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The Fashoda Meet Theorem for Rectangles

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Summary. Here, the so called Fashoda Meet Theorem is proven in the case of rectangles. All cases of proper location of arcs are listed up, and it is shown that the theorem is valid in each case. Such a list of cases will be useful when one wants to apply the theorem.

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The articles [1], [6], [15], [17], [5], [2], [3], [16], [7], [14], [13], [10], [11], [8], [4], [9], and [12] provide the notation and terminology for this paper.

One can prove the following propositions:

(1) For all real numbers a, b, d and for every point p of $\mathcal{E}^2_{\mathrm{T}}$ such that $a < b$ and $p_{\mathbf{2}} = d$ and $a \leq p_{\mathbf{1}}$ and $p_{\mathbf{1}} \leq b$ holds $p \in \mathcal{L}([a, d], [b, d])$.

(2) Let n be a natural number, P be a subset of $\mathcal{E}^n_{\mathrm{T}}$, and p_1, p_2 be points of $\mathcal{E}^n_{\mathrm{T}}$. Suppose P is an arc from p_1 to p_2. Then there exists a map f from $\mathbb{I}$ into $\mathcal{E}^n_{\mathrm{T}}$ such that f is continuous and one-to-one and $\operatorname{rng} f = P$ and $f(0) = p_1$ and $f(1) = p_2$.

(3) Let p_1, p_2 be points of $\mathcal{E}^2_{\mathrm{T}}$ and b, c, d be real numbers. If $(p_1)_{\mathbf{1}} < b$ and $(p_1)_{\mathbf{1}} = (p_2)_{\mathbf{1}}$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$, then $p_1 \leq_{\mathrm{Rectangle}((p_1)_{\mathbf{1}}, b, c, d)} p_2$.

(4) Let p_1, p_2 be points of $\mathcal{E}^2_{\mathrm{T}}$ and b, c be real numbers. Suppose $(p_1)_{\mathbf{1}} < b$ and $c < (p_2)_{\mathbf{2}}$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq (p_2)_{\mathbf{2}}$ and $(p_1)_{\mathbf{1}} \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$. Then $p_1 \leq_{\mathrm{Rectangle}((p_1)_{\mathbf{1}}, b, c, (p_2)_{\mathbf{2}})} p_2$.

(5) Let p_1, p_2 be points of $\mathcal{E}^2_{\mathrm{T}}$ and c, d be real numbers. Suppose $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $c < d$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $c \leq (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$. Then $p_1 \leq_{\mathrm{Rectangle}((p_1)_{\mathbf{1}}, (p_2)_{\mathbf{1}}, c, d)} p_2$.

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(6) Let p_1, p_2 be points of $\mathcal{E}^2_{\rm T}$ and b, d be real numbers. If $(p_2)_{\mathbf{2}} < d$ and $(p_2)_{\mathbf{2}} \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$, then $p_1 \leq_{{\rm Rectangle}((p_1)_{\mathbf{1}},b,(p_2)_{\mathbf{2}},d)} p_2$.

(7) Let p_1, p_2 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$. Then $p_1 \leq_{{\rm Rectangle}(a,b,c,d)} p_2$.

(8) Let p_1, p_2 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{1}} = b$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} \leq b$ and $c \leq (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$. Then $p_1 \leq_{{\rm Rectangle}(a,b,c,d)} p_2$.

(9) Let p_1, p_2 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} \leq b$ and $a < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$. Then $p_1 \leq_{{\rm Rectangle}(a,b,c,d)} p_2$.

(10) Let p_1, p_2 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = b$ and $(p_2)_{\mathbf{1}} = b$ and $c \leq (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} < (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$. Then $p_1 \leq_{{\rm Rectangle}(a,b,c,d)} p_2$.

(11) Let p_1, p_2 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = b$ and $(p_2)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$. Then $p_1 \leq_{{\rm Rectangle}(a,b,c,d)} p_2$.

(12) Let p_1, p_2 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = c$ and $(p_2)_{\mathbf{2}} = c$ and $a < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} \leq b$. Then $p_1 \leq_{{\rm Rectangle}(a,b,c,d)} p_2$.

(13) Let p_1, p_2 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{1}} = b$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} \leq b$ and $c \leq (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$. Then $p_1 \leq_{{\rm Rectangle}(a,b,c,d)} p_2$.

