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Several Differentiable Formulas of Special Functions. Part II

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Summary. In this article, we give several other differentiable formulas of special functions.

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The papers [11], [13], [14], [1], [8], [10], [2], [4], [7], [5], [6], [9], [15], [3], and [12] provide the notation and terminology for this paper.

For simplicity, we use the following convention: x, a denote real numbers, n denotes a natural number, Z denotes an open subset of $\mathbb{R}$, and f, f_1, f_2 denote partial functions from $\mathbb{R}$ to $\mathbb{R}$.

One can prove the following propositions:

(1) If $a > 0$, then $\exp(x \cdot \log_e a) = a^x_{\mathbb{R}}$.

(2) If $a > 0$, then $\exp(-x \cdot \log_e a) = a^{-x}_{\mathbb{R}}$.

(3) Suppose $Z \subseteq \mathrm{dom}(f_1 - f_2)$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $f_2 = \frac{2}{\mathbb{Z}}$. Then $f_1 - f_2$ is differentiable on Z and for every x such that $x \in Z$ holds $(f_1 - f_2)'_{\restriction Z}(x) = -2 \cdot x$.

(4) Suppose $Z \subseteq \mathrm{dom}(\frac{f_1+f_2}{f_1-f_2})$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $(f_1 - f_2)(x) \neq 0$. Then $\frac{f_1+f_2}{f_1-f_2}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{f_1+f_2}{f_1-f_2})'_{\restriction Z}(x) = \frac{4 \cdot a^{\mathbf{2}} \cdot x}{(a^{\mathbf{2}} - x^2)^{\mathbf{2}}}$.

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(5) Suppose $Z \subseteq \mathrm{dom}\, f$ and $f = \log\_(e) \cdot \frac{f_1+f_2}{f_1-f_2}$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $(f_1-f_2)(x) > 0$ and $a \neq 0$. Then f is differentiable on Z and for every x such that $x \in Z$ holds $f'_{\restriction Z}(x) = \frac{4 \cdot a^{\mathbf{2}} \cdot x}{a^4 - x^4}$.

(6) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{4 \cdot a^{\mathbf{2}}}\, f)$ and $f = \log\_(e) \cdot \frac{f_1+f_2}{f_1-f_2}$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $(f_1 - f_2)(x) > 0$ and $a \neq 0$. Then $\frac{1}{4 \cdot a^{\mathbf{2}}}\, f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{1}{4 \cdot a^{\mathbf{2}}}\, f)'_{\restriction Z}(x) = \frac{x}{a^4 - x^4}$.

(7) Suppose $Z \subseteq \mathrm{dom}(\frac{f_1}{f_2+f_1})$ and $f_1 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_2(x) = 1$ and $x \neq 0$. Then $\frac{f_1}{f_2+f_1}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{f_1}{f_2+f_1})'_{\restriction Z}(x) = \frac{2 \cdot x}{(1+x^{\mathbf{2}})^{\mathbf{2}}}$.

(8) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{2}\, f)$ and $f = \log\_(e) \cdot \frac{f_1}{f_2+f_1}$ and $f_1 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_2(x) = 1$ and $x \neq 0$. Then $\frac{1}{2}\, f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{1}{2}\, f)'_{\restriction Z}(x) = \frac{1}{x \cdot (1+x^{\mathbf{2}})}$.

(9) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot \frac{n}{\mathbb{Z}})$ and for every x such that $x \in Z$ holds $x > 0$. Then $\log\_(e) \cdot \frac{n}{\mathbb{Z}}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot \frac{n}{\mathbb{Z}})'_{\restriction Z}(x) = \frac{n}{x}$.

(10) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{f_2} + \log\_(e) \cdot \frac{f_1}{f_2})$ and for every x such that $x \in Z$ holds $f_2(x) = x$ and $f_2(x) > 0$ and $f_1(x) = x - 1$ and $f_1(x) > 0$. Then $\frac{1}{f_2} + \log\_(e) \cdot \frac{f_1}{f_2}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{1}{f_2} + \log\_(e) \cdot \frac{f_1}{f_2})'_{\restriction Z}(x) = \frac{1}{x^{\mathbf{2}} \cdot (x-1)}$.

