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Propositional Calculus for Boolean Valued Functions. Part VII

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Summary. In this paper, we proved some elementary propositional calculus formulae for Boolean valued functions.

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The notation and terminology used in this paper have been introduced in the following articles: [4], [3], [2], and [1].

We use the following convention: Y is a non empty set and a, b, c, d are elements of $\mathit{Boolean}^Y$.

The following propositions are true:

(1) $\neg(a \Rightarrow b) = a \wedge \neg b$.

(2) $\neg b \Rightarrow \neg a \Rightarrow a \Rightarrow b = \mathit{true}(Y)$.

(3) $a \Rightarrow b = \neg b \Rightarrow \neg a$.

(4) $a \Leftrightarrow b = \neg a \Leftrightarrow \neg b$.

(5) $a \Rightarrow b = a \Rightarrow a \wedge b$.

(6) $a \Leftrightarrow b = a \vee b \Rightarrow a \wedge b$.

(7) $a \Leftrightarrow \neg a = \mathit{false}(Y)$.

(8) $a \Rightarrow b \Rightarrow c = b \Rightarrow a \Rightarrow c$.

(9) $a \Rightarrow b \Rightarrow c = a \Rightarrow b \Rightarrow a \Rightarrow c$.

(10) $a \Leftrightarrow b = a \oplus \neg b$.

(11) $a \wedge (b \oplus c) = a \wedge b \oplus a \wedge c$.

(12) $a \Leftrightarrow b = \neg(a \oplus b)$.

(13) $a \oplus a = \mathit{false}(Y)$.

(14) $a \oplus \neg a = \mathit{true}(Y)$.

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(15) $a \Rightarrow b \Rightarrow b \Rightarrow a = b \Rightarrow a$.

(16) $(a \vee b) \wedge (\neg a \vee \neg b) = \neg a \wedge b \vee a \wedge \neg b$.

(17) $a \wedge b \vee \neg a \wedge \neg b = (\neg a \vee b) \wedge (a \vee \neg b)$.

(18) $a \oplus (b \oplus c) = (a \oplus b) \oplus c$.

(19) $a \Leftrightarrow b \Leftrightarrow c = a \Leftrightarrow b \Leftrightarrow c$.

(20) $\neg\neg a \Rightarrow a = \mathit{true}(Y)$.

(21) $(a \Rightarrow b) \wedge a \Rightarrow b = \mathit{true}(Y)$.

(22) $a \Rightarrow \neg a \Rightarrow a = \mathit{true}(Y)$.

(23) $\neg a \Rightarrow a \Leftrightarrow a = \mathit{true}(Y)$.

(24) $a \vee (a \Rightarrow b) = \mathit{true}(Y)$.

(25) $(a \Rightarrow b) \vee (c \Rightarrow a) = \mathit{true}(Y)$.

(26) $(a \Rightarrow b) \vee (\neg a \Rightarrow b) = \mathit{true}(Y)$.

(27) $(a \Rightarrow b) \vee (a \Rightarrow \neg b) = \mathit{true}(Y)$.

(28) $\neg a \Rightarrow \neg b \Leftrightarrow b \Rightarrow a = \mathit{true}(Y)$.

(29) $a \Rightarrow b \Rightarrow a \Rightarrow c \Rightarrow b \Rightarrow b = \mathit{true}(Y)$.

(30) $a \Rightarrow b = a \Leftrightarrow a \wedge b$.

(31) $a \Rightarrow b = \mathit{true}(Y)$ and $b \Rightarrow a = \mathit{true}(Y)$ iff $a = b$.

(32) $a = \neg a \Rightarrow a$.

(33) $a \Rightarrow a \Rightarrow b \Rightarrow a = \mathit{true}(Y)$.

(34) $a = a \Rightarrow b \Rightarrow a$.

(35) $a = (b \Rightarrow a) \wedge (\neg b \Rightarrow a)$.

(36) $a \wedge b = \neg(a \Rightarrow \neg b)$.

(37) $a \vee b = \neg a \Rightarrow b$.

(38) $a \vee b = a \Rightarrow b \Rightarrow b$.

(39) $a \Rightarrow b \Rightarrow a \Rightarrow a = \mathit{true}(Y)$.

(40) $a \Rightarrow b \Rightarrow c \Rightarrow d \Rightarrow b \Rightarrow a \Rightarrow d \Rightarrow c = \mathit{true}(Y)$.

(41) $(a \Rightarrow b) \wedge a \wedge c \Rightarrow b = \mathit{true}(Y)$.

(42) $b \Rightarrow c \Rightarrow a \wedge b \Rightarrow c = \mathit{true}(Y)$.

(43) $a \wedge b \Rightarrow c \Rightarrow a \wedge b \Rightarrow c \wedge b = \mathit{true}(Y)$.

(44) $a \Rightarrow b \Rightarrow a \wedge c \Rightarrow b \wedge c = \mathit{true}(Y)$.

(45) $(a \Rightarrow b) \wedge (a \wedge c) \Rightarrow b \wedge c = \mathit{true}(Y)$.

(46) $a \wedge (a \Rightarrow b) \wedge (b \Rightarrow c) \Subset c$.

(47) $(a \vee b) \wedge (a \Rightarrow c) \wedge (b \Rightarrow c) \Subset \neg a \Rightarrow b \vee c$.

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References

[1] Shunichi Kobayashi and Kui Jia. A theory of Boolean valued functions and partitions. *Formalized Mathematics*, 7(**2**):249–254, 1998.
[2] Andrzej Trybulec. Function domains and Frænkel operator. *Formalized Mathematics*, 1(**3**):495–500, 1990.
[3] Edmund Woronowicz. Interpretation and satisfiability in the first order logic. *Formalized Mathematics*, 1(**4**):739–743, 1990.
[4] Edmund Woronowicz. Many–argument relations. *Formalized Mathematics*, 1(**4**):733–737, 1990.

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