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Circled Sets, Circled Hull, and Circled Family

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Summary. In this article, we prove some basic properties of the circled sets. We also define the circled hull, and give the definition of a circled family.

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The articles [15], [19], [14], [3], [4], [12], [5], [11], [13], [18], [9], [8], [2], [17], [16], [6], [1], [7], and [10] provide the terminology and notation for this paper.

1. Circled Sets

One can prove the following proposition

(1) For every real linear space V and for all circled subsets A, B of V holds $A - B$ is circled.

Let V be a real linear space and let M, N be circled subsets of V. Note that $M - N$ is circled.

Next we state the proposition

(2) Let V be a non empty RLS structure and M be a subset of V. Then M is circled if and only if for every vector u of V and for every real number r such that $|r| \leq 1$ and $u \in M$ holds $r \cdot u \in M$.

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Let V be a non empty RLS structure and let M be a subset of V. Let us observe that M is circled if and only if:

(Def. 1) For every vector u of V and for every real number r such that $|r| \leq 1$ and $u \in M$ holds $r \cdot u \in M$.

The following propositions are true:

(3) Let V be a real linear space, M be a subset of V, and r be a real number. If M is circled, then $r \cdot M$ is circled.

(4) Let V be a real linear space, M_1, M_2 be subsets of V, and r_1, r_2 be real numbers. If M_1 is circled and M_2 is circled, then $r_1 \cdot M_1 + r_2 \cdot M_2$ is circled.

(5) Let V be a real linear space, M_1, M_2, M_3 be subsets of V, and r_1, r_2, r_3 be real numbers. Suppose M_1 is circled and M_2 is circled and M_3 is circled. Then $r_1 \cdot M_1 + r_2 \cdot M_2 + r_3 \cdot M_3$ is circled.

(6) For every real linear space V holds $\mathrm{Up}(\mathbf{0}_V)$ is circled.

(7) For every real linear space V holds $\mathrm{Up}(\Omega_V)$ is circled.

(8) For every real linear space V and for all circled subsets M, N of V holds $M \cap N$ is circled.

(9) For every real linear space V and for all circled subsets M, N of V holds $M \cup N$ is circled.

2. Circled Hull and Circled Family

Let V be a non empty RLS structure and let M be a subset of V. The functor Circled-Family M yields a family of subsets of V and is defined as follows:

(Def. 2) For every subset N of V holds $N \in$ Circled-Family M iff N is circled and $M \subseteq N$.

Let V be a real linear space and let M be a subset of V. The functor Cir M yielding a circled subset of V is defined by:

(Def. 3) $\mathrm{Cir}\, M = \bigcap \text{Circled-Family}\, M$.

Let V be a real linear space and let M be a subset of V. Note that Circled-Family M is non empty.

We now state several propositions:

(10) For every real linear space V and for all subsets M_1, M_2 of V such that $M_1 \subseteq M_2$ holds Circled-Family $M_2 \subseteq$ Circled-Family M_1.

(11) For every real linear space V and for all subsets M_1, M_2 of V such that $M_1 \subseteq M_2$ holds $\mathrm{Cir}\, M_1 \subseteq \mathrm{Cir}\, M_2$.

(12) For every real linear space V and for every subset M of V holds $M \subseteq \mathrm{Cir}\, M$.

(13) Let V be a real linear space, M be a subset of V, and N be a circled subset of V. If $M \subseteq N$, then $\mathrm{Cir}\, M \subseteq N$.

(14) For every real linear space V and for every circled subset M of V holds $\mathrm{Cir}\, M = M$.

(15) For every real linear space V holds $\mathrm{Cir}(\emptyset_V) = \emptyset$.

(16) For every real linear space V and for every subset M of V and for every real number r holds $r \cdot \mathrm{Cir}\, M = \mathrm{Cir}(r \cdot M)$.

3. Basic Properties of Combination

Let V be a real linear space and let L be a linear combination of V. We say that L is circled if and only if the condition (Def. 4) is satisfied.

