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The Ordering of Points on a Curve. Part IV[1]

Artur Korniłowicz
University of Białystok

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The notation and terminology used in this paper are introduced in the following articles: [19], [21], [22], [2], [3], [10], [20], [13], [14], [18], [6], [17], [5], [11], [1], [7], [8], [4], [9], [16], [12], and [15].

1. Preliminaries

For simplicity, we adopt the following rules: n denotes an element of $\mathbb{N}$, V denotes a subset of the carrier of $\mathcal{E}^n_{\mathrm{T}}$, s, s_1, s_2, t, t_1, t_2 denote points of $\mathcal{E}^n_{\mathrm{T}}$, C denotes a simple closed curve, P denotes a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$, and a, p, p_1, p_2, q, q_1, q_2 denote points of $\mathcal{E}^2_{\mathrm{T}}$.

Next we state several propositions:

(1) For all real numbers a, b holds $(a-b)^{\mathbf{2}} = (b-a)^{\mathbf{2}}$.

(2) Let S, T be non empty topological spaces, f be a map from S into T, and A be a subset of T. If f is a homeomorphism and A is connected, then $f^{-1}(A)$ is connected.

(3) Let S, T be non empty topological structures, f be a map from S into T, and A be a subset of T. If f is a homeomorphism and A is compact, then $f^{-1}(A)$ is compact.

(4) $\mathrm{proj2}^\circ\,\mathrm{NorthHalfline}\,a$ is lower bounded.

(5) $\mathrm{proj2}^\circ\,\mathrm{SouthHalfline}\,a$ is upper bounded.

(6) $\mathrm{proj1}^\circ\,\mathrm{WestHalfline}\,a$ is upper bounded.

(7) $\mathrm{proj1}^\circ\,\mathrm{EastHalfline}\,a$ is lower bounded.

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Let us consider a. One can verify the following observations:

* $\mathrm{proj2}^\circ\,\mathrm{NorthHalfline}\,a$ is non empty,
* $\mathrm{proj2}^\circ\,\mathrm{SouthHalfline}\,a$ is non empty,
* $\mathrm{proj1}^\circ\,\mathrm{WestHalfline}\,a$ is non empty, and
* $\mathrm{proj1}^\circ\,\mathrm{EastHalfline}\,a$ is non empty.

Next we state four propositions:

(8) $\inf(\mathrm{proj2}^\circ\,\mathrm{NorthHalfline}\,a) = a_{\mathbf{2}}$.

(9) $\sup(\mathrm{proj2}^\circ\,\mathrm{SouthHalfline}\,a) = a_{\mathbf{2}}$.

(10) $\sup(\mathrm{proj1}^\circ\,\mathrm{WestHalfline}\,a) = a_{\mathbf{1}}$.

(11) $\inf(\mathrm{proj1}^\circ\,\mathrm{EastHalfline}\,a) = a_{\mathbf{1}}$.

Let us consider a. One can verify the following observations:

* $\mathrm{NorthHalfline}\,a$ is closed,
* $\mathrm{SouthHalfline}\,a$ is closed,
* $\mathrm{EastHalfline}\,a$ is closed, and
* $\mathrm{WestHalfline}\,a$ is closed.

One can prove the following propositions:

(12) If $a \in \mathrm{BDD}\,P$, then $\mathrm{NorthHalfline}\,a \not\subseteq \mathrm{UBD}\,P$.

(13) If $a \in \mathrm{BDD}\,P$, then $\mathrm{SouthHalfline}\,a \not\subseteq \mathrm{UBD}\,P$.

(14) If $a \in \mathrm{BDD}\,P$, then $\mathrm{EastHalfline}\,a \not\subseteq \mathrm{UBD}\,P$.

(15) If $a \in \mathrm{BDD}\,P$, then $\mathrm{WestHalfline}\,a \not\subseteq \mathrm{UBD}\,P$.

(16) Let P be a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ and p_1, p_2, q be points of $\mathcal{E}^2_{\mathrm{T}}$. If P is an arc from p_1 to p_2 and $q \neq p_2$, then $p_2 \notin \mathrm{LSegment}(P, p_1, p_2, q)$.

(17) Let P be a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ and p_1, p_2, q be points of $\mathcal{E}^2_{\mathrm{T}}$. If P is an arc from p_1 to p_2 and $q \neq p_1$, then $p_1 \notin \mathrm{RSegment}(P, p_1, p_2, q)$.

(18) Let C be a simple closed curve, P be a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$, and p_1, p_2 be points of $\mathcal{E}^2_{\mathrm{T}}$. Suppose P is an arc from p_1 to p_2 and $P \subseteq C$. Then there exists a non empty subset R of $\mathcal{E}^2_{\mathrm{T}}$ such that R is an arc from p_1 to p_2 and $P \cup R = C$ and $P \cap R = \{p_1, p_2\}$.

(19) Let P be a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ and p_1, p_2, q_1, q_2 be points of $\mathcal{E}^2_{\mathrm{T}}$. Suppose P is an arc from p_1 to p_2 and $q_1 \in P$ and $q_2 \in P$ and $q_1 \neq p_1$ and $q_1 \neq p_2$ and $q_2 \neq p_1$ and $q_2 \neq p_2$ and $q_1 \neq q_2$. Then there exists a non empty subset Q of $\mathcal{E}^2_{\mathrm{T}}$ such that Q is an arc from q_1 to q_2 and $Q \subseteq P$ and Q misses $\{p_1, p_2\}$.

