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# Propositional Calculus for Boolean Valued Functions. Part VIII

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**Summary.** In this paper, we proved some elementary propositional calculus formulae for Boolean valued functions.

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The articles [5], [6], [8], [7], [9], [1], [4], [3], and [2] provide the notation and terminology for this paper.

In this paper $Y$ denotes a non empty set and $a$, $b$, $c$ denote elements of $Boolean^Y$.

Let $p$, $q$ be boolean-valued functions. The functor $p$ 'nand' $q$ yielding a function is defined as follows:

(Def. 1) $\operatorname{dom}(p \text{ 'nand' } q) = \operatorname{dom} p \cap \operatorname{dom} q$ and for every set $x$ such that $x \in \operatorname{dom}(p \text{ 'nand' } q)$ holds $(p \text{ 'nand' } q)(x) = p(x) \text{ 'nand' } q(x)$.

Let us observe that the functor $p$ 'nand' $q$ is commutative. The functor $p$ 'nor' $q$ yielding a function is defined as follows:

(Def. 2) $\operatorname{dom}(p \text{ 'nor' } q) = \operatorname{dom} p \cap \operatorname{dom} q$ and for every set $x$ such that $x \in \operatorname{dom}(p \text{ 'nor' } q)$ holds $(p \text{ 'nor' } q)(x) = p(x) \text{ 'nor' } q(x)$.

Let us note that the functor $p$ 'nor' $q$ is commutative.

Let $p$, $q$ be boolean-valued functions. Note that $p$ 'nand' $q$ is boolean-valued and $p$ 'nor' $q$ is boolean-valued.

Let $A$ be a non empty set and let $p$, $q$ be elements of $Boolean^A$. Then $p$ 'nand' $q$ is an element of $Boolean^A$ and it can be characterized by the condition:

(Def. 3) For every element $x$ of $A$ holds $(p \text{ 'nand' } q)(x) = p(x) \text{ 'nand' } q(x)$.




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Then $p$ 'nor' $q$ is an element of $Boolean^A$ and it can be characterized by the condition:

(Def. 4) For every element $x$ of $A$ holds $(p \text{ 'nor' } q)(x) = p(x) \text{ 'nor' } q(x)$.

Let us consider $Y$ and let $a$, $b$ be elements of BVF($Y$). Then $a$ 'nand' $b$ is an element of BVF($Y$). Then $a$ 'nor' $b$ is an element of BVF($Y$).

We now state a number of propositions:

(1) $a \text{ 'nand' } b = \neg(a \wedge b)$.

(2) $a \text{ 'nor' } b = \neg(a \vee b)$.

(3) $\mathit{true}(Y) \text{ 'nand' } a = \neg a$.

(4) $\mathit{false}(Y) \text{ 'nand' } a = \mathit{true}(Y)$.

(5) $\mathit{false}(Y) \text{ 'nand' } \mathit{false}(Y) = \mathit{true}(Y)$ and $\mathit{false}(Y) \text{ 'nand' } \mathit{true}(Y) = \mathit{true}(Y)$ and $\mathit{true}(Y) \text{ 'nand' } \mathit{true}(Y) = \mathit{false}(Y)$.

(6) $a \text{ 'nand' } a = \neg a$ and $\neg(a \text{ 'nand' } a) = a$.

(7) $\neg(a \text{ 'nand' } b) = a \wedge b$.

(8) $a \text{ 'nand' } \neg a = \mathit{true}(Y)$ and $\neg(a \text{ 'nand' } \neg a) = \mathit{false}(Y)$.

(9) $a \text{ 'nand' } b \wedge c = \neg(a \wedge b \wedge c)$.

(10) $a \text{ 'nand' } b \wedge c = a \wedge b \text{ 'nand' } c$.

(11) $a \text{ 'nand' } (b \vee c) = \neg(a \wedge b) \wedge \neg(a \wedge c)$.

(12) $a \text{ 'nand' } (b \oplus c) = a \wedge b \Leftrightarrow a \wedge c$.

