FORMALIZED MATHEMATICS
Volume 13, Number 3, Pages 427–434
University of Białystok, 2005

Several Differentiable Formulas of Special Functions

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Summary. In this article, we give several differentiable formulas of special functions. There are some specific composite functions consisting of rational functions, irrational functions, trigonometric functions, exponential functions or logarithmic functions.

MML identifier: `FDIFF_4`, version: `7.5.01 4.39.921`

The notation and terminology used in this paper have been introduced in the following articles: [13], [15], [16], [1], [4], [10], [12], [3], [6], [9], [7], [8], [11], [17], [5], [14], and [2].

For simplicity, we follow the rules: x, a, b, c denote real numbers, n denotes a natural number, Z denotes an open subset of $\mathbb{R}$, and f, f_1, f_2 denote partial functions from $\mathbb{R}$ to $\mathbb{R}$.

One can prove the following propositions:

(1) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a + x$ and $f(x) > 0$. Then $\log\_(e) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot f)'_{\restriction Z}(x) = \frac{1}{a+x}$.

(2) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = x - a$ and $f(x) > 0$. Then $\log\_(e) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot f)'_{\restriction Z}(x) = \frac{1}{x-a}$.

(3) Suppose $Z \subseteq \mathrm{dom}(-\log\_(e) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a - x$ and $f(x) > 0$. Then $-\log\_(e) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $(-\log\_(e) \cdot f)'_{\restriction Z}(x) = \frac{1}{a-x}$.

ISSN 1426–2630

(4) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z - a\,f)$ and $f = \log\_(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = a + x$ and $f_1(x) > 0$. Then $\mathrm{id}_Z - a\,f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\mathrm{id}_Z - a\,f)'_{\restriction Z}(x) = \frac{x}{a+x}$.

(5) Suppose $Z \subseteq \mathrm{dom}((2 \cdot a)\,f - \mathrm{id}_Z)$ and $f = \log\_(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = a + x$ and $f_1(x) > 0$. Then $(2 \cdot a)\,f - \mathrm{id}_Z$ is differentiable on Z and for every x such that $x \in Z$ holds $((2 \cdot a)\,f - \mathrm{id}_Z)'_{\restriction Z}(x) = \frac{a-x}{a+x}$.

(6) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z - (2 \cdot a)\,f)$ and $f = \log\_(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = x + a$ and $f_1(x) > 0$. Then $\mathrm{id}_Z - (2 \cdot a)\,f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\mathrm{id}_Z - (2 \cdot a)\,f)'_{\restriction Z}(x) = \frac{x-a}{x+a}$.

(7) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z + (2 \cdot a)\,f)$ and $f = \log\_(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = x - a$ and $f_1(x) > 0$. Then $\mathrm{id}_Z + (2 \cdot a)\,f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\mathrm{id}_Z + (2 \cdot a)\,f)'_{\restriction Z}(x) = \frac{x+a}{x-a}$.

(8) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z + (a - b)\,f)$ and $f = \log\_(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = x + b$ and $f_1(x) > 0$. Then $\mathrm{id}_Z + (a - b)\,f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\mathrm{id}_Z + (a - b)\,f)'_{\restriction Z}(x) = \frac{x+a}{x+b}$.

(9) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z + (a + b)\,f)$ and $f = \log\_(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = x - b$ and $f_1(x) > 0$. Then $\mathrm{id}_Z + (a + b)\,f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\mathrm{id}_Z + (a + b)\,f)'_{\restriction Z}(x) = \frac{x+a}{x-b}$.

(10) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z - (a + b)\,f)$ and $f = \log\_(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = x + b$ and $f_1(x) > 0$. Then $\mathrm{id}_Z - (a + b)\,f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\mathrm{id}_Z - (a + b)\,f)'_{\restriction Z}(x) = \frac{x-a}{x+b}$.

(11) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z + (b - a)\,f)$ and $f = \log\_(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = x - b$ and $f_1(x) > 0$. Then $\mathrm{id}_Z + (b - a)\,f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\mathrm{id}_Z + (b - a)\,f)'_{\restriction Z}(x) = \frac{x-a}{x-b}$.