(14) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{1}} = a$ and $(p_4)_{\mathbf{1}} = a$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} < (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \leq d$. Then p_1, p_2, p_3, p_4 are in this order on ${\rm Rectangle}(a,b,c,d)$.

(15) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{1}} = a$ and $(p_4)_{\mathbf{2}} = d$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and $a \leq (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on ${\rm Rectangle}(a,b,c,d)$.

(16) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{1}} = a$ and $(p_4)_{\mathbf{1}} = b$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \leq d$. Then p_1, p_2, p_3, p_4 are in this order on ${\rm Rectangle}(a,b,c,d)$.

(17) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{1}} = a$ and

$(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(18) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{2}} = d$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $a \leq (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(19) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{1}} = b$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $a \leq (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \leq d$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(20) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $a \leq (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(21) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{1}} = b$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(22) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $c \leq (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(23) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(24) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{2}} = d$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(25) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{1}} = b$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and

$(p_3)_{\mathbf{1}} \leq b$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \leq d$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(26) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(27) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{1}} = b$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(28) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $(p_1)_{\mathbf{1}} \neq (p_3)_{\mathbf{1}}$ and $(p_4)_{\mathbf{2}} \neq (p_2)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq (p_2)_{\mathbf{2}}$ and $(p_1)_{\mathbf{1}} \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq (p_3)_{\mathbf{1}}$ and $(p_4)_{\mathbf{2}} \leq (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq (p_2)_{\mathbf{2}}$ and $(p_1)_{\mathbf{1}} < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq (p_3)_{\mathbf{1}}$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle$((p_1)_{\mathbf{1}}, (p_3)_{\mathbf{1}}, (p_4)_{\mathbf{2}}, (p_2)_{\mathbf{2}})$.

(29) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(30) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{1}} = b$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $d \geq (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} > (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} > (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \geq c$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(31) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $d \geq (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} > (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \geq c$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(32) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $c \leq (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(33) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = c$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on

Rectangle(a, b, c, d).

(34) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{2}} = d$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(35) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{1}} = b$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \leq d$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(36) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(37) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{1}} = b$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(38) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $c \leq (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(39) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(40) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{1}} = b$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} \leq b$ and $d \geq (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} > (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} > (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \geq c$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(41) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} \leq b$ and $d \geq (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} > (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \geq c$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on Rectangle(a, b, c, d).

(42) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} \leq b$ and $c \leq (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on $\mathrm{Rectangle}(a, b, c, d)$.

(43) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = c$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} \leq b$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on $\mathrm{Rectangle}(a, b, c, d)$.

(44) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = b$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{1}} = b$ and $d \geq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} > (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} > (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} > (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \geq c$. Then p_1, p_2, p_3, p_4 are in this order on $\mathrm{Rectangle}(a, b, c, d)$.

(45) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = b$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{2}} = c$ and $d \geq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} > (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} > (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \geq c$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$. Then p_1, p_2, p_3, p_4 are in this order on $\mathrm{Rectangle}(a, b, c, d)$.

(46) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = b$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $d \geq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} > (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \geq c$ and $b \geq (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} > (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} > a$. Then p_1, p_2, p_3, p_4 are in this order on $\mathrm{Rectangle}(a, b, c, d)$.

(47) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = b$ and $(p_2)_{\mathbf{2}} = c$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $b \geq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} > (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} > (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} > a$. Then p_1, p_2, p_3, p_4 are in this order on $\mathrm{Rectangle}(a, b, c, d)$.

(48) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$ and a, b, c, d be real numbers. Suppose $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = c$ and $(p_2)_{\mathbf{2}} = c$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $b \geq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} > (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} > (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} > (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} > a$. Then p_1, p_2, p_3, p_4 are in this order on $\mathrm{Rectangle}(a, b, c, d)$.

(49) Let A, B, C, D be real numbers and h, g be maps from $\mathcal{E}^2_{\rm T}$ into $\mathcal{E}^2_{\rm T}$. Suppose $A > 0$ and $C > 0$ and $h = \mathrm{AffineMap}(A, B, C, D)$ and $g = \mathrm{AffineMap}(\frac{1}{A}, -\frac{B}{A}, \frac{1}{C}, -\frac{D}{C})$. Then $g = h^{-1}$ and $h = g^{-1}$.