(Def. 4) There exists a finite sequence F of elements of the carrier of V such that

(i) F is one-to-one,

(ii) $\mathrm{rng}\, F =$ the support of L, and

(iii) there exists a finite sequence f of elements of $\mathbb{R}$ such that $\mathrm{len}\, f = \mathrm{len}\, F$ and $\sum f = 1$ and for every natural number n such that $n \in \mathrm{dom}\, f$ holds $f(n) = L(F(n))$ and $f(n) \geq 0$.

The following propositions are true:

(17) Let V be a real linear space and L be a linear combination of V. If L is circled, then the support of $L \neq \emptyset$.

(18) Let V be a real linear space, L be a linear combination of V, and v be a vector of V. If L is circled and $L(v) \leq 0$, then $v \notin$ the support of L.

(19) For every real linear space V and for every linear combination L of V such that L is circled holds $L \neq \mathbf{0}_{\mathrm{LC}_V}$.

(20) For every real linear space V holds there exists a linear combination of V which is circled.

Let V be a real linear space. One can check that there exists a linear combination of V which is circled.

Let V be a real linear space. A circled combination of V is a circled linear combination of V.

We now state the proposition

(21) For every real linear space V and for every non empty subset M of V holds there exists a linear combination of M which is circled.

Let V be a real linear space and let M be a non empty subset of V. Note that there exists a linear combination of M which is circled.

Let V be a real linear space and let M be a non empty subset of V. A circled combination of M is a circled linear combination of M.

Let V be a real linear space. The functor circledComb V is defined as follows:

(Def. 5) For every set L holds $L \in$ circledComb V iff L is a circled combination of V.

Let V be a real linear space and let M be a non empty subset of V. The functor circledComb M is defined by:

(Def. 6) For every set L holds $L \in$ circledComb M iff L is a circled combination of M.

The following propositions are true:

(22) Let V be a real linear space and v be a vector of V. Then there exists a circled combination L of V such that $\sum L = v$ and for every non empty subset A of V such that $v \in A$ holds L is a circled combination of A.

(23) Let V be a real linear space and v_1, v_2 be vectors of V. Suppose $v_1 \neq v_2$. Then there exists a circled combination L of V such that for every non empty subset A of V if $\{v_1, v_2\} \subseteq A$, then L is a circled combination of A.

(24) Let V be a real linear space, L_1, L_2 be circled combinations of V, and a, b be real numbers. Suppose $a \cdot b > 0$. Then the support of $a \cdot L_1 + b \cdot L_2 =$ (the support of $a \cdot L_1$) $\cup$ (the support of $b \cdot L_2$).

(25) Let V be a real linear space, v be a vector of V, and L be a linear combination of V. If L is circled and the support of $L = \{v\}$, then $L(v) = 1$ and $\sum L = L(v) \cdot v$.

(26) Let V be a real linear space, v_1, v_2 be vectors of V, and L be a linear combination of V. Suppose L is circled and the support of $L = \{v_1, v_2\}$ and $v_1 \neq v_2$. Then $L(v_1) + L(v_2) = 1$ and $L(v_1) \geq 0$ and $L(v_2) \geq 0$ and $\sum L = L(v_1) \cdot v_1 + L(v_2) \cdot v_2$.

(27) Let V be a real linear space, v be a vector of V, and L be a linear combination of $\{v\}$. If L is circled, then $L(v) = 1$ and $\sum L = L(v) \cdot v$.

(28) Let V be a real linear space, v_1, v_2 be vectors of V, and L be a linear combination of $\{v_1, v_2\}$. Suppose $v_1 \neq v_2$ and L is circled. Then $L(v_1) + L(v_2) = 1$ and $L(v_1) \geq 0$ and $L(v_2) \geq 0$ and $\sum L = L(v_1) \cdot v_1 + L(v_2) \cdot v_2$.

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