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Formulas and Identities of Trigonometric Functions

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Summary. In this article, we concentrated especially on addition formulas of fundamental trigonometric functions, and their identities.

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The articles [1] and [2] provide the notation and terminology for this paper.

In this paper t_1, t_2, t_3, t_4 denote real numbers.

Let us consider t_1. The functor $\tan t_1$ yielding a real number is defined by:

(Def. 1) $\tan t_1 = \frac{\sin t_1}{\cos t_1}$.

Let us consider t_1. The functor $\cot t_1$ yields a real number and is defined by:

(Def. 2) $\cot t_1 = \frac{\cos t_1}{\sin t_1}$.

Let us consider t_1. The functor $\operatorname{cosec} t_1$ yielding a real number is defined as follows:

(Def. 3) $\operatorname{cosec} t_1 = \frac{1}{\sin t_1}$.

Let us consider t_1. The functor $\sec t_1$ yielding a real number is defined by:

(Def. 4) $\sec t_1 = \frac{1}{\cos t_1}$.

Next we state a number of propositions:

(1) $\tan t_1 = \frac{1}{\cot t_1}$.

(2) $\tan(-t_1) = -\tan t_1$.

(3) $\operatorname{cosec}(-t_1) = -\frac{1}{\sin t_1}$.

(4) $\cot(-t_1) = -\cot t_1$.

(5) If $\cos t_2 \neq 0$, then $\cos t_2 \cdot \sec t_2 = 1$.

(6) $\sin t_1 \cdot \sin t_1 = 1 - \cos t_1 \cdot \cos t_1$.

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(7) $\cos t_1 \cdot \cos t_1 = 1 - \sin t_1 \cdot \sin t_1$.

(8) If $\cos t_1 \neq 0$, then $\sin t_1 = \cos t_1 \cdot \tan t_1$.

(9) $\sin(t_2 - t_3) = \sin t_2 \cdot \cos t_3 - \cos t_2 \cdot \sin t_3$.

(10) $\cos(t_2 - t_3) = \cos t_2 \cdot \cos t_3 + \sin t_2 \cdot \sin t_3$.

(11) If $\cos t_2 \neq 0$ and $\cos t_3 \neq 0$, then $\tan(t_2 + t_3) = \frac{\tan t_2 + \tan t_3}{1 - \tan t_2 \cdot \tan t_3}$.

(12) If $\cos t_2 \neq 0$ and $\cos t_3 \neq 0$, then $\tan(t_2 - t_3) = \frac{\tan t_2 - \tan t_3}{1 + \tan t_2 \cdot \tan t_3}$.

(13) If $\sin t_2 \neq 0$ and $\sin t_3 \neq 0$, then $\cot(t_2 + t_3) = \frac{\cot t_2 \cdot \cot t_3 - 1}{\cot t_3 + \cot t_2}$.

(14) If $\sin t_2 \neq 0$ and $\sin t_3 \neq 0$, then $\cot(t_2 - t_3) = \frac{\cot t_2 \cdot \cot t_3 + 1}{\cot t_3 - \cot t_2}$.

(15) If $\cos t_2 \neq 0$ and $\cos t_3 \neq 0$ and $\cos t_4 \neq 0$, then $\sin(t_2 + t_3 + t_4) = \cos t_2 \cdot \cos t_3 \cdot \cos t_4 \cdot ((\tan t_2 + \tan t_3 + \tan t_4) - \tan t_2 \cdot \tan t_3 \cdot \tan t_4)$.

(16) If $\cos t_2 \neq 0$ and $\cos t_3 \neq 0$ and $\cos t_4 \neq 0$, then $\cos(t_2 + t_3 + t_4) = \cos t_2 \cdot \cos t_3 \cdot \cos t_4 \cdot (1 - \tan t_3 \cdot \tan t_4 - \tan t_4 \cdot \tan t_2 - \tan t_2 \cdot \tan t_3)$.

(17) If $\cos t_2 \neq 0$ and $\cos t_3 \neq 0$ and $\cos t_4 \neq 0$, then $\tan(t_2 + t_3 + t_4) = \frac{(\tan t_2 + \tan t_3 + \tan t_4) - \tan t_2 \cdot \tan t_3 \cdot \tan t_4}{1 - \tan t_3 \cdot \tan t_4 - \tan t_4 \cdot \tan t_2 - \tan t_2 \cdot \tan t_3}$.