(50) Let A, B, C, D be real numbers and h be a map from $\mathcal{E}^2_{\rm T}$ into $\mathcal{E}^2_{\rm T}$. Suppose $A > 0$ and $C > 0$ and $h = \mathrm{AffineMap}(A, B, C, D)$. Then h is a homeomorphism and for all points p_1, p_2 of $\mathcal{E}^2_{\rm T}$ such that $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ holds $h(p_1)_{\mathbf{1}} < h(p_2)_{\mathbf{1}}$.

(51) Let A, B, C, D be real numbers and h be a map from $\mathcal{E}^2_{\mathrm{T}}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose $A > 0$ and $C > 0$ and $h = \mathrm{AffineMap}(A, B, C, D)$. Then h is a homeomorphism and for all points p_1, p_2 of $\mathcal{E}^2_{\mathrm{T}}$ such that $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ holds $h(p_1)_{\mathbf{2}} < h(p_2)_{\mathbf{2}}$.

(52) Let a, b, c, d be real numbers, h be a map from $\mathcal{E}^2_{\mathrm{T}}$ into $\mathcal{E}^2_{\mathrm{T}}$, and f be a map from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose $a < b$ and $c < d$ and $h = \mathrm{AffineMap}(\frac{2}{b-a}, -\frac{b+a}{b-a}, \frac{2}{d-c}, -\frac{d+c}{d-c})$ and $\mathrm{rng}\, f \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\mathrm{rng}(h \cdot f) \subseteq \mathrm{ClosedInsideOfRectangle}(-1, 1, -1, 1)$.

(53) Let a, b, c, d be real numbers, h be a map from $\mathcal{E}^2_{\mathrm{T}}$ into $\mathcal{E}^2_{\mathrm{T}}$, and f be a map from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose $a < b$ and $c < d$ and $h = \mathrm{AffineMap}(\frac{2}{b-a}, -\frac{b+a}{b-a}, \frac{2}{d-c}, -\frac{d+c}{d-c})$ and f is continuous and one-to-one. Then $h \cdot f$ is continuous and one-to-one.

(54) Let a, b, c, d be real numbers, h be a map from $\mathcal{E}^2_{\mathrm{T}}$ into $\mathcal{E}^2_{\mathrm{T}}$, f be a map from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$, and O be a point of $\mathbb{I}$. Suppose $a < b$ and $c < d$ and $h = \mathrm{AffineMap}(\frac{2}{b-a}, -\frac{b+a}{b-a}, \frac{2}{d-c}, -\frac{d+c}{d-c})$ and $f(O)_{\mathbf{1}} = a$. Then $(h \cdot f)(O)_{\mathbf{1}} = -1$.

(55) Let a, b, c, d be real numbers, h be a map from $\mathcal{E}^2_{\mathrm{T}}$ into $\mathcal{E}^2_{\mathrm{T}}$, f be a map from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$, and I be a point of $\mathbb{I}$. Suppose $a < b$ and $c < d$ and $h = \mathrm{AffineMap}(\frac{2}{b-a}, -\frac{b+a}{b-a}, \frac{2}{d-c}, -\frac{d+c}{d-c})$ and $f(I)_{\mathbf{2}} = d$. Then $(h \cdot f)(I)_{\mathbf{2}} = 1$.

(56) Let a, b, c, d be real numbers, h be a map from $\mathcal{E}^2_{\mathrm{T}}$ into $\mathcal{E}^2_{\mathrm{T}}$, f be a map from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$, and I be a point of $\mathbb{I}$. Suppose $a < b$ and $c < d$ and $h = \mathrm{AffineMap}(\frac{2}{b-a}, -\frac{b+a}{b-a}, \frac{2}{d-c}, -\frac{d+c}{d-c})$ and $f(I)_{\mathbf{1}} = b$. Then $(h \cdot f)(I)_{\mathbf{1}} = 1$.

(57) Let a, b, c, d be real numbers, h be a map from $\mathcal{E}^2_{\mathrm{T}}$ into $\mathcal{E}^2_{\mathrm{T}}$, f be a map from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$, and I be a point of $\mathbb{I}$. Suppose $a < b$ and $c < d$ and $h = \mathrm{AffineMap}(\frac{2}{b-a}, -\frac{b+a}{b-a}, \frac{2}{d-c}, -\frac{d+c}{d-c})$ and $f(I)_{\mathbf{2}} = c$. Then $(h \cdot f)(I)_{\mathbf{2}} = -1$.

