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On the Sets Inhabited by Numbers[1]

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Summary. The information that all members of a set enjoy a property expressed by an adjective can be processed in a systematic way. The purpose of the work is to find out how to do that. If it works, 'membered' will become a reserved word and the work with it will be automated. I have chosen *membered* rather than *inhabited* because of the compatibility with the Automath terminology. The phrase τ *inhabits* θ could be translated to τ **is** θ in Mizar.

MML Identifier: `MEMBERED`.

The articles [6], [8], [4], [5], [3], [7], [1], and [2] provide the notation and terminology for this paper.

In this paper x, X, F denote sets.

Let X be a set. We say that X is complex-membered if and only if:

(Def. 1) If $x \in X$, then x is complex.

We say that X is real-membered if and only if:

(Def. 2) If $x \in X$, then x is real.

We say that X is rational-membered if and only if:

(Def. 3) If $x \in X$, then x is rational.

We say that X is integer-membered if and only if:

(Def. 4) If $x \in X$, then x is integer.

We say that X is natural-membered if and only if:

(Def. 5) If $x \in X$, then x is natural.

One can check the following observations:

* every set which is natural-membered is also integer-membered,
* every set which is integer-membered is also rational-membered,

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* every set which is rational-membered is also real-membered, and
* every set which is real-membered is also complex-membered.

Let us observe that there exists a set which is non empty and natural-membered.

One can verify the following observations:

* every subset of $\mathbb{C}$ is complex-membered,
* every subset of $\mathbb{R}$ is real-membered,
* every subset of $\mathbb{Q}$ is rational-membered,
* every subset of $\mathbb{Z}$ is integer-membered, and
* every subset of $\mathbb{N}$ is natural-membered.

One can verify the following observations:

* $\mathbb{C}$ is complex-membered,
* $\mathbb{R}$ is real-membered,
* $\mathbb{Q}$ is rational-membered,
* $\mathbb{Z}$ is integer-membered, and
* $\mathbb{N}$ is natural-membered.

Next we state several propositions:

(1) If X is complex-membered, then $X \subseteq \mathbb{C}$.

(2) If X is real-membered, then $X \subseteq \mathbb{R}$.

(3) If X is rational-membered, then $X \subseteq \mathbb{Q}$.

(4) If X is integer-membered, then $X \subseteq \mathbb{Z}$.

(5) If X is natural-membered, then $X \subseteq \mathbb{N}$.

Let X be a complex-membered set. One can check that every element of X is complex.

Let X be a real-membered set. One can verify that every element of X is real.

Let X be a rational-membered set. Note that every element of X is rational.

Let X be an integer-membered set. One can verify that every element of X is integer.

Let X be a natural-membered set. Observe that every element of X is natural.

For simplicity, we follow the rules: c, c_1, c_2, c_3 are complex numbers, r, r_1, r_2, r_3 are real numbers, w, w_1, w_2, w_3 are rational numbers, i, i_1, i_2, i_3 are integer numbers, and n, n_1, n_2, n_3 are natural numbers.

We now state a number of propositions:

(6) For every non empty complex-membered set X there exists c such that $c \in X$.

(7) For every non empty real-membered set X there exists r such that $r \in X$.

(8) For every non empty rational-membered set X there exists w such that $w \in X$.

(9) For every non empty integer-membered set X there exists i such that $i \in X$.

(10) For every non empty natural-membered set X there exists n such that $n \in X$.

(11) For every complex-membered set X such that for every c holds $c \in X$ holds $X = \mathbb{C}$.

(12) For every real-membered set X such that for every r holds $r \in X$ holds $X = \mathbb{R}$.

(13) For every rational-membered set X such that for every w holds $w \in X$ holds $X = \mathbb{Q}$.

(14) For every integer-membered set X such that for every i holds $i \in X$ holds $X = \mathbb{Z}$.

(15) For every natural-membered set X such that for every n holds $n \in X$ holds $X = \mathbb{N}$.

(16) For every complex-membered set Y such that $X \subseteq Y$ holds X is complex-membered.

(17) For every real-membered set Y such that $X \subseteq Y$ holds X is real-membered.

(18) For every rational-membered set Y such that $X \subseteq Y$ holds X is rational-membered.

(19) For every integer-membered set Y such that $X \subseteq Y$ holds X is integer-membered.

