FORMALIZED MATHEMATICS
Volume 11, Number 1, 2003
University of Białystok

Processes in Petri Nets

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Summary. Sequential and concurrent compositions of processes in Petri nets are introduced. A process is formalized as a set of (possible), so called, firing sequences. In the definition of the sequential composition the standard concatenation is used

$$R_1 \operatorname{before} R_2 = \{p_1 {}^\frown p_2 : p_1 \in R_1 \ \wedge \ p_2 \in R_2\}$$

The definition of the concurrent composition is more complicated

$$R_1 \operatorname{concur} R_2 = \{q_1 \cup q_2 : q_1 \text{ misses } q_2 \ \wedge \ \operatorname{Seq} q_1 \in R_1 \ \wedge \ \operatorname{Seq} q_2 \in R_2\}$$

For example,

$$\{\langle t_0 \rangle\} \operatorname{concur} \{\langle t_1, t_2 \rangle\} = \{\langle t_0, t_1, t_2 \rangle, \langle t_1, t_0, t_2 \rangle, \langle t_1, t_2, t_0 \rangle\}$$

The basic properties of the compositions are shown.

MML Identifier: PNPROC_1.

The notation and terminology used in this paper are introduced in the following papers: [14], [13], [18], [6], [17], [9], [1], [3], [7], [12], [2], [10], [15], [5], [16], [8], [11], and [4].

ISSN 1426–2630

1. Preliminaries

We adopt the following rules: i is a natural number and x, x_1, x_2, y_1, y_2 are sets.

Next we state three propositions:

(1) If $i > 0$, then $\{\langle i, x\rangle\}$ is a finite subsequence.

(2) For every finite subsequence q holds $q = \emptyset$ iff $\operatorname{Seq} q = \emptyset$.

(3) For every finite subsequence q such that $q = \{\langle i, x\rangle\}$ holds $\operatorname{Seq} q = \langle x\rangle$.

Let us observe that every finite subsequence is finite.

We now state several propositions:

(4) For every finite subsequence q such that $\operatorname{Seq} q = \langle x\rangle$ there exists i such that $q = \{\langle i, x\rangle\}$.

(5) If $\{\langle x_1, y_1\rangle, \langle x_2, y_2\rangle\}$ is a finite sequence, then $x_1 = 1$ and $x_2 = 1$ and $y_1 = y_2$ or $x_1 = 1$ and $x_2 = 2$ or $x_1 = 2$ and $x_2 = 1$.

(6) $\langle x_1, x_2\rangle = \{\langle 1, x_1\rangle, \langle 2, x_2\rangle\}$.

(7) For every finite subsequence p holds $\overline{\overline{p}} = \operatorname{len} \operatorname{Seq} p$.

(8) For all binary relations P, R such that $\operatorname{dom} P$ misses $\operatorname{dom} R$ holds P misses R.

(9) For all sets X, Y and for all binary relations P, R such that X misses Y holds $P{\restriction}X$ misses $R{\restriction}Y$.

(10) For all functions f, g, h such that $f \subseteq h$ and $g \subseteq h$ and f misses g holds $\operatorname{dom} f$ misses $\operatorname{dom} g$.

(11) For every set Y and for every binary relation R holds $Y{\restriction}R \subseteq R{\restriction}R^{-1}(Y)$.

(12) For every set Y and for every function f holds $Y{\restriction}f = f{\restriction}f^{-1}(Y)$.

2. Markings on Petri Nets

Let P be a set. A function is called a marking of P if:

(Def. 1) $\operatorname{dom} \mathrm{it} = P$ and $\operatorname{rng} \mathrm{it} \subseteq \mathbb{N}$.

We adopt the following convention: P, p, x denote sets, m_1, m_2, m_3, m_4, m denote markings of P, and i, j, j_1, k denote natural numbers.

Let P be a set, let m be a marking of P, and let p be a set. Then $m(p)$ is a natural number. We introduce the m multitude of p as a synonym of $m(p)$.

