$ are in this order on $P$, then $c$, $d$, $a$, $b$ are in this order on $P$.

(12) If $a$, $b$, $c$, $d$ are in this order on $P$, then $d$, $a$, $b$, $c$ are in this order on $P$.

(13) Suppose $a \neq b$ and $a$, $b$, $c$, $d$ are in this order on $P$. Then there exists $e$ such that $e \neq a$ and $e \neq b$ and $a$, $e$, $b$, $c$ are in this order on $P$.

(14) Suppose $a \neq b$ and $a$, $b$, $c$, $d$ are in this order on $P$. Then there exists $e$ such that $e \neq a$ and $e \neq b$ and $a$, $e$, $b$, $d$ are in this order on $P$.

(15) Suppose $b \neq c$ and $a$, $b$, $c$, $d$ are in this order on $P$. Then there exists $e$ such that $e \neq b$ and $e \neq c$ and $a$, $b$, $e$, $c$ are in this order on $P$.

(16) Suppose $b \neq c$ and $a$, $b$, $c$, $d$ are in this order on $P$. Then there exists $e$ such that $e \neq b$ and $e \neq c$ and $b$, $e$, $c$, $d$ are in this order on $P$.

(17) Suppose $c \neq d$ and $a$, $b$, $c$, $d$ are in this order on $P$. Then there exists $e$ such that $e \neq c$ and $e \neq d$ and $a$, $c$, $e$, $d$ are in this order on $P$.

(18) Suppose $c \neq d$ and $a$, $b$, $c$, $d$ are in this order on $P$. Then there exists $e$ such that $e \neq c$ and $e \neq d$ and $b$, $c$, $e$, $d$ are in this order on $P$.

(19) Suppose $d \neq a$ and $a$, $b$, $c$, $d$ are in this order on $P$. Then there exists $e$ such that $e \neq d$ and $e \neq a$ and $a$, $b$, $d$, $e$ are in this order on $P$.

(20) Suppose $d \neq a$ and $a$, $b$, $c$, $d$ are in this order on $P$. Then there exists $e$ such that $e \neq d$ and $e \neq a$ and $a$, $c$, $d$, $e$ are in this order on $P$.

(21) Suppose $a \neq c$ and $a \neq d$ and $b \neq d$ and $a$, $b$, $c$, $d$ are in this order on $P$ and $b$, $a$, $c$, $d$ are in this order on $P$. Then $a = b$.

(22) Suppose $a \neq b$ and $b \neq c$ and $c \neq d$ and $a$, $b$, $c$, $d$ are in this order on $P$ and $c$, $b$, $a$, $d$ are in this order on $P$. Then $a = c$.

(23) Suppose $a \neq b$ and $a \neq c$ and $b \neq d$ and $a$, $b$, $c$, $d$ are in this order on $P$ and $d$, $b$, $c$, $a$ are in this order on $P$. Then $a = d$.

(24) Suppose $a \neq c$ and $a \neq d$ and $b \neq d$ and $a$, $b$, $c$, $d$ are in this order on $P$ and $a$, $c$, $b$, $d$ are in this order on $P$. Then $b = c$.

(25) Suppose $a \neq b$ and $b \neq c$ and $c \neq d$ and $a$, $b$, $c$, $d$ are in this order on $P$ and $a$, $d$, $c$, $b$ are in this order on $P$. Then $b = d$.

(26) Suppose $a \neq b$ and $a \neq c$ and $b \neq d$ and $a$, $b$, $c$, $d$ are in this order on $P$ and $a$, $b$, $d$, $c$ are in this order on $P$. Then $c = d$.

(27) Suppose $a \in C$ and $b \in C$ and $c \in C$ and $d \in C$. Then

(i) $a$, $b$, $c$, $d$ are in this order on $C$, or

(ii) $a$, $b$, $d$, $c$ are in this order on $C$, or

(iii) $a$, $c$, $b$, $d$ are in this order on $C$, or
(iv) $a$, $c$, $d$, $b$ are in this order on $C$, or
(v) $a$, $d$, $b$, $c$ are in this order on $C$, or
(vi) $a$, $d$, $c$, $b$ are in this order on $C$.

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