(12) Suppose $Z \subseteq \mathrm{dom}(f_1 + c\,f_2)$ and for every x such that $x \in Z$ holds $f_1(x) = a + b \cdot x$ and $f_2 = \frac{2}{\mathbb{Z}}$. Then $f_1 + c\,f_2$ is differentiable on Z and for every x such that $x \in Z$ holds $(f_1 + c\,f_2)'_{\restriction Z}(x) = b + 2 \cdot c \cdot x$.

(13) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot (f_1 + c\,f_2))$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a + b \cdot x$ and $(f_1 + c\,f_2)(x) > 0$. Then $\log\_(e) \cdot (f_1 + c\,f_2)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot (f_1 + c\,f_2))'_{\restriction Z}(x) = \frac{b+2 \cdot c \cdot x}{a+b \cdot x+c \cdot x^2}$.

(14) Suppose $Z \subseteq \mathrm{dom}\,f$ and for every x such that $x \in Z$ holds $f(x) = a + x$ and $f(x) \neq 0$. Then $\frac{1}{f}$ is differentiable on Z and for every x such that

$x \in Z$ holds $(\frac{1}{f})'_{\restriction Z}(x) = -\frac{1}{(a+x)^{\mathbf{2}}}$.

(15) Suppose $Z \subseteq \mathrm{dom}((-1)\,\frac{1}{f})$ and for every x such that $x \in Z$ holds $f(x) = a + x$ and $f(x) \neq 0$. Then $(-1)\,\frac{1}{f}$ is differentiable on Z and for every x such that $x \in Z$ holds $((-1)\,\frac{1}{f})'_{\restriction Z}(x) = \frac{1}{(a+x)^{\mathbf{2}}}$.

(16) Suppose $Z \subseteq \mathrm{dom}\, f$ and for every x such that $x \in Z$ holds $f(x) = a - x$ and $f(x) \neq 0$. Then $\frac{1}{f}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{1}{f})'_{\restriction Z}(x) = \frac{1}{(a-x)^{\mathbf{2}}}$.

(17) Suppose $Z \subseteq \mathrm{dom}(f_1 + f_2)$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $f_2 = \frac{2}{\mathbb{Z}}$. Then $f_1 + f_2$ is differentiable on Z and for every x such that $x \in Z$ holds $(f_1 + f_2)'_{\restriction Z}(x) = 2 \cdot x$.

(18) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot (f_1 + f_2))$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $(f_1 + f_2)(x) > 0$. Then $\log\_(e) \cdot (f_1 + f_2)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot (f_1 + f_2))'_{\restriction Z}(x) = \frac{2 \cdot x}{a^{\mathbf{2}} + x^2}$.

(19) Suppose $Z \subseteq \mathrm{dom}(-\log\_(e) \cdot (f_1 - f_2))$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $(f_1 - f_2)(x) > 0$. Then $-\log\_(e) \cdot (f_1 - f_2)$ is differentiable on Z and for every x such that $x \in Z$ holds $(-\log\_(e) \cdot (f_1 - f_2))'_{\restriction Z}(x) = \frac{2 \cdot x}{a^{\mathbf{2}} - x^2}$.

(20) Suppose $Z \subseteq \mathrm{dom}(f_1 + f_2)$ and for every x such that $x \in Z$ holds $f_1(x) = a$ and $f_2 = \frac{3}{\mathbb{Z}}$. Then $f_1 + f_2$ is differentiable on Z and for every x such that $x \in Z$ holds $(f_1 + f_2)'_{\restriction Z}(x) = 3 \cdot x^2$.

(21) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot (f_1 + f_2))$ and $f_2 = \frac{3}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a$ and $(f_1 + f_2)(x) > 0$. Then $\log\_(e) \cdot (f_1 + f_2)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot (f_1 + f_2))'_{\restriction Z}(x) = \frac{3 \cdot x^2}{a + x^3}$.