The scheme *MarkingLambda* deals with a set $\mathcal{A}$ and a unary functor $\mathcal{F}$ yielding a natural number, and states that:

> There exists a marking m of $\mathcal{A}$ such that for every p such that $p \in \mathcal{A}$ holds the m multitude of $p = \mathcal{F}(p)$

for all values of the parameters.

Let us consider P, m_1, m_2. Let us observe that $m_1 = m_2$ if and only if:

(Def. 2) For every p such that $p \in P$ holds the m_1 multitude of $p =$ the m_2 multitude of p.

Let us consider P. The functor $\{\}_P$ yielding a marking of P is defined by:

(Def. 3) $\{\}_P = P \longmapsto 0$.

Let P be a set and let m_1, m_2 be markings of P. The predicate $m_1 \subseteq m_2$ is defined by:

(Def. 4) For every set p such that $p \in P$ holds the m_1 multitude of $p \leqslant$ the m_2 multitude of p.

Let us note that the predicate $m_1 \subseteq m_2$ is reflexive.

We now state two propositions:

(13) $\{\}_P \subseteq m$.

(14) If $m_1 \subseteq m_2$ and $m_2 \subseteq m_3$, then $m_1 \subseteq m_3$.

Let P be a set and let m_1, m_2 be markings of P. The functor $m_1 + m_2$ yields a marking of P and is defined as follows:

(Def. 5) For every set p such that $p \in P$ holds the $m_1 + m_2$ multitude of $p =$ (the m_1 multitude of p) + (the m_2 multitude of p).

Let us notice that the functor $m_1 + m_2$ is commutative.

The following proposition is true

(15) $m + \{\}_P = m$.

Let P be a set and let m_1, m_2 be markings of P. Let us assume that $m_2 \subseteq m_1$. The functor $m_1 - m_2$ yielding a marking of P is defined by:

(Def. 6) For every set p such that $p \in P$ holds the $m_1 - m_2$ multitude of $p =$ (the m_1 multitude of p) − (the m_2 multitude of p).

One can prove the following propositions:

(16) $m_1 \subseteq m_1 + m_2$.

(17) $m - \{\}_P = m$.

(18) If $m_1 \subseteq m_2$ and $m_2 \subseteq m_3$, then $m_3 - m_2 \subseteq m_3 - m_1$.

(19) $(m_1 + m_2) - m_2 = m_1$.

(20) If $m \subseteq m_1$ and $m_1 \subseteq m_2$, then $m_1 - m \subseteq m_2 - m$.

(21) If $m_1 \subseteq m_2$, then $(m_2 + m_3) - m_1 = (m_2 - m_1) + m_3$.

(22) If $m_1 \subseteq m_2$ and $m_2 \subseteq m_1$, then $m_1 = m_2$.

(23) $(m_1 + m_2) + m_3 = m_1 + (m_2 + m_3)$.

(24) If $m_1 \subseteq m_2$ and $m_3 \subseteq m_4$, then $m_1 + m_3 \subseteq m_2 + m_4$.

(25) If $m_1 \subseteq m_2$, then $m_2 - m_1 \subseteq m_2$.

(26) If $m_1 \subseteq m_2$ and $m_3 \subseteq m_4$ and $m_4 \subseteq m_1$, then $m_1 - m_4 \subseteq m_2 - m_3$.

(27) If $m_1 \subseteq m_2$, then $m_2 = (m_2 - m_1) + m_1$.

(28) $(m_1 + m_2) - m_1 = m_2$.

(29) If $m_2 + m_3 \subseteq m_1$, then $m_1 - m_2 - m_3 = m_1 - (m_2 + m_3)$.

(30) If $m_3 \subseteq m_2$ and $m_2 \subseteq m_1$, then $m_1 - (m_2 - m_3) = (m_1 - m_2) + m_3$.

(31) $m \in \mathbb{N}^P$.

(32) If $x \in \mathbb{N}^P$, then x is a marking of P.