2. Two Special Points on a Simple Closed Curve

Let us consider p, P. The functor $\mathrm{North\text{-}Bound}(p, P)$ yields a point of $\mathcal{E}^2_{\mathrm{T}}$ and is defined by:

(Def. 1) North-Bound$(p, P) = [p_{\mathbf{1}}, \inf(\text{proj2}^{\circ}(P \cap \text{NorthHalfline}\, p))]$.

The functor South-Bound(p, P) yields a point of $\mathcal{E}^2_{\rm T}$ and is defined by:

(Def. 2) South-Bound$(p, P) = [p_{\mathbf{1}}, \sup(\text{proj2}^{\circ}(P \cap \text{SouthHalfline}\, p))]$.

One can prove the following propositions:

(20) (North-Bound$(p, P))_{\mathbf{1}} = p_{\mathbf{1}}$ and (South-Bound$(p, P))_{\mathbf{1}} = p_{\mathbf{1}}$.

(21) (North-Bound$(p, P))_{\mathbf{2}} = \inf(\text{proj2}^{\circ}(P \cap \text{NorthHalfline}\, p))$ and (South-Bound$(p, P))_{\mathbf{2}} = \sup(\text{proj2}^{\circ}(P \cap \text{SouthHalfline}\, p))$.

(22) For every compact subset C of $\mathcal{E}^2_{\rm T}$ such that $p \in \text{BDD}\, C$ holds North-Bound$(p, C) \in C$ and North-Bound$(p, C) \in$ NorthHalfline p and South-Bound$(p, C) \in C$ and South-Bound$(p, C) \in$ SouthHalfline p.

(23) For every compact subset C of $\mathcal{E}^2_{\rm T}$ such that $p \in \text{BDD}\, C$ holds (South-Bound$(p, C))_{\mathbf{2}} < p_{\mathbf{2}}$ and $p_{\mathbf{2}} <$ (North-Bound$(p, C))_{\mathbf{2}}$.

(24) For every compact subset C of $\mathcal{E}^2_{\rm T}$ such that $p \in \text{BDD}\, C$ holds $\inf(\text{proj2}^{\circ}(C \cap \text{NorthHalfline}\, p)) > \sup(\text{proj2}^{\circ}(C \cap \text{SouthHalfline}\, p))$.

(25) For every compact subset C of $\mathcal{E}^2_{\rm T}$ such that $p \in \text{BDD}\, C$ holds South-Bound$(p, C) \neq$ North-Bound(p, C).

(26) For every subset C of the carrier of $\mathcal{E}^2_{\rm T}$ holds $\mathcal{L}$(North-Bound(p, C), South-Bound(p, C)) is vertical.

(27) For every compact subset C of $\mathcal{E}^2_{\rm T}$ such that $p \in \text{BDD}\, C$ holds $\mathcal{L}$(North-Bound(p, C), South-Bound(p, C)) $\cap\, C =$ {North-Bound(p, C), South-Bound(p, C)}.

(28) Let C be a compact subset of $\mathcal{E}^2_{\rm T}$. Suppose $p \in \text{BDD}\, C$ and $q \in \text{BDD}\, C$ and $p_{\mathbf{1}} \neq q_{\mathbf{1}}$. Then North-Bound$(p, C)$, South-Bound$(q, C)$, North-Bound$(q, C)$, South-Bound$(p, C)$ are mutually different.

3. An Order of Points on a Simple Closed Curve

Let us consider n, V, s_1, s_2, t_1, t_2. We say that s_1, s_2 separate t_1, t_2 on V if and only if:

(Def. 3) For every subset A of the carrier of $\mathcal{E}^n_{\rm T}$ such that A is an arc from s_1 to s_2 and $A \subseteq V$ holds A meets $\{t_1, t_2\}$.

We introduce s_1, s_2 are neighbours wrt t_1, t_2 on V as an antonym of s_1, s_2 separate t_1, t_2 on V.

We now state a number of propositions:

(29) t, t separate s_1, s_2 on V.

(30) If s_1, s_2 separate t_1, t_2 on V, then s_2, s_1 separate t_1, t_2 on V.

(31) If s_1, s_2 separate t_1, t_2 on V, then s_1, s_2 separate t_2, t_1 on V.

(32) s, t_1 separate s, t_2 on V.

(33) t_1, s separate t_2, s on V.

(34) t_1, s separate s, t_2 on V.

(35) s, t_1 separate t_2, s on V.

(36) Let p_1, p_2, q be points of $\mathcal{E}^2_{\rm T}$. Suppose $q \in C$ and $p_1 \in C$ and $p_2 \in C$ and $p_1 \neq p_2$ and $p_1 \neq q$ and $p_2 \neq q$. Then p_1, p_2 are neighbours wrt q, q on C.

(37) If $p_1 \neq p_2$ and $p_1 \in C$ and $p_2 \in C$, then if p_1, p_2 separate q_1, q_2 on C, then q_1, q_2 separate p_1, p_2 on C.

(38) Suppose $p_1 \in C$ and $p_2 \in C$ and $q_1 \in C$ and $p_1 \neq p_2$ and $q_1 \neq p_1$ and $q_1 \neq p_2$ and $q_2 \neq p_1$ and $q_2 \neq p_2$. Then p_1, p_2 are neighbours wrt q_1, q_2 on C or p_1, q_1 are neighbours wrt p_2, q_2 on C.

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