(20) For every natural-membered set Y such that $X \subseteq Y$ holds X is natural-membered.

One can verify that $\emptyset$ is natural-membered.

One can verify that every set which is empty is also natural-membered.

Let us consider c. One can verify that $\{c\}$ is complex-membered.

Let us consider r. One can verify that $\{r\}$ is real-membered.

Let us consider w. One can check that $\{w\}$ is rational-membered.

Let us consider i. One can verify that $\{i\}$ is integer-membered.

Let us consider n. Observe that $\{n\}$ is natural-membered.

Let us consider c_1, c_2. Note that $\{c_1, c_2\}$ is complex-membered.

Let us consider r_1, r_2. One can check that $\{r_1, r_2\}$ is real-membered.

Let us consider w_1, w_2. Observe that $\{w_1, w_2\}$ is rational-membered.

Let us consider i_1, i_2. One can verify that $\{i_1, i_2\}$ is integer-membered.

Let us consider n_1, n_2. Observe that $\{n_1, n_2\}$ is natural-membered.

Let us consider c_1, c_2, c_3. One can verify that $\{c_1, c_2, c_3\}$ is complex-membered.

Let us consider r_1, r_2, r_3. One can verify that $\{r_1, r_2, r_3\}$ is real-membered.

Let us consider w_1, w_2, w_3. Observe that $\{w_1, w_2, w_3\}$ is rational-membered.

Let us consider i_1, i_2, i_3. One can verify that $\{i_1, i_2, i_3\}$ is integer-membered.

Let us consider n_1, n_2, n_3. One can check that $\{n_1, n_2, n_3\}$ is natural-membered.

Let X be a complex-membered set. Note that every subset of X is complex-membered.

Let X be a real-membered set. One can verify that every subset of X is real-membered.

Let X be a rational-membered set. One can check that every subset of X is rational-membered.

Let X be an integer-membered set. Observe that every subset of X is integer-membered.

Let X be a natural-membered set. One can verify that every subset of X is natural-membered.

Let X, Y be complex-membered sets. Note that $X \cup Y$ is complex-membered.

Let X, Y be real-membered sets. Observe that $X \cup Y$ is real-membered.

Let X, Y be rational-membered sets. Note that $X \cup Y$ is rational-membered.

Let X, Y be integer-membered sets. Note that $X \cup Y$ is integer-membered.

Let X, Y be natural-membered sets. Observe that $X \cup Y$ is natural-membered.

Let X be a complex-membered set and let Y be a set. Note that $X \cap Y$ is complex-membered and $Y \cap X$ is complex-membered.

Let X be a real-membered set and let Y be a set. Note that $X \cap Y$ is real-membered and $Y \cap X$ is real-membered.

Let X be a rational-membered set and let Y be a set. Observe that $X \cap Y$ is rational-membered and $Y \cap X$ is rational-membered.

Let X be an integer-membered set and let Y be a set. Note that $X \cap Y$ is integer-membered and $Y \cap X$ is integer-membered.

Let X be a natural-membered set and let Y be a set. Observe that $X \cap Y$ is natural-membered and $Y \cap X$ is natural-membered.

Let X be a complex-membered set and let Y be a set. Note that $X \setminus Y$ is complex-membered.

Let X be a real-membered set and let Y be a set. Note that $X \setminus Y$ is real-membered.

Let X be a rational-membered set and let Y be a set. Observe that $X \setminus Y$ is rational-membered.

Let X be an integer-membered set and let Y be a set. Observe that $X \setminus Y$ is integer-membered.

Let X be a natural-membered set and let Y be a set. Observe that $X \setminus Y$ is natural-membered.

Let X, Y be complex-membered sets. Note that $X \mathbin{\dot{-}} Y$ is complex-membered.

Let X, Y be real-membered sets. One can check that $X \mathbin{\dot{-}} Y$ is real-membered.

Let X, Y be rational-membered sets. Note that $X \mathbin{\dot{-}} Y$ is rational-membered.

Let X, Y be integer-membered sets. One can check that $X \dot{-} Y$ is integer-membered.

Let X, Y be natural-membered sets. One can verify that $X \dot{-} Y$ is natural-membered.

Let X, Y be complex-membered sets. Let us observe that $X \subseteq Y$ if and only if:

(Def. 6) If $c \in X$, then $c \in Y$.