(22) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot \frac{f_1}{f_2})$ and for every x such that $x \in Z$ holds $f_1(x) = a + x$ and $f_1(x) > 0$ and $f_2(x) = a - x$ and $f_2(x) > 0$. Then $\log\_(e) \cdot \frac{f_1}{f_2}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot \frac{f_1}{f_2})'_{\restriction Z}(x) = \frac{2 \cdot a}{a^{\mathbf{2}} - x^{\mathbf{2}}}$.

(23) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot \frac{f_1}{f_2})$ and for every x such that $x \in Z$ holds $f_1(x) = x - a$ and $f_1(x) > 0$ and $f_2(x) = x + a$ and $f_2(x) > 0$. Then $\log\_(e) \cdot \frac{f_1}{f_2}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot \frac{f_1}{f_2})'_{\restriction Z}(x) = \frac{2 \cdot a}{x^{\mathbf{2}} - a^{\mathbf{2}}}$.

(24) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot \frac{f_1}{f_2})$ and for every x such that $x \in Z$ holds $f_1(x) = x - a$ and $f_1(x) > 0$ and $f_2(x) = x - b$ and $f_2(x) > 0$. Then $\log\_(e) \cdot \frac{f_1}{f_2}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot \frac{f_1}{f_2})'_{\restriction Z}(x) = \frac{a-b}{(x-a) \cdot (x-b)}$.

(25) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{a-b}\, f)$ and $f = \log\_(e) \cdot \frac{f_1}{f_2}$ and for every x such that

$x \in Z$ holds $f_1(x) = x - a$ and $f_1(x) > 0$ and $f_2(x) = x - b$ and $f_2(x) > 0$ and $a - b \neq 0$. Then $\frac{1}{a-b} f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{1}{a-b} f)'_{\restriction Z}(x) = \frac{1}{(x-a)\cdot(x-b)}$.

(26) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot \frac{f_1}{f_2})$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = x - a$ and $f_1(x) > 0$ and $f_2(x) > 0$ and $x \neq 0$. Then $\log\_(e) \cdot \frac{f_1}{f_2}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot \frac{f_1}{f_2})'_{\restriction Z}(x) = \frac{2\cdot a - x}{x\cdot(x-a)}$.

(27) Suppose $Z \subseteq \mathrm{dom}((\frac{3}{2}_{\mathbb{R}}) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a + x$ and $f(x) > 0$. Then $(\frac{3}{2}_{\mathbb{R}}) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $((\frac{3}{2}_{\mathbb{R}}) \cdot f)'_{\restriction Z}(x) = \frac{3}{2} \cdot (a + x)^{\frac{1}{2}}_{\mathbb{R}}$.

(28) Suppose $Z \subseteq \mathrm{dom}(\frac{2}{3}\,((\frac{3}{2}_{\mathbb{R}}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = a + x$ and $f(x) > 0$. Then $\frac{2}{3}\,((\frac{3}{2}_{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{2}{3}\,((\frac{3}{2}_{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (a + x)^{\frac{1}{2}}_{\mathbb{R}}$.

(29) Suppose $Z \subseteq \mathrm{dom}((-\frac{2}{3})\,((\frac{3}{2}_{\mathbb{R}}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = a - x$ and $f(x) > 0$. Then $(-\frac{2}{3})\,((\frac{3}{2}_{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $((-\frac{2}{3})\,((\frac{3}{2}_{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (a - x)^{\frac{1}{2}}_{\mathbb{R}}$.

(30) Suppose $Z \subseteq \mathrm{dom}(2\,((\frac{1}{2}_{\mathbb{R}}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = a + x$ and $f(x) > 0$. Then $2\,((\frac{1}{2}_{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $(2\,((\frac{1}{2}_{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (a + x)^{-\frac{1}{2}}_{\mathbb{R}}$.