3. Transitions and Firing

Let us consider P. Transition of P is defined by:

(Def. 7) There exist m_1, m_2 such that it $= \langle m_1, m_2 \rangle$.

In the sequel t, t_1, t_2 denote transitions of P.

Let us consider P, t. Then $t_{\mathbf{1}}$ is a marking of P. We introduce Pre t as a synonym of $t_{\mathbf{1}}$. $t_{\mathbf{2}}$ is a marking of P. We introduce Post t as a synonym of $t_{\mathbf{2}}$.

Let us consider P, m, t. The functor $\mathrm{fire}(t, m)$ yielding a marking of P is defined by:

(Def. 8) $\mathrm{fire}(t, m) = \begin{cases} (m - \mathrm{Pre}\ t) + \mathrm{Post}\ t, & \text{if } \mathrm{Pre}\ t \subseteq m, \\ m, & \text{otherwise.} \end{cases}$

The following proposition is true

(33) If $\mathrm{Pre}\ t_1 + \mathrm{Pre}\ t_2 \subseteq m$, then $\mathrm{fire}(t_2, \mathrm{fire}(t_1, m)) = (m - \mathrm{Pre}\ t_1 - \mathrm{Pre}\ t_2) + \mathrm{Post}\ t_1 + \mathrm{Post}\ t_2$.

Let us consider P, t. The functor fire t yielding a function is defined by:

(Def. 9) $\mathrm{dom}\,\mathrm{fire}\ t = \mathbb{N}^P$ and for every marking m of P holds $(\mathrm{fire}\ t)(m) = \mathrm{fire}(t, m)$.

Next we state two propositions:

(34) $\mathrm{rng}\,\mathrm{fire}\ t \subseteq \mathbb{N}^P$.

(35) $\mathrm{fire}(t_2, \mathrm{fire}(t_1, m)) = (\mathrm{fire}\ t_2 \cdot \mathrm{fire}\ t_1)(m)$.

Let us consider P. A non empty set is called a Petri net over P if:

(Def. 10) For every set x such that $x \in$ it holds x is a transition of P.

In the sequel N denotes a Petri net over P.

Let us consider P, N. We see that the element of N is a transition of P.

In the sequel e, e_1, e_2 denote elements of N.

4. Firing Sequences of Transitions

Let us consider P, N. A firing-sequence of N is an element of N^*.

In the sequel C, C_1, C_2 are firing-sequences of N.

Let us consider P, N, C. The functor fire C yielding a function is defined by the condition (Def. 11).

(Def. 11) There exists a function yielding finite sequence F such that fire $C = \mathrm{compose}_{\mathbb{N}^P} F$ and $\mathrm{len}\, F = \mathrm{len}\, C$ and for every natural number i such that $i \in \mathrm{dom}\, C$ holds $F(i) = \mathrm{fire}\,(C_i$ **qua** element of $N)$.

The following propositions are true:

(36) $\operatorname{fire}(\varepsilon_N) = \mathrm{id}_{\mathbb{N}^P}$.

(37) $\operatorname{fire}\langle e\rangle = \operatorname{fire} e$.

(38) $\operatorname{fire} e \cdot \mathrm{id}_{\mathbb{N}^P} = \operatorname{fire} e$.

(39) $\operatorname{fire}\langle e_1, e_2\rangle = \operatorname{fire} e_2 \cdot \operatorname{fire} e_1$.

(40) $\operatorname{dom}\operatorname{fire} C = \mathbb{N}^P$ and $\operatorname{rng}\operatorname{fire} C \subseteq \mathbb{N}^P$.

(41) $\operatorname{fire}(C_1 \frown C_2) = \operatorname{fire} C_2 \cdot \operatorname{fire} C_1$.

(42) $\operatorname{fire}(C \frown \langle e\rangle) = \operatorname{fire} e \cdot \operatorname{fire} C$.

Let us consider P, N, C, m. The functor $\operatorname{fire}(C, m)$ yielding a marking of P is defined as follows:

(Def. 12) $\operatorname{fire}(C, m) = (\operatorname{fire} C)(m)$.