(11) Suppose $Z \subseteq \mathrm{dom}(\exp \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = x \cdot \log_e a$ and $a > 0$. Then $\exp \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\exp \cdot f)'_{\restriction Z}(x) = (a^x_{\mathbb{R}}) \cdot \log_e a$.

(12) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{\log_e a}\, ((\exp \cdot f_1)\, f_2))$ and for every x such that $x \in Z$ holds $f_1(x) = x \cdot \log_e a$ and $f_2(x) = x - \frac{1}{\log_e a}$ and $a > 0$ and $a \neq 1$. Then $\frac{1}{\log_e a}\, ((\exp \cdot f_1)\, f_2)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{1}{\log_e a}\, ((\exp \cdot f_1)\, f_2))'_{\restriction Z}(x) = x \cdot a^x_{\mathbb{R}}$.

(13) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{1+\log_e a}\, ((\exp \cdot f)\ \exp))$ and for every x such that $x \in Z$ holds $f(x) = x \cdot \log_e a$ and $a > 0$ and $a \neq \frac{1}{e}$. Then $\frac{1}{1+\log_e a}\, ((\exp \cdot f)\ \exp)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{1}{1+\log_e a}\, ((\exp \cdot f)\ \exp))'_{\restriction Z}(x) = (a^x_{\mathbb{R}}) \cdot \exp(x)$.

(14) Suppose $Z \subseteq \mathrm{dom}(\exp \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = -x$. Then $\exp \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\exp \cdot f)'_{\restriction Z}(x) = -\exp(-x)$.

(15) Suppose $Z \subseteq \mathrm{dom}(f_1\, (\exp \cdot f_2))$ and for every x such that $x \in Z$ holds $f_1(x) = -x - 1$ and $f_2(x) = -x$. Then $f_1\, (\exp \cdot f_2)$ is differentiable on Z and for every x such that $x \in Z$ holds $(f_1\, (\exp \cdot f_2))'_{\restriction Z}(x) = \frac{x}{\exp x}$.

(16) Suppose $Z \subseteq \mathrm{dom}(-\exp \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = -x \cdot \log_e a$ and $a > 0$. Then $-\exp \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $(-\exp \cdot f)'_{\restriction Z}(x) = (a_{\mathbb{R}}^{-x}) \cdot \log_e a$.

(17) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{\log_e a}\,((-\exp \cdot f_1)\, f_2))$ and for every x such that $x \in Z$ holds $f_1(x) = -x \cdot \log_e a$ and $f_2(x) = x + \frac{1}{\log_e a}$ and $a > 0$ and $a \neq 1$. Then $\frac{1}{\log_e a}\,((-\exp \cdot f_1)\, f_2)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{1}{\log_e a}\,((-\exp \cdot f_1)\, f_2))'_{\restriction Z}(x) = \frac{x}{a_{\mathbb{R}}^{x}}$.

(18) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{\log_e a - 1}\,\frac{\exp \cdot f}{\exp})$ and for every x such that $x \in Z$ holds $f(x) = x \cdot \log_e a$ and $a > 0$ and $a \neq e$. Then $\frac{1}{\log_e a - 1}\,\frac{\exp \cdot f}{\exp}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{1}{\log_e a - 1}\,\frac{\exp \cdot f}{\exp})'_{\restriction Z}(x) = \frac{a_{\mathbb{R}}^{x}}{\exp(x)}$.

(19) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{1 - \log_e a}\,\frac{\exp}{\exp \cdot f})$ and for every x such that $x \in Z$ holds $f(x) = x \cdot \log_e a$ and $a > 0$ and $a \neq e$. Then $\frac{1}{1 - \log_e a}\,\frac{\exp}{\exp \cdot f}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{1}{1 - \log_e a}\,\frac{\exp}{\exp \cdot f})'_{\restriction Z}(x) = \frac{\exp(x)}{a_{\mathbb{R}}^{x}}$.

(20) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot (\exp + f))$ and for every x such that $x \in Z$ holds $f(x) = 1$. Then $\log\_(e) \cdot (\exp + f)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot (\exp + f))'_{\restriction Z}(x) = \frac{\exp(x)}{\exp(x) + 1}$.