(18) If $\sin t_2 \neq 0$ and $\sin t_3 \neq 0$ and $\sin t_4 \neq 0$, then $\cot(t_2 + t_3 + t_4) = \frac{\cot t_2 \cdot \cot t_3 \cdot \cot t_4 - \cot t_2 - \cot t_3 - \cot t_4}{(\cot t_3 \cdot \cot t_4 + \cot t_4 \cdot \cot t_2 + \cot t_2 \cdot \cot t_3) - 1}$.

(19) $\sin t_2 + \sin t_3 = 2 \cdot (\cos(\frac{t_2 - t_3}{2}) \cdot \sin(\frac{t_2 + t_3}{2}))$.

(20) $\sin t_2 - \sin t_3 = 2 \cdot (\cos(\frac{t_2 + t_3}{2}) \cdot \sin(\frac{t_2 - t_3}{2}))$.

(21) $\cos t_2 + \cos t_3 = 2 \cdot (\cos(\frac{t_2 + t_3}{2}) \cdot \cos(\frac{t_2 - t_3}{2}))$.

(22) $\cos t_2 - \cos t_3 = -2 \cdot (\sin(\frac{t_2 + t_3}{2}) \cdot \sin(\frac{t_2 - t_3}{2}))$.

(23) If $\cos t_2 \neq 0$ and $\cos t_3 \neq 0$, then $\tan t_2 + \tan t_3 = \frac{\sin(t_2 + t_3)}{\cos t_2 \cdot \cos t_3}$.

(24) If $\cos t_2 \neq 0$ and $\cos t_3 \neq 0$, then $\tan t_2 - \tan t_3 = \frac{\sin(t_2 - t_3)}{\cos t_2 \cdot \cos t_3}$.

(25) If $\cos t_2 \neq 0$ and $\sin t_3 \neq 0$, then $\tan t_2 + \cot t_3 = \frac{\cos(t_2 - t_3)}{\cos t_2 \cdot \sin t_3}$.

(26) If $\cos t_2 \neq 0$ and $\sin t_3 \neq 0$, then $\tan t_2 - \cot t_3 = -\frac{\cos(t_2 + t_3)}{\cos t_2 \cdot \sin t_3}$.

(27) If $\sin t_2 \neq 0$ and $\sin t_3 \neq 0$, then $\cot t_2 + \cot t_3 = \frac{\sin(t_2 + t_3)}{\sin t_2 \cdot \sin t_3}$.

(28) If $\sin t_2 \neq 0$ and $\sin t_3 \neq 0$, then $\cot t_2 - \cot t_3 = -\frac{\sin(t_2 - t_3)}{\sin t_2 \cdot \sin t_3}$.

(29) $\sin(t_2 + t_3) + \sin(t_2 - t_3) = 2 \cdot (\sin t_2 \cdot \cos t_3)$.

(30) $\sin(t_2 + t_3) - \sin(t_2 - t_3) = 2 \cdot (\cos t_2 \cdot \sin t_3)$.

(31) $\cos(t_2 + t_3) + \cos(t_2 - t_3) = 2 \cdot (\cos t_2 \cdot \cos t_3)$.

(32) $\cos(t_2 + t_3) - \cos(t_2 - t_3) = -2 \cdot (\sin t_2 \cdot \sin t_3)$.

(33) $\sin t_2 \cdot \sin t_3 = -\frac{1}{2} \cdot (\cos(t_2 + t_3) - \cos(t_2 - t_3))$.

(34) $\sin t_2 \cdot \cos t_3 = \frac{1}{2} \cdot (\sin(t_2 + t_3) + \sin(t_2 - t_3))$.

(35) $\cos t_2 \cdot \sin t_3 = \frac{1}{2} \cdot (\sin(t_2 + t_3) - \sin(t_2 - t_3))$.

(36) $\cos t_2 \cdot \cos t_3 = \frac{1}{2} \cdot (\cos(t_2 + t_3) + \cos(t_2 - t_3))$.