5. Sequential Composition

Let us consider P, N. A process in N is a subset of N^*.

In the sequel R, R_1, R_2, R_3, P_1, P_2 denote processes in N.

One can verify that every function which is finite sequence-like is also finite subsequence-like.

Let us consider P, N, R_1, R_2. The functor $R_1 \operatorname{before} R_2$ yields a process in N and is defined by:

(Def. 13) $R_1 \operatorname{before} R_2 = \{C_1 \frown C_2 : C_1 \in R_1 \ \wedge\ C_2 \in R_2\}$.

Let us consider P, N and let R_1, R_2 be non empty processes in N. One can verify that $R_1 \operatorname{before} R_2$ is non empty.

One can prove the following propositions:

(43) $(R_1 \cup R_2) \operatorname{before} R = (R_1 \operatorname{before} R) \cup (R_2 \operatorname{before} R)$.

(44) $R \operatorname{before}(R_1 \cup R_2) = (R \operatorname{before} R_1) \cup (R \operatorname{before} R_2)$.

(45) $\{C_1\} \operatorname{before}\{C_2\} = \{C_1 \frown C_2\}$.

(46) $\{C_1, C_2\} \operatorname{before}\{C\} = \{C_1 \frown C, C_2 \frown C\}$.

(47) $\{C\} \operatorname{before}\{C_1, C_2\} = \{C \frown C_1, C \frown C_2\}$.

6. Concurrent Composition

Let us consider P, N, R_1, R_2. The functor $R_1 \operatorname{concur} R_2$ yielding a process in N is defined as follows:

(Def. 14) $R_1 \operatorname{concur} R_2 = \{C : \bigvee_{q_1, q_2 : \text{finite subsequence}} (C = q_1 \cup q_2 \ \wedge\ q_1 \text{ misses } q_2 \ \wedge\ \operatorname{Seq} q_1 \in R_1 \ \wedge\ \operatorname{Seq} q_2 \in R_2)\}$.

Let us observe that the functor $R_1 \operatorname{concur} R_2$ is commutative.

Next we state four propositions:

(48) $(R_1 \cup R_2)$ concur $R = (R_1$ concur $R) \cup (R_2$ concur $R)$.

(49) $\{\langle e_1 \rangle\}$ concur $\{\langle e_2 \rangle\} = \{\langle e_1, e_2 \rangle, \langle e_2, e_1 \rangle\}$.

(50) $\{\langle e_1 \rangle, \langle e_2 \rangle\}$ concur $\{\langle e \rangle\} = \{\langle e_1, e \rangle, \langle e_2, e \rangle, \langle e, e_1 \rangle, \langle e, e_2 \rangle\}$.

(51) $(R_1$ before $R_2)$ before $R_3 = R_1$ before$(R_2$ before $R_3)$.

Let p be a finite subsequence and let i be a natural number. The functor $\mathrm{Shift}^i\, p$ yielding a finite subsequence is defined as follows:

(Def. 15) $\mathrm{dom}\,\mathrm{Shift}^i\, p = \{i + k; k$ ranges over natural numbers: $k \in \mathrm{dom}\, p\}$ and for every natural number j such that $j \in \mathrm{dom}\, p$ holds $(\mathrm{Shift}^i\, p)(i + j) = p(j)$.

In the sequel q, q_1, q_2 denote finite subsequences.

One can prove the following propositions:

(52) $\mathrm{Shift}^0\, q = q$.

(53) $\mathrm{Shift}^{i+j}\, q = \mathrm{Shift}^i\, \mathrm{Shift}^j\, q$.

(54) For every finite sequence p such that $p \neq \emptyset$ holds $\mathrm{dom}\,\mathrm{Shift}^i\, p = \{j_1 : i + 1 \leqslant j_1 \ \wedge \ j_1 \leqslant i + \mathrm{len}\, p\}$.

(55) For every finite subsequence q holds $q = \emptyset$ iff $\mathrm{Shift}^i\, q = \emptyset$.