(21) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot (\exp - f))$ and for every x such that $x \in Z$ holds $f(x) = 1$ and $(\exp - f)(x) > 0$. Then $\log\_(e) \cdot (\exp - f)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot (\exp - f))'_{\restriction Z}(x) = \frac{\exp(x)}{\exp(x) - 1}$.

(22) Suppose $Z \subseteq \mathrm{dom}(-\log\_(e) \cdot (f - \exp))$ and for every x such that $x \in Z$ holds $f(x) = 1$ and $(f - \exp)(x) > 0$. Then $-\log\_(e) \cdot (f - \exp)$ is differentiable on Z and for every x such that $x \in Z$ holds $(-\log\_(e) \cdot (f - \exp))'_{\restriction Z}(x) = \frac{\exp(x)}{1 - \exp(x)}$.

(23) Suppose $Z \subseteq \mathrm{dom}((\frac{2}{\mathbb{Z}}) \cdot \exp + f)$ and for every x such that $x \in Z$ holds $f(x) = 1$. Then $(\frac{2}{\mathbb{Z}}) \cdot \exp + f$ is differentiable on Z and for every x such that $x \in Z$ holds $((\frac{2}{\mathbb{Z}}) \cdot \exp + f)'_{\restriction Z}(x) = 2 \cdot \exp(2 \cdot x)$.

(24) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{2}\,(\log\_(e) \cdot f))$ and $f = (\frac{2}{\mathbb{Z}}) \cdot \exp + f_1$ and for every x such that $x \in Z$ holds $f_1(x) = 1$. Then $\frac{1}{2}\,(\log\_(e) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{1}{2}\,(\log\_(e) \cdot f))'_{\restriction Z}(x) = \frac{\exp x}{\exp x + \exp(-x)}$.

(25) Suppose $Z \subseteq \mathrm{dom}((\frac{2}{\mathbb{Z}}) \cdot \exp - f)$ and for every x such that $x \in Z$ holds $f(x) = 1$. Then $(\frac{2}{\mathbb{Z}}) \cdot \exp - f$ is differentiable on Z and for every x such that $x \in Z$ holds $((\frac{2}{\mathbb{Z}}) \cdot \exp - f)'_{\restriction Z}(x) = 2 \cdot \exp(2 \cdot x)$.

(26) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{2}\,(\log\_(e) \cdot f))$ and $f = (\frac{2}{\mathbb{Z}}) \cdot \exp - f_1$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f(x) > 0$. Then $\frac{1}{2}\,(\log\_(e) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{1}{2}\,(\log\_(e) \cdot f))'_{\restriction Z}(x) = \frac{\exp x}{\exp x - \exp(-x)}$.

(27) Suppose $Z \subseteq \mathrm{dom}((\frac{2}{\mathbb{Z}}) \cdot (\exp - f))$ and for every x such that $x \in Z$ holds $f(x) = 1$. Then $(\frac{2}{\mathbb{Z}}) \cdot (\exp - f)$ is differentiable on Z and for every x such that $x \in Z$ holds $((\frac{2}{\mathbb{Z}}) \cdot (\exp - f))'_{\restriction Z}(x) = 2 \cdot \exp(x) \cdot (\exp(x) - 1)$.

(28) Suppose $Z \subseteq \mathrm{dom}\, f$ and $f = \log\_(e) \cdot \frac{(\frac{2}{\mathbb{Z}}) \cdot (\exp - f_1)}{\exp}$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $(\exp - f_1)(x) > 0$. Then f is differentiable on Z and for every x such that $x \in Z$ holds $f'_{\restriction Z}(x) = \frac{\exp(x)+1}{\exp(x)-1}$.

(29) Suppose $Z \subseteq \mathrm{dom}((\frac{2}{\mathbb{Z}}) \cdot (\exp + f))$ and for every x such that $x \in Z$ holds $f(x) = 1$. Then $(\frac{2}{\mathbb{Z}}) \cdot (\exp + f)$ is differentiable on Z and for every x such that $x \in Z$ holds $((\frac{2}{\mathbb{Z}}) \cdot (\exp + f))'_{\restriction Z}(x) = 2 \cdot \exp(x) \cdot (\exp(x) + 1)$.