(58) Let a, b, c, d be real numbers, h be a map from $\mathcal{E}^2_{\mathrm{T}}$ into $\mathcal{E}^2_{\mathrm{T}}$, f be a map from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$, and O, I be points of $\mathbb{I}$. Suppose $a < b$ and $c < d$ and $h = \mathrm{AffineMap}(\frac{2}{b-a}, -\frac{b+a}{b-a}, \frac{2}{d-c}, -\frac{d+c}{d-c})$ and $c \leq f(O)_{\mathbf{2}}$ and $f(O)_{\mathbf{2}} < f(I)_{\mathbf{2}}$ and $f(I)_{\mathbf{2}} \leq d$. Then $-1 \leq (h \cdot f)(O)_{\mathbf{2}}$ and $(h \cdot f)(O)_{\mathbf{2}} < (h \cdot f)(I)_{\mathbf{2}}$ and $(h \cdot f)(I)_{\mathbf{2}} \leq 1$.

(59) Let a, b, c, d be real numbers, h be a map from $\mathcal{E}^2_{\mathrm{T}}$ into $\mathcal{E}^2_{\mathrm{T}}$, f be a map from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$, and O, I be points of $\mathbb{I}$. Suppose $a < b$ and $c < d$ and $h = \mathrm{AffineMap}(\frac{2}{b-a}, -\frac{b+a}{b-a}, \frac{2}{d-c}, -\frac{d+c}{d-c})$ and $c \leq f(O)_{\mathbf{2}}$ and $f(O)_{\mathbf{2}} \leq d$ and $a \leq f(I)_{\mathbf{1}}$ and $f(I)_{\mathbf{1}} \leq b$. Then $-1 \leq (h \cdot f)(O)_{\mathbf{2}}$ and $(h \cdot f)(O)_{\mathbf{2}} \leq 1$ and $-1 \leq (h \cdot f)(I)_{\mathbf{1}}$ and $(h \cdot f)(I)_{\mathbf{1}} \leq 1$.

(60) Let a, b, c, d be real numbers, h be a map from $\mathcal{E}^2_{\mathrm{T}}$ into $\mathcal{E}^2_{\mathrm{T}}$, f be a map from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$, and O, I be points of $\mathbb{I}$. Suppose $a < b$ and $c < d$ and $h = \mathrm{AffineMap}(\frac{2}{b-a}, -\frac{b+a}{b-a}, \frac{2}{d-c}, -\frac{d+c}{d-c})$ and $c \leq f(O)_{\mathbf{2}}$ and $f(O)_{\mathbf{2}} \leq d$ and $c \leq f(I)_{\mathbf{2}}$ and $f(I)_{\mathbf{2}} \leq d$. Then $-1 \leq (h \cdot f)(O)_{\mathbf{2}}$ and $(h \cdot f)(O)_{\mathbf{2}} \leq 1$ and $-1 \leq (h \cdot f)(I)_{\mathbf{2}}$ and $(h \cdot f)(I)_{\mathbf{2}} \leq 1$.

(61) Let a, b, c, d be real numbers, h be a map from $\mathcal{E}^2_{\mathrm{T}}$ into $\mathcal{E}^2_{\mathrm{T}}$, f be a map from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$, and O, I be points of $\mathbb{I}$. Suppose $a < b$ and $c < d$ and $h = \mathrm{AffineMap}(\frac{2}{b-a}, -\frac{b+a}{b-a}, \frac{2}{d-c}, -\frac{d+c}{d-c})$ and $c \leq f(O)_{\mathbf{2}}$ and $f(O)_{\mathbf{2}} \leq d$ and $a < f(I)_{\mathbf{1}}$ and $f(I)_{\mathbf{1}} \leq b$. Then $-1 \leq (h \cdot f)(O)_{\mathbf{2}}$ and $(h \cdot f)(O)_{\mathbf{2}} \leq 1$ and $-1 < (h \cdot f)(I)_{\mathbf{1}}$ and $(h \cdot f)(I)_{\mathbf{1}} \leq 1$.