(56) Let q be a finite subsequence. Then there exists a finite subsequence s_1 such that $\mathrm{dom}\, s_1 = \mathrm{dom}\, q$ and $\mathrm{rng}\, s_1 = \mathrm{dom}\,\mathrm{Shift}^i\, q$ and for every k such that $k \in \mathrm{dom}\, q$ holds $s_1(k) = i + k$ and s_1 is one-to-one.

(57) For every finite subsequence q holds $\overline{\overline{q}} = \overline{\overline{\mathrm{Shift}^i\, q}}$.

(58) For every finite sequence p holds $\mathrm{dom}\, p = \mathrm{dom}\,\mathrm{Seq}\,\mathrm{Shift}^i\, p$.

(59) For every finite sequence p such that $k \in \mathrm{dom}\, p$ holds $(\mathrm{Sgm}\,\mathrm{dom}\,\mathrm{Shift}^i\, p)(k) = i + k$.

(60) For every finite sequence p such that $k \in \mathrm{dom}\, p$ holds $(\mathrm{Seq}\,\mathrm{Shift}^i\, p)(k) = p(k)$.

(61) For every finite sequence p holds $\mathrm{Seq}\,\mathrm{Shift}^i\, p = p$.

In the sequel p_1, p_2 are finite sequences.

One can prove the following propositions:

(62) $\mathrm{dom}(p_1 \cup \mathrm{Shift}^{\mathrm{len}\, p_1}\, p_2) = \mathrm{Seg}(\mathrm{len}\, p_1 + \mathrm{len}\, p_2)$.

(63) For every finite sequence p_1 and for every finite subsequence p_2 such that $\mathrm{len}\, p_1 \leqslant i$ holds $\mathrm{dom}\, p_1$ misses $\mathrm{dom}\,\mathrm{Shift}^i\, p_2$.

(64) For all finite sequences p_1, p_2 holds $p_1 \frown p_2 = p_1 \cup \mathrm{Shift}^{\mathrm{len}\, p_1}\, p_2$.

(65) For every finite sequence p_1 and for every finite subsequence p_2 such that $i \geqslant \mathrm{len}\, p_1$ holds p_1 misses $\mathrm{Shift}^i\, p_2$.

(66) $(R_1$ concur $R_2)$ concur $R_3 = R_1$ concur$(R_2$ concur $R_3)$.

(67) R_1 before $R_2 \subseteq R_1$ concur R_2.

(68) If $R_1 \subseteq P_1$ and $R_2 \subseteq P_2$, then R_1 before $R_2 \subseteq P_1$ before P_2.

(69) If $R_1 \subseteq P_1$ and $R_2 \subseteq P_2$, then R_1 concur $R_2 \subseteq P_1$ concur P_2.

(70) For all finite subsequences p, q such that $q \subseteq p$ holds $\mathrm{Shift}^i\, q \subseteq \mathrm{Shift}^i\, p$.

(71) For all finite sequences p_1, p_2 holds $\mathrm{Shift}^{\mathrm{len}\, p_1} p_2 \subseteq p_1 ^\frown p_2$.

(72) If $\mathrm{dom}\, q_1$ misses $\mathrm{dom}\, q_2$, then $\mathrm{dom}\,\mathrm{Shift}^i q_1$ misses $\mathrm{dom}\,\mathrm{Shift}^i q_2$.

(73) For all finite subsequences q, q_1, q_2 such that $q = q_1 \cup q_2$ and q_1 misses q_2 holds $\mathrm{Shift}^i q_1 \cup \mathrm{Shift}^i q_2 = \mathrm{Shift}^i q$.

(74) For every finite subsequence q holds $\mathrm{dom}\,\mathrm{Seq}\, q = \mathrm{dom}\,\mathrm{Seq}\,\mathrm{Shift}^i q$.

(75) For every finite subsequence q such that $k \in \mathrm{dom}\,\mathrm{Seq}\, q$ there exists j such that $j = (\mathrm{Sgm}\,\mathrm{dom}\, q)(k)$ and $(\mathrm{Sgm}\,\mathrm{dom}\,\mathrm{Shift}^i q)(k) = i + j$.