(13) $a \text{ 'nand' } (b \text{ 'nand' } c) = \neg a \vee b \wedge c$ and $a \text{ 'nand' } (b \text{ 'nand' } c) = a \Rightarrow b \wedge c$.

(14) $a \text{ 'nand' } (b \text{ 'nor' } c) = \neg a \vee b \vee c$ and $a \text{ 'nand' } (b \text{ 'nor' } c) = a \Rightarrow b \vee c$.

(15) $a \text{ 'nand' } (b \Leftrightarrow c) = a \Rightarrow b \oplus c$.

(16) $a \text{ 'nand' } a \wedge b = a \text{ 'nand' } b$.

(17) $a \text{ 'nand' } (a \vee b) = \neg a \wedge \neg(a \wedge b)$.

(18) $a \text{ 'nand' } (a \Leftrightarrow b) = a \Rightarrow a \oplus b$.

(19) $a \text{ 'nand' } (a \text{ 'nand' } b) = \neg a \vee b$ and $a \text{ 'nand' } (a \text{ 'nand' } b) = a \Rightarrow b$.

(20) $a \text{ 'nand' } (a \text{ 'nor' } b) = \mathit{true}(Y)$.

(21) $a \text{ 'nand' } (a \Leftrightarrow b) = \neg a \vee \neg b$.

(22) $a \wedge b = a \text{ 'nand' } b \text{ 'nand' } (a \text{ 'nand' } b)$.

(23) $a \text{ 'nand' } b \text{ 'nand' } (a \text{ 'nand' } c) = a \wedge (b \vee c)$.

(24) $a \text{ 'nand' } (b \Rightarrow c) = (\neg a \vee b) \wedge \neg(a \wedge c)$.

(25) $a \text{ 'nand' } (a \Rightarrow b) = \neg(a \wedge b)$.

(26) $\mathit{true}(Y) \text{ 'nor' } a = \mathit{false}(Y)$.

(27) $\mathit{false}(Y) \text{ 'nor' } a = \neg a$.

(28) $\mathit{false}(Y) \text{ 'nor' } \mathit{false}(Y) = \mathit{true}(Y)$ and $\mathit{false}(Y) \text{ 'nor' } \mathit{true}(Y) = \mathit{false}(Y)$ and $\mathit{true}(Y) \text{ 'nor' } \mathit{true}(Y) = \mathit{false}(Y)$.

(29) $a \text{ 'nor' } a = \neg a$ and $\neg(a \text{ 'nor' } a) = a$.

(30) $\neg(a \,'\mathrm{nor}'\, b) = a \vee b$.

(31) $a \,'\mathrm{nor}'\, \neg a = \mathit{false}(Y)$ and $\neg(a \,'\mathrm{nor}'\, \neg a) = \mathit{true}(Y)$.

(32) $\neg a \wedge (a \oplus b) = \neg a \wedge b$.

(33) $a \,'\mathrm{nor}'\, b \wedge c = \neg(a \vee b) \vee \neg(a \vee c)$.

(34) $a \,'\mathrm{nor}'\, (b \vee c) = \neg(a \vee b \vee c)$.

(35) $a \,'\mathrm{nor}'\, (b \Leftrightarrow c) = \neg a \wedge (b \oplus c)$.

(36) $a \,'\mathrm{nor}'\, (b \Rightarrow c) = \neg a \wedge b \wedge \neg c$.

(37) $a \,'\mathrm{nor}'\, (b \,'\mathrm{nand}'\, c) = \neg a \wedge b \wedge c$.

(38) $a \,'\mathrm{nor}'\, (b \,'\mathrm{nor}'\, c) = \neg a \wedge (b \vee c)$.

(39) $a \,'\mathrm{nor}'\, a \wedge b = \neg(a \wedge (a \vee b))$.

(40) $a \,'\mathrm{nor}'\, (a \vee b) = \neg(a \vee b)$.

(41) $a \,'\mathrm{nor}'\, (a \Leftrightarrow b) = \neg a \wedge b$.