(62) Let a, b, c, d be real numbers, h be a map from $\mathcal{E}^2_{\mathrm{T}}$ into $\mathcal{E}^2_{\mathrm{T}}$, f be a map from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$, and O, I be points of $\mathbb{I}$. Suppose $a < b$ and $c < d$ and $h = \mathrm{AffineMap}(\frac{2}{b-a}, -\frac{b+a}{b-a}, \frac{2}{d-c}, -\frac{d+c}{d-c})$ and $a \leq f(O)_{\mathbf{1}}$ and $f(O)_{\mathbf{1}} < f(I)_{\mathbf{1}}$ and $f(I)_{\mathbf{1}} \leq b$. Then $-1 \leq (h \cdot f)(O)_{\mathbf{1}}$ and $(h \cdot f)(O)_{\mathbf{1}} < (h \cdot f)(I)_{\mathbf{1}}$ and $(h \cdot f)(I)_{\mathbf{1}} \leq 1$.

(63) Let a, b, c, d be real numbers, h be a map from $\mathcal{E}^2_{\mathrm{T}}$ into $\mathcal{E}^2_{\mathrm{T}}$, f be a map from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$, and O, I be points of $\mathbb{I}$. Suppose $a < b$ and $c < d$ and $h = \mathrm{AffineMap}(\frac{2}{b-a}, -\frac{b+a}{b-a}, \frac{2}{d-c}, -\frac{d+c}{d-c})$ and $a \leq f(O)_{\mathbf{1}}$ and $f(O)_{\mathbf{1}} \leq b$ and $c \leq f(I)_{\mathbf{2}}$ and $f(I)_{\mathbf{2}} \leq d$. Then $-1 \leq (h \cdot f)(O)_{\mathbf{1}}$ and $(h \cdot f)(O)_{\mathbf{1}} \leq 1$ and $-1 \leq (h \cdot f)(I)_{\mathbf{2}}$ and $(h \cdot f)(I)_{\mathbf{2}} \leq 1$.

(64) Let a, b, c, d be real numbers, h be a map from $\mathcal{E}^2_{\mathrm{T}}$ into $\mathcal{E}^2_{\mathrm{T}}$, f be a map from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$, and O, I be points of $\mathbb{I}$. Suppose $a < b$ and $c < d$ and $h = \mathrm{AffineMap}(\frac{2}{b-a}, -\frac{b+a}{b-a}, \frac{2}{d-c}, -\frac{d+c}{d-c})$ and $a \leq f(O)_{\mathbf{1}}$ and $f(O)_{\mathbf{1}} \leq b$ and $a < f(I)_{\mathbf{1}}$ and $f(I)_{\mathbf{1}} \leq b$. Then $-1 \leq (h \cdot f)(O)_{\mathbf{1}}$ and $(h \cdot f)(O)_{\mathbf{1}} \leq 1$ and $-1 < (h \cdot f)(I)_{\mathbf{1}}$ and $(h \cdot f)(I)_{\mathbf{1}} \leq 1$.

(65) Let a, b, c, d be real numbers, h be a map from $\mathcal{E}^2_{\mathrm{T}}$ into $\mathcal{E}^2_{\mathrm{T}}$, f be a map from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$, and O, I be points of $\mathbb{I}$. Suppose $a < b$ and $c < d$ and $h = \mathrm{AffineMap}(\frac{2}{b-a}, -\frac{b+a}{b-a}, \frac{2}{d-c}, -\frac{d+c}{d-c})$ and $d \geq f(O)_{\mathbf{2}}$ and $f(O)_{\mathbf{2}} > f(I)_{\mathbf{2}}$ and $f(I)_{\mathbf{2}} \geq c$. Then $1 \geq (h \cdot f)(O)_{\mathbf{2}}$ and $(h \cdot f)(O)_{\mathbf{2}} > (h \cdot f)(I)_{\mathbf{2}}$ and $(h \cdot f)(I)_{\mathbf{2}} \geq -1$.