(76) For every finite subsequence q such that $k \in \mathrm{dom}\,\mathrm{Seq}\, q$ holds $(\mathrm{Seq}\,\mathrm{Shift}^i q)(k) = (\mathrm{Seq}\, q)(k)$.

(77) For every finite subsequence q holds $\mathrm{Seq}\, q = \mathrm{Seq}\,\mathrm{Shift}^i q$.

(78) For every finite subsequence q such that $\mathrm{dom}\, q \subseteq \mathrm{Seg}\, k$ holds $\mathrm{dom}\,\mathrm{Shift}^i q \subseteq \mathrm{Seg}(i + k)$.

(79) Let p be a finite sequence and q_1, q_2 be finite subsequences. If $q_1 \subseteq p$, then there exists a finite subsequence s_1 such that $s_1 = q_1 \cup \mathrm{Shift}^{\mathrm{len}\, p} q_2$.

(80) Let p_1, p_2 be finite sequences and q_1, q_2 be finite subsequences. Suppose $q_1 \subseteq p_1$ and $q_2 \subseteq p_2$. Then there exists a finite subsequence s_1 such that $s_1 = q_1 \cup \mathrm{Shift}^{\mathrm{len}\, p_1} q_2$ and $\mathrm{dom}\,\mathrm{Seq}\, s_1 = \mathrm{Seg}(\mathrm{len}\,\mathrm{Seq}\, q_1 + \mathrm{len}\,\mathrm{Seq}\, q_2)$.

(81) Let p_1, p_2 be finite sequences and q_1, q_2 be finite subsequences. Suppose $q_1 \subseteq p_1$ and $q_2 \subseteq p_2$. Then there exists a finite subsequence s_1 such that $s_1 = q_1 \cup \mathrm{Shift}^{\mathrm{len}\, p_1} q_2$ and $\mathrm{dom}\,\mathrm{Seq}\, s_1 = \mathrm{Seg}(\mathrm{len}\,\mathrm{Seq}\, q_1 + \mathrm{len}\,\mathrm{Seq}\, q_2)$ and $\mathrm{Seq}\, s_1 = \mathrm{Seq}\, q_1 \cup \mathrm{Shift}^{\mathrm{len}\,\mathrm{Seq}\, q_1} \mathrm{Seq}\, q_2$.

(82) Let p_1, p_2 be finite sequences and q_1, q_2 be finite subsequences. Suppose $q_1 \subseteq p_1$ and $q_2 \subseteq p_2$. Then there exists a finite subsequence s_1 such that $s_1 = q_1 \cup \mathrm{Shift}^{\mathrm{len}\, p_1} q_2$ and $(\mathrm{Seq}\, q_1) ^\frown (\mathrm{Seq}\, q_2) = \mathrm{Seq}\, s_1$.

(83) $(R_1 \,\mathrm{concur}\, R_2)\,\mathrm{before}(P_1 \,\mathrm{concur}\, P_2) \subseteq (R_1 \,\mathrm{before}\, P_1)\,\mathrm{concur}(R_2 \,\mathrm{before}\, P_2)$.

Let us consider P, N and let R_1, R_2 be non empty processes in N. Note that $R_1 \,\mathrm{concur}\, R_2$ is non empty.

7. Neutral Process

Let us consider P and let N be a Petri net over P. The neutral process in N yields a non empty process in N and is defined as follows:

(Def. 16) The neutral process in $N = \{\varepsilon_N\}$.

Let us consider P, let N be a Petri net over P, and let t be an element of N. The elementary process with t yielding a non empty process in N is defined as follows:

(Def. 17) The elementary process with $t = \{\langle t \rangle\}$.

One can prove the following propositions:

(84) (The neutral process in N) $\mathrm{before}\, R = R$.

(85) R before the neutral process in $N = R$.

(86) (The neutral process in N) concur $R = R$.

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Received December 20, 2002