Let X, Y be real-membered sets. Let us observe that $X \subseteq Y$ if and only if:

(Def. 7) If $r \in X$, then $r \in Y$.

Let X, Y be rational-membered sets. Let us observe that $X \subseteq Y$ if and only if:

(Def. 8) If $w \in X$, then $w \in Y$.

Let X, Y be integer-membered sets. Let us observe that $X \subseteq Y$ if and only if:

(Def. 9) If $i \in X$, then $i \in Y$.

Let X, Y be natural-membered sets. Let us observe that $X \subseteq Y$ if and only if:

(Def. 10) If $n \in X$, then $n \in Y$.

Let X, Y be complex-membered sets. Let us observe that $X = Y$ if and only if:

(Def. 11) $c \in X$ iff $c \in Y$.

Let X, Y be real-membered sets. Let us observe that $X = Y$ if and only if:

(Def. 12) $r \in X$ iff $r \in Y$.

Let X, Y be rational-membered sets. Let us observe that $X = Y$ if and only if:

(Def. 13) $w \in X$ iff $w \in Y$.

Let X, Y be integer-membered sets. Let us observe that $X = Y$ if and only if:

(Def. 14) $i \in X$ iff $i \in Y$.

Let X, Y be natural-membered sets. Let us observe that $X = Y$ if and only if:

(Def. 15) $n \in X$ iff $n \in Y$.

Let X, Y be complex-membered sets. Let us observe that X meets Y if and only if:

(Def. 16) There exists c such that $c \in X$ and $c \in Y$.

Let X, Y be real-membered sets. Let us observe that X meets Y if and only if:

(Def. 17) There exists r such that $r \in X$ and $r \in Y$.

Let X, Y be rational-membered sets. Let us observe that X meets Y if and only if:

(Def. 18) There exists w such that $w \in X$ and $w \in Y$.

Let X, Y be integer-membered sets. Let us observe that X meets Y if and only if:

(Def. 19) There exists i such that $i \in X$ and $i \in Y$.

Let X, Y be natural-membered sets. Let us observe that X meets Y if and only if:

(Def. 20) There exists n such that $n \in X$ and $n \in Y$.

One can prove the following propositions:

(21) If for every X such that $X \in F$ holds X is complex-membered, then $\bigcup F$ is complex-membered.

(22) If for every X such that $X \in F$ holds X is real-membered, then $\bigcup F$ is real-membered.

(23) If for every X such that $X \in F$ holds X is rational-membered, then $\bigcup F$ is rational-membered.

(24) If for every X such that $X \in F$ holds X is integer-membered, then $\bigcup F$ is integer-membered.

(25) If for every X such that $X \in F$ holds X is natural-membered, then $\bigcup F$ is natural-membered.

(26) For every X such that $X \in F$ and X is complex-membered holds $\bigcap F$ is complex-membered.

(27) For every X such that $X \in F$ and X is real-membered holds $\bigcap F$ is real-membered.

(28) For every X such that $X \in F$ and X is rational-membered holds $\bigcap F$ is rational-membered.

(29) For every X such that $X \in F$ and X is integer-membered holds $\bigcap F$ is integer-membered.

(30) For every X such that $X \in F$ and X is natural-membered holds $\bigcap F$ is natural-membered.

In this article we present several logical schemes. The scheme *CM Separation* concerns a unary predicate $\mathcal{P}$, and states that:

> There exists a complex-membered set X such that for every c holds $c \in X$ iff $\mathcal{P}[c]$

for all values of the parameters.

The scheme *RM Separation* concerns a unary predicate $\mathcal{P}$, and states that:

> There exists a real-membered set X such that for every r holds $r \in X$ iff $\mathcal{P}[r]$

for all values of the parameters.

The scheme *WM Separation* concerns a unary predicate $\mathcal{P}$, and states that:

There exists a rational-membered set X such that for every w holds $w \in X$ iff $\mathcal{P}[w]$

for all values of the parameters.

The scheme *IM Separation* concerns a unary predicate $\mathcal{P}$, and states that:

There exists an integer-membered set X such that for every i holds $i \in X$ iff $\mathcal{P}[i]$

for all values of the parameters.

The scheme *NM Separation* concerns a unary predicate $\mathcal{P}$, and states that:

There exists a natural-membered set X such that for every n holds $n \in X$ iff $\mathcal{P}[n]$

for all values of the parameters.

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