(66) Let a, b, c, d be real numbers, h be a map from $\mathcal{E}^2_{\mathrm{T}}$ into $\mathcal{E}^2_{\mathrm{T}}$, f be a map from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$, and O, I be points of $\mathbb{I}$. Suppose $a < b$ and $c < d$ and $h = \mathrm{AffineMap}(\frac{2}{b-a}, -\frac{b+a}{b-a}, \frac{2}{d-c}, -\frac{d+c}{d-c})$ and $c \leq f(O)_{\mathbf{2}}$ and $f(O)_{\mathbf{2}} \leq d$ and $a < f(I)_{\mathbf{1}}$ and $f(I)_{\mathbf{1}} \leq b$. Then $-1 \leq (h \cdot f)(O)_{\mathbf{2}}$ and $(h \cdot f)(O)_{\mathbf{2}} \leq 1$ and $-1 < (h \cdot f)(I)_{\mathbf{1}}$ and $(h \cdot f)(I)_{\mathbf{1}} \leq 1$.

(67) Let a, b, c, d be real numbers, h be a map from $\mathcal{E}^2_{\mathrm{T}}$ into $\mathcal{E}^2_{\mathrm{T}}$, f be a map from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$, and O, I be points of $\mathbb{I}$. Suppose $a < b$ and $c < d$ and $h = \mathrm{AffineMap}(\frac{2}{b-a}, -\frac{b+a}{b-a}, \frac{2}{d-c}, -\frac{d+c}{d-c})$ and $a < f(I)_{\mathbf{1}}$ and $f(I)_{\mathbf{1}} < f(O)_{\mathbf{1}}$ and $f(O)_{\mathbf{1}} \leq b$. Then $-1 < (h \cdot f)(I)_{\mathbf{1}}$ and $(h \cdot f)(I)_{\mathbf{1}} < (h \cdot f)(O)_{\mathbf{1}}$ and $(h \cdot f)(O)_{\mathbf{1}} \leq 1$.

One can prove the following propositions:

(68) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{1}} = a$ and $(p_4)_{\mathbf{1}} = a$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} < (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \leq d$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is

continuous and one-to-one and $\mathrm{rng}\, f \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $\mathrm{rng}\, g \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\mathrm{rng}\, f$ meets $\mathrm{rng}\, g$.

(69) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{1}} = a$ and $(p_4)_{\mathbf{1}} = a$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} < (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \leq d$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(70) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{1}} = a$ and $(p_4)_{\mathbf{2}} = d$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and $a \leq (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\mathrm{rng}\, f \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $\mathrm{rng}\, g \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\mathrm{rng}\, f$ meets $\mathrm{rng}\, g$.

(71) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{1}} = a$ and $(p_4)_{\mathbf{2}} = d$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and $a \leq (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(72) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{1}} = a$ and $(p_4)_{\mathbf{1}} = b$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \leq d$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\mathrm{rng}\, f \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $\mathrm{rng}\, g \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\mathrm{rng}\, f$ meets $\mathrm{rng}\, g$.

(73) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{1}} = a$ and $(p_4)_{\mathbf{1}} = b$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \leq d$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(74) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{1}} = a$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$

and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \operatorname{ClosedInsideOfRectangle}(a,b,c,d)$ and $\operatorname{rng} g \subseteq \operatorname{ClosedInsideOfRectangle}(a,b,c,d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(75) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{1}} = a$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \operatorname{ClosedInsideOfRectangle}(a,b,c,d)$ and $Q \subseteq \operatorname{ClosedInsideOfRectangle}(a,b,c,d)$. Then P meets Q.

(76) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{2}} = d$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $a \leq (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \operatorname{ClosedInsideOfRectangle}(a,b,c,d)$ and $\operatorname{rng} g \subseteq \operatorname{ClosedInsideOfRectangle}(a,b,c,d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(77) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{2}} = d$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $a \leq (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \operatorname{ClosedInsideOfRectangle}(a,b,c,d)$ and $Q \subseteq \operatorname{ClosedInsideOfRectangle}(a,b,c,d)$. Then P meets Q.

(78) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{1}} = b$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $a \leq (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \leq d$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \operatorname{ClosedInsideOfRectangle}(a,b,c,d)$ and $\operatorname{rng} g \subseteq \operatorname{ClosedInsideOfRectangle}(a,b,c,d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(79) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{1}} = b$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $a \leq (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \leq d$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \operatorname{ClosedInsideOfRectangle}(a,b,c,d)$ and $Q \subseteq \operatorname{ClosedInsideOfRectangle}(a,b,c,d)$. Then P meets Q.