(31) Suppose $Z \subseteq \mathrm{dom}((-2)\,((\frac{1}{2}_{\mathbb{R}}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = a - x$ and $f(x) > 0$. Then $(-2)\,((\frac{1}{2}_{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $((-2)\,((\frac{1}{2}_{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (a - x)^{-\frac{1}{2}}_{\mathbb{R}}$.

(32) Suppose $Z \subseteq \mathrm{dom}(\frac{2}{3\cdot b}\,((\frac{3}{2}_{\mathbb{R}}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = a + b \cdot x$ and $b \neq 0$ and $f(x) > 0$. Then $\frac{2}{3\cdot b}\,((\frac{3}{2}_{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{2}{3\cdot b}\,((\frac{3}{2}_{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (a + b \cdot x)^{\frac{1}{2}}_{\mathbb{R}}$.

(33) Suppose $Z \subseteq \mathrm{dom}((-\frac{2}{3\cdot b})\,((\frac{3}{2}_{\mathbb{R}}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = a - b \cdot x$ and $b \neq 0$ and $f(x) > 0$. Then $(-\frac{2}{3\cdot b})\,((\frac{3}{2}_{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $((-\frac{2}{3\cdot b})\,((\frac{3}{2}_{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (a - b \cdot x)^{\frac{1}{2}}_{\mathbb{R}}$.

(34) Suppose $Z \subseteq \mathrm{dom}((\frac{1}{2}_{\mathbb{R}}) \cdot f)$ and $f = f_1 + f_2$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $f(x) > 0$. Then $(\frac{1}{2}_{\mathbb{R}}) \cdot f$ is differentiable on

Z and for every x such that $x \in Z$ holds $((\frac{1}{2}_{\mathbb{R}}) \cdot f)'_{\restriction Z}(x) = x \cdot (a^{\mathbf{2}} + x^2)_{\mathbb{R}}^{-\frac{1}{2}}$.

(35) Suppose $Z \subseteq \mathrm{dom}(-(\frac{1}{2}_{\mathbb{R}}) \cdot f)$ and $f = f_1 - f_2$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $f(x) > 0$. Then $-(\frac{1}{2}_{\mathbb{R}}) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $(-(\frac{1}{2}_{\mathbb{R}}) \cdot f)'_{\restriction Z}(x) = x \cdot (a^{\mathbf{2}} - x^2)_{\mathbb{R}}^{-\frac{1}{2}}$.

(36) Suppose $Z \subseteq \mathrm{dom}(2\,((\frac{1}{2}_{\mathbb{R}}) \cdot f))$ and $f = f_1 + f_2$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = x$ and $f(x) > 0$. Then $2\,((\frac{1}{2}_{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $(2\,((\frac{1}{2}_{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (2 \cdot x + 1) \cdot (x^2 + x)_{\mathbb{R}}^{-\frac{1}{2}}$.

(37) Suppose $Z \subseteq \mathrm{dom}((\text{the function sin}) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$. Then

(i) (the function sin) $\cdot f$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function sin}) \cdot f)'_{\restriction Z}(x) = a \cdot (\text{the function cos})(a \cdot x + b)$.

(38) Suppose $Z \subseteq \mathrm{dom}((\text{the function cos}) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$. Then

(i) (the function cos) $\cdot f$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function cos}) \cdot f)'_{\restriction Z}(x) = -a \cdot (\text{the function sin})(a \cdot x + b)$.

(39) Suppose that for every x such that $x \in Z$ holds $(\text{the function cos})(x) \neq 0$. Then

(i) $\frac{1}{\text{the function cos}}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{\text{the function cos}})'_{\restriction Z}(x) = \frac{(\text{the function sin})(x)}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(40) Suppose that for every x such that $x \in Z$ holds $(\text{the function sin})(x) \neq 0$. Then

(i) $\frac{1}{\text{the function sin}}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{\text{the function sin}})'_{\restriction Z}(x) = -\frac{(\text{the function cos})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(41) Suppose $Z \subseteq \mathrm{dom}((\text{the function sin})\ (\text{the function cos}))$. Then

(i) (the function sin) (the function cos) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function sin})\ (\text{the function cos}))'_{\restriction Z}(x) = \cos(2 \cdot x)$.