(37) $\sin t_2 \cdot \sin t_3 \cdot \sin t_4 = \frac{1}{4} \cdot ((\sin((t_2 + t_3) - t_4) + \sin((t_3 + t_4) - t_2) + \sin((t_4 + t_2) - t_3)) - \sin(t_2 + t_3 + t_4))$.

(38) $\sin t_2 \cdot \sin t_3 \cdot \cos t_4 = \frac{1}{4} \cdot ((-\cos((t_2 + t_3) - t_4) + \cos((t_3 + t_4) - t_2) + \cos((t_4 + t_2) - t_3)) - \cos(t_2 + t_3 + t_4))$.

(39) $\sin t_2 \cdot \cos t_3 \cdot \cos t_4 = \frac{1}{4} \cdot ((\sin((t_2 + t_3) - t_4) - \sin((t_3 + t_4) - t_2)) + \sin((t_4 + t_2) - t_3) + \sin(t_2 + t_3 + t_4))$.

(40) $\cos t_2 \cdot \cos t_3 \cdot \cos t_4 = \frac{1}{4} \cdot (\cos((t_2 + t_3) - t_4) + \cos((t_3 + t_4) - t_2) + \cos((t_4 + t_2) - t_3) + \cos(t_2 + t_3 + t_4))$.

(41) $\sin(t_2 + t_3) \cdot \sin(t_2 - t_3) = \sin t_2 \cdot \sin t_2 - \sin t_3 \cdot \sin t_3$.

(42) $\sin(t_2 + t_3) \cdot \sin(t_2 - t_3) = \cos t_3 \cdot \cos t_3 - \cos t_2 \cdot \cos t_2$.

(43) $\sin(t_2 + t_3) \cdot \cos(t_2 - t_3) = \sin t_2 \cdot \cos t_2 + \sin t_3 \cdot \cos t_3$.

(44) $\cos(t_2 + t_3) \cdot \sin(t_2 - t_3) = \sin t_2 \cdot \cos t_2 - \sin t_3 \cdot \cos t_3$.

(45) $\cos(t_2 + t_3) \cdot \cos(t_2 - t_3) = \cos t_2 \cdot \cos t_2 - \sin t_3 \cdot \sin t_3$.

(46) $\cos(t_2 + t_3) \cdot \cos(t_2 - t_3) = \cos t_3 \cdot \cos t_3 - \sin t_2 \cdot \sin t_2$.

(47) If $\cos t_2 \neq 0$ and $\cos t_3 \neq 0$, then $\frac{\sin(t_2+t_3)}{\sin(t_2-t_3)} = \frac{\tan t_2+\tan t_3}{\tan t_2-\tan t_3}$.

(48) If $\cos t_2 \neq 0$ and $\cos t_3 \neq 0$, then $\frac{\cos(t_2+t_3)}{\cos(t_2-t_3)} = \frac{1-\tan t_2 \cdot \tan t_3}{1+\tan t_2 \cdot \tan t_3}$.

(49) $\frac{\sin t_2+\sin t_3}{\sin t_2-\sin t_3} = \tan(\frac{t_2+t_3}{2}) \cdot \cot(\frac{t_2-t_3}{2})$.

(50) If $\cos(\frac{t_2-t_3}{2}) \neq 0$, then $\frac{\sin t_2+\sin t_3}{\cos t_2+\cos t_3} = \tan(\frac{t_2+t_3}{2})$.

(51) If $\cos(\frac{t_2+t_3}{2}) \neq 0$, then $\frac{\sin t_2-\sin t_3}{\cos t_2+\cos t_3} = \tan(\frac{t_2-t_3}{2})$.

(52) If $\sin(\frac{t_2+t_3}{2}) \neq 0$, then $\frac{\sin t_2+\sin t_3}{\cos t_3-\cos t_2} = \cot(\frac{t_2-t_3}{2})$.

(53) If $\sin(\frac{t_2-t_3}{2}) \neq 0$, then $\frac{\sin t_2-\sin t_3}{\cos t_3-\cos t_2} = \cot(\frac{t_2+t_3}{2})$.

(54) $\frac{\cos t_2+\cos t_3}{\cos t_2-\cos t_3} = \cot(\frac{t_2+t_3}{2}) \cdot \cot(\frac{t_3-t_2}{2})$.

References

[1] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.

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