(80) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\rm T}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\bf 1} = a$ and $(p_2)_{\bf 1} = a$ and $(p_3)_{\bf 2} = d$ and $(p_4)_{\bf 2} = c$ and $c \leq (p_1)_{\bf 2}$ and $(p_1)_{\bf 2} < (p_2)_{\bf 2}$ and $(p_2)_{\bf 2} \leq d$ and $a \leq (p_3)_{\bf 1}$ and $(p_3)_{\bf 1} \leq b$ and $a < (p_4)_{\bf 1}$ and $(p_4)_{\bf 1} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(81) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\rm T}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\bf 1} = a$ and $(p_2)_{\bf 1} = a$ and $(p_3)_{\bf 2} = d$ and $(p_4)_{\bf 2} = c$ and $c \leq (p_1)_{\bf 2}$ and $(p_1)_{\bf 2} < (p_2)_{\bf 2}$ and $(p_2)_{\bf 2} \leq d$ and $a \leq (p_3)_{\bf 1}$ and $(p_3)_{\bf 1} \leq b$ and $a < (p_4)_{\bf 1}$ and $(p_4)_{\bf 1} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(82) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\rm T}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\bf 1} = a$ and $(p_2)_{\bf 1} = a$ and $(p_3)_{\bf 1} = b$ and $(p_4)_{\bf 1} = b$ and $c \leq (p_1)_{\bf 2}$ and $(p_1)_{\bf 2} < (p_2)_{\bf 2}$ and $(p_2)_{\bf 2} \leq d$ and $c \leq (p_4)_{\bf 2}$ and $(p_4)_{\bf 2} < (p_3)_{\bf 2}$ and $(p_3)_{\bf 2} \leq d$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(83) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\rm T}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\bf 1} = a$ and $(p_2)_{\bf 1} = a$ and $(p_3)_{\bf 1} = b$ and $(p_4)_{\bf 1} = b$ and $c \leq (p_1)_{\bf 2}$ and $(p_1)_{\bf 2} < (p_2)_{\bf 2}$ and $(p_2)_{\bf 2} \leq d$ and $c \leq (p_4)_{\bf 2}$ and $(p_4)_{\bf 2} < (p_3)_{\bf 2}$ and $(p_3)_{\bf 2} \leq d$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(84) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\rm T}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\bf 1} = a$ and $(p_2)_{\bf 1} = a$ and $(p_3)_{\bf 1} = b$ and $(p_4)_{\bf 2} = c$ and $c \leq (p_1)_{\bf 2}$ and $(p_1)_{\bf 2} < (p_2)_{\bf 2}$ and $(p_2)_{\bf 2} \leq d$ and $c \leq (p_3)_{\bf 2}$ and $(p_3)_{\bf 2} \leq d$ and $a < (p_4)_{\bf 1}$ and $(p_4)_{\bf 1} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(85) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\rm T}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\rm T}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\bf 1} = a$ and $(p_2)_{\bf 1} = a$ and $(p_3)_{\bf 1} = b$ and $(p_4)_{\bf 2} = c$ and $c \leq (p_1)_{\bf 2}$ and $(p_1)_{\bf 2} < (p_2)_{\bf 2}$ and $(p_2)_{\bf 2} \leq d$ and $c \leq (p_3)_{\bf 2}$ and $(p_3)_{\bf 2} \leq d$ and

$a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(86) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\mathrm{rng}\, f \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $\mathrm{rng}\, g \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\mathrm{rng}\, f$ meets $\mathrm{rng}\, g$.

(87) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = a$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(88) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{2}} = d$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\mathrm{rng}\, f \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $\mathrm{rng}\, g \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\mathrm{rng}\, f$ meets $\mathrm{rng}\, g$.

(89) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{2}} = d$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(90) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{1}} = b$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \leq d$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\mathrm{rng}\, f \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $\mathrm{rng}\, g \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\mathrm{rng}\, f$ meets $\mathrm{rng}\, g$.

(91) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P,

Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{1}} = b$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \leq d$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(92) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\mathrm{rng}\, f \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $\mathrm{rng}\, g \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\mathrm{rng}\, f$ meets $\mathrm{rng}\, g$.

(93) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(94) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{1}} = b$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\mathrm{rng}\, f \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $\mathrm{rng}\, g \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\mathrm{rng}\, f$ meets $\mathrm{rng}\, g$.