(42) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot (\text{the function cos}))$ and for every x such that $x \in Z$ holds $(\text{the function cos})(x) > 0$. Then $\log\_(e) \cdot (\text{the function cos})$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot (\text{the function cos}))'_{\restriction Z}(x) = -\tan x$.

(43) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot (\text{the function sin}))$ and for every x such that $x \in Z$ holds (the function sin)$(x) > 0$. Then $\log\_(e) \cdot (\text{the function sin})$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot (\text{the function sin}))'_{\restriction Z}(x) = \cot x$.

(44) Suppose $Z \subseteq \mathrm{dom}((-\mathrm{id}_Z)\,(\text{the function cos}))$. Then

(i) $(-\mathrm{id}_Z)\,(\text{the function cos})$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((-\mathrm{id}_Z)\,(\text{the function cos}))'_{\restriction Z}(x) = -(\text{the function cos})(x) + x \cdot (\text{the function sin})(x)$.

(45) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z\,(\text{the function sin}))$. Then

(i) $\mathrm{id}_Z\,(\text{the function sin})$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\mathrm{id}_Z\,(\text{the function sin}))'_{\restriction Z}(x) = (\text{the function sin})(x) + x \cdot (\text{the function cos})(x)$.

(46) Suppose $Z \subseteq \mathrm{dom}((-\mathrm{id}_Z)\,(\text{the function cos})+\text{the function sin})$. Then

(i) $(-\mathrm{id}_Z)\,(\text{the function cos})+\text{the function sin}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((-\mathrm{id}_Z)\,(\text{the function cos})+\text{the function sin})'_{\restriction Z}(x) = x \cdot (\text{the function sin})(x)$.

(47) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z\,(\text{the function sin})+\text{the function cos})$. Then

(i) $\mathrm{id}_Z\,(\text{the function sin})+\text{the function cos}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\mathrm{id}_Z\,(\text{the function sin})+\text{the function cos})'_{\restriction Z}(x) = x \cdot (\text{the function cos})(x)$.

(48) Suppose $Z \subseteq \mathrm{dom}(2\,((^{\frac{1}{2}}_{\mathbb{R}}) \cdot (\text{the function sin})))$ and for every x such that $x \in Z$ holds (the function sin)$(x) > 0$. Then

(i) $2\,((^{\frac{1}{2}}_{\mathbb{R}}) \cdot (\text{the function sin}))$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(2\,((^{\frac{1}{2}}_{\mathbb{R}}) \cdot (\text{the function sin})))'_{\restriction Z}(x) = (\text{the function cos})(x) \cdot (\text{the function sin})(x)_{\mathbb{R}}^{-\frac{1}{2}}$.

(49) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{2}\,((^{2}_{\mathbb{Z}}) \cdot (\text{the function sin})))$. Then

(i) $\frac{1}{2}\,((^{2}_{\mathbb{Z}}) \cdot (\text{the function sin}))$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{2}\,((^{2}_{\mathbb{Z}}) \cdot (\text{the function sin})))'_{\restriction Z}(x) = (\text{the function sin})(x) \cdot (\text{the function cos})(x)$.

(50) Suppose that

(i) $Z \subseteq \mathrm{dom}((\text{the function sin})+\frac{1}{2}\,((^{2}_{\mathbb{Z}}) \cdot (\text{the function sin})))$, and

(ii) for every x such that $x \in Z$ holds (the function sin)$(x) > 0$ and (the function sin)$(x) < 1$.

Then

(iii) $(\text{the function sin})+\frac{1}{2}\,((^{2}_{\mathbb{Z}}) \cdot (\text{the function sin}))$ is differentiable on Z, and

(iv) for every x such that $x \in Z$ holds $((\text{the function sin})+\frac{1}{2}\,((^{2}_{\mathbb{Z}}) \cdot (\text{the function sin})))'_{\restriction Z}(x) = \frac{(\text{the function cos})(x)^3}{1-(\text{the function sin})(x)}$.