(42) $a \,'\mathrm{nor}'\, (a \Rightarrow b) = \mathit{false}(Y)$.

(43) $a \,'\mathrm{nor}'\, (a \,'\mathrm{nand}'\, b) = \mathit{false}(Y)$.

(44) $a \,'\mathrm{nor}'\, (a \,'\mathrm{nor}'\, b) = \neg a \wedge b$.

(45) $\mathit{false}(Y) \Leftrightarrow \mathit{false}(Y) = \mathit{true}(Y)$.

(46) $\mathit{false}(Y) \Leftrightarrow \mathit{true}(Y) = \mathit{false}(Y)$.

(47) $\mathit{true}(Y) \Leftrightarrow \mathit{true}(Y) = \mathit{true}(Y)$.

(48) $a \Leftrightarrow a = \mathit{true}(Y)$ and $\neg(a \Leftrightarrow a) = \mathit{false}(Y)$.

(49) $a \Leftrightarrow a \vee b = a \vee \neg b$.

(50) $a \wedge (b \,'\mathrm{nand}'\, c) = a \wedge \neg b \vee a \wedge \neg c$.

(51) $a \vee (b \,'\mathrm{nand}'\, c) = a \vee \neg b \vee \neg c$.

(52) $a \oplus (b \,'\mathrm{nand}'\, c) = \neg a \wedge \neg(b \wedge c) \vee a \wedge b \wedge c$.

(53) $a \Leftrightarrow b \,'\mathrm{nand}'\, c = a \wedge \neg(b \wedge c) \vee \neg a \wedge b \wedge c$.

(54) $a \Rightarrow b \,'\mathrm{nand}'\, c = \neg(a \wedge b \wedge c)$.

(55) $a \,'\mathrm{nor}'\, (b \,'\mathrm{nand}'\, c) = \neg(a \vee \neg b \vee \neg c)$.

(56) $a \wedge (a \,'\mathrm{nand}'\, b) = a \wedge \neg b$.

(57) $a \vee (a \,'\mathrm{nand}'\, b) = \mathit{true}(Y)$.

(58) $a \oplus (a \,'\mathrm{nand}'\, b) = \neg a \vee b$.

(59) $a \Leftrightarrow a \,'\mathrm{nand}'\, b = a \wedge \neg b$.

(60) $a \Rightarrow a \,'\mathrm{nand}'\, b = \neg(a \wedge b)$.

(61) $a \,'\mathrm{nor}'\, (a \,'\mathrm{nand}'\, b) = \mathit{false}(Y)$.

(62) $a \wedge (b \,'\mathrm{nor}'\, c) = a \wedge \neg b \wedge \neg c$.

(63) $a \vee (b \,'\mathrm{nor}'\, c) = (a \vee \neg b) \wedge (a \vee \neg c)$.

(64) $a \oplus (b \,'\mathrm{nor}'\, c) = (a \vee \neg(b \vee c)) \wedge (\neg a \vee b \vee c)$.

(65) $a \Leftrightarrow b \,'\mathrm{nor}'\, c = (a \vee b \vee c) \wedge (\neg a \vee \neg(b \vee c))$.

(66) $a \Rightarrow b \,'\mathrm{nor}'\, c = \neg(a \wedge (b \vee c))$.

(67) $a\ '\text{nand}'\ (b\ '\text{nor}'\ c) = \neg a \vee b \vee c.$

(68) $a \wedge (a\ '\text{nor}'\ b) = \mathit{false}(Y).$

(69) $a \vee (a\ '\text{nor}'\ b) = a \vee \neg b.$

(70) $a \oplus (a\ '\text{nor}'\ b) = a \vee \neg b.$

(71) $a \Leftrightarrow a\ '\text{nor}'\ b = \neg a \wedge b.$

(72) $a \Rightarrow a\ '\text{nor}'\ b = \neg(a \vee a \wedge b).$

(73) $a\ '\text{nand}'\ (a\ '\text{nor}'\ b) = \mathit{true}(Y).$

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