(95) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{1}} = b$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(96) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $c \leq (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and

$g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(97) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $c \leq (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(98) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(99) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a \leq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(100) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{1}} = b$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(101) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{1}} = b$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(102) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and

$(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $c \leq (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(103) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $c \leq (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} < (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(104) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $c \leq (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(105) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $c \leq (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(106) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = c$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(107) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = a$ and $(p_2)_{\mathbf{2}} = c$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$

and $Q \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(108) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{2}} = d$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\mathrm{rng}\, f \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $\mathrm{rng}\, g \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\mathrm{rng}\, f$ meets $\mathrm{rng}\, g$.

(109) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{2}} = d$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(110) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{1}} = b$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \leq d$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\mathrm{rng}\, f \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $\mathrm{rng}\, g \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\mathrm{rng}\, f$ meets $\mathrm{rng}\, g$.

(111) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{1}} = b$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \leq d$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(112) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\mathrm{rng}\, f \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $\mathrm{rng}\, g \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\mathrm{rng}\, f$ meets $\mathrm{rng}\, g$.

(113) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = d$ and $(p_4)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and

$(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(114) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{1}} = b$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(115) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{1}} = b$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $c \leq (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} < (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(116) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $c \leq (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(117) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $c \leq (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(118) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(119) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = d$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(120) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{1}} = b$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} \leq b$ and $d \geq (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} > (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} > (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \geq c$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(121) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{1}} = b$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} \leq b$ and $d \geq (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} > (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} > (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \geq c$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(122) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} \leq b$ and $d \geq (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} > (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \geq c$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(123) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} \leq b$ and $d \geq (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} > (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \geq c$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \mathrm{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(124) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} \leq b$ and $c \leq (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continu-

ous and one-to-one and $\operatorname{rng} f \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(125) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} \leq b$ and $c \leq (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \leq d$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(126) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = c$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} \leq b$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(127) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = d$ and $(p_2)_{\mathbf{2}} = c$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $a \leq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} \leq b$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} < (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} < (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} \leq b$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(128) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = b$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{1}} = b$ and $d \geq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} > (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} > (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} > (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \geq c$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(129) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = b$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{1}} = b$ and $d \geq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} > (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} > (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} > (p_4)_{\mathbf{2}}$ and $(p_4)_{\mathbf{2}} \geq c$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(130) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = b$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{1}} = b$ and $(p_4)_{\mathbf{2}} = c$ and $d \geq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} > (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} > (p_3)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} \geq c$ and $a < (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} \leq b$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and

$g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

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(132) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = b$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $d \geq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} > (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \geq c$ and $b \geq (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} > (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} > a$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(133) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = b$ and $(p_2)_{\mathbf{1}} = b$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $d \geq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} > (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} \geq c$ and $b \geq (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} > (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} > a$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

(134) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and f, g be maps from $\mathbb{I}$ into $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{1}} = b$ and $(p_2)_{\mathbf{2}} = c$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $c \leq (p_1)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} \leq d$ and $b \geq (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} > (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} > (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} > a$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \text{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

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$(p_2)_{\mathbf{1}} > (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} > (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} > a$ and $f(0) = p_1$ and $f(1) = p_3$ and $g(0) = p_2$ and $g(1) = p_4$ and f is continuous and one-to-one and g is continuous and one-to-one and $\operatorname{rng} f \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$ and $\operatorname{rng} g \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(137) Let p_1, p_2, p_3, p_4 be points of $\mathcal{E}^2_{\mathrm{T}}$, a, b, c, d be real numbers, and P, Q be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $a < b$ and $c < d$ and $(p_1)_{\mathbf{2}} = c$ and $(p_2)_{\mathbf{2}} = c$ and $(p_3)_{\mathbf{2}} = c$ and $(p_4)_{\mathbf{2}} = c$ and $b \geq (p_1)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} > (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} > (p_3)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} > (p_4)_{\mathbf{1}}$ and $(p_4)_{\mathbf{1}} > a$ and P is an arc from p_1 to p_3 and Q is an arc from p_2 to p_4 and $P \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$ and $Q \subseteq \operatorname{ClosedInsideOfRectangle}(a, b, c, d)$. Then P meets Q.

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