(51) Suppose that

(i) $Z \subseteq \mathrm{dom}(\frac{1}{2}\,((^{2}_{\mathbb{Z}}) \cdot (\text{the function sin}))-\text{the function cos})$, and

(ii) for every x such that $x \in Z$ holds (the function sin)$(x) > 0$ and (the function cos)$(x) < 1$.

Then

(iii) $\frac{1}{2}$ $((\frac{2}{\mathbb{Z}})$ · (the function sin))−the function cos is differentiable on Z, and

(iv) for every x such that $x \in Z$ holds ($\frac{1}{2}$ $((\frac{2}{\mathbb{Z}})$ · (the function sin))−the function cos$)'_{\restriction Z}(x) = \frac{(\text{the function sin})(x)^3}{1-(\text{the function cos})(x)}$.

(52) Suppose that

(i) $Z \subseteq$ dom((the function sin)$-\frac{1}{2}$ $((\frac{2}{\mathbb{Z}})$ · (the function sin))), and

(ii) for every x such that $x \in Z$ holds (the function sin)$(x) > 0$ and (the function sin)$(x) > -1$.

Then

(iii) (the function sin)$-\frac{1}{2}$ $((\frac{2}{\mathbb{Z}})$ · (the function sin)) is differentiable on Z, and

(iv) for every x such that $x \in Z$ holds ((the function sin)$-\frac{1}{2}$ $((\frac{2}{\mathbb{Z}})$ · (the function sin)))$'_{\restriction Z}(x) = \frac{(\text{the function cos})(x)^3}{1+(\text{the function sin})(x)}$.

(53) Suppose that

(i) $Z \subseteq$ dom(−the function cos $-\frac{1}{2}$ $((\frac{2}{\mathbb{Z}})$ · (the function sin))), and

(ii) for every x such that $x \in Z$ holds (the function sin)$(x) > 0$ and (the function cos)$(x) > -1$.

Then

(iii) −the function cos $-\frac{1}{2}$ $((\frac{2}{\mathbb{Z}})$ · (the function sin)) is differentiable on Z, and

(iv) for every x such that $x \in Z$ holds (−the function cos $-\frac{1}{2}$ $((\frac{2}{\mathbb{Z}})$ · (the function sin)))$'_{\restriction Z}(x) = \frac{(\text{the function sin})(x)^3}{1+(\text{the function cos})(x)}$.

(54) Suppose $Z \subseteq$ dom($\frac{1}{n}$ $((\frac{n}{\mathbb{Z}})$ · (the function sin))) and $n > 0$. Then

(i) $\frac{1}{n}$ $((\frac{n}{\mathbb{Z}})$ · (the function sin)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ($\frac{1}{n}$ $((\frac{n}{\mathbb{Z}})$ · (the function sin)))$'_{\restriction Z}(x) =$ ((the function sin)$(x)^{n-1}_{\mathbb{Z}}$) · (the function cos)(x).

(55) Suppose $Z \subseteq \mathrm{dom}(\exp f)$ and for every x such that $x \in Z$ holds $f(x) = x - 1$. Then $\exp f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\exp f)'_{\restriction Z}(x) = x \cdot \exp(x)$.

(56) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot \frac{\exp}{\exp + f})$ and for every x such that $x \in Z$ holds $f(x) = 1$. Then $\log\_(e) \cdot \frac{\exp}{\exp + f}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot \frac{\exp}{\exp + f})'_{\restriction Z}(x) = \frac{1}{\exp(x)+1}$.

(57) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot \frac{\exp - f}{\exp})$ and for every x such that $x \in Z$ holds $f(x) = 1$ and $(\exp - f)(x) > 0$. Then $\log\_(e) \cdot \frac{\exp - f}{\exp}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot \frac{\exp - f}{\exp})'_{\restriction Z}(x) = \frac{1}{\exp(x)-1}$.

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Received July 6, 2005