(30) Suppose $Z \subseteq \mathrm{dom}\, f$ and $f = \log\_(e) \cdot \frac{(\frac{2}{\mathbb{Z}}) \cdot (\exp + f_1)}{\exp}$ and for every x such that $x \in Z$ holds $f_1(x) = 1$. Then f is differentiable on Z and for every x such that $x \in Z$ holds $f'_{\restriction Z}(x) = \frac{\exp(x)-1}{\exp(x)+1}$.

(31) Suppose $Z \subseteq \mathrm{dom}((\frac{2}{\mathbb{Z}}) \cdot (f - \exp))$ and for every x such that $x \in Z$ holds $f(x) = 1$. Then $(\frac{2}{\mathbb{Z}}) \cdot (f - \exp)$ is differentiable on Z and for every x such that $x \in Z$ holds $((\frac{2}{\mathbb{Z}}) \cdot (f - \exp))'_{\restriction Z}(x) = -2 \cdot \exp(x) \cdot (1 - \exp(x))$.

(32) Suppose $Z \subseteq \mathrm{dom}\, f$ and $f = \log\_(e) \cdot \frac{\exp}{(\frac{2}{\mathbb{Z}}) \cdot (f_1 - \exp)}$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $(f_1 - \exp)(x) > 0$. Then f is differentiable on Z and for every x such that $x \in Z$ holds $f'_{\restriction Z}(x) = \frac{1+\exp(x)}{1-\exp(x)}$.

(33) Suppose $Z \subseteq \mathrm{dom}\, f$ and $f = \log\_(e) \cdot \frac{\exp}{(\frac{2}{\mathbb{Z}}) \cdot (f_1 + \exp)}$ and for every x such that $x \in Z$ holds $f_1(x) = 1$. Then f is differentiable on Z and for every x such that $x \in Z$ holds $f'_{\restriction Z}(x) = \frac{1-\exp(x)}{1+\exp(x)}$.

(34) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot f)$ and $f = \exp + \exp \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = -x$. Then $\log\_(e) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot f)'_{\restriction Z}(x) = \frac{\exp x - \exp(-x)}{\exp x + \exp(-x)}$.

(35) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot f)$ and $f = \exp - \exp \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = -x$ and $f(x) > 0$. Then $\log\_(e) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot f)'_{\restriction Z}(x) = \frac{\exp x + \exp(-x)}{\exp x - \exp(-x)}$.

(36) Suppose $Z \subseteq \mathrm{dom}(\frac{2}{3}\, ((\frac{\frac{3}{2}}{\mathbb{R}}) \cdot (f + \exp)))$ and for every x such that $x \in Z$ holds $f(x) = 1$. Then $\frac{2}{3}\, ((\frac{\frac{3}{2}}{\mathbb{R}}) \cdot (f + \exp))$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{2}{3}\, ((\frac{\frac{3}{2}}{\mathbb{R}}) \cdot (f + \exp)))'_{\restriction Z}(x) = \exp(x) \cdot (1 + \exp(x))^{\frac{1}{2}}_{\mathbb{R}}$.

(37) Suppose $Z \subseteq \mathrm{dom}(\frac{2}{3 \cdot \log_e a}\, ((\frac{\frac{3}{2}}{\mathbb{R}}) \cdot (f + \exp \cdot f_1)))$ and for every x such that $x \in Z$ holds $f(x) = 1$ and $f_1(x) = x \cdot \log_e a$ and $a > 0$ and $a \neq 1$. Then $\frac{2}{3 \cdot \log_e a}\, ((\frac{\frac{3}{2}}{\mathbb{R}}) \cdot (f + \exp \cdot f_1))$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{2}{3 \cdot \log_e a}\, ((\frac{\frac{3}{2}}{\mathbb{R}}) \cdot (f + \exp \cdot f_1)))'_{\restriction Z}(x) = (a^{x}_{\mathbb{R}}) \cdot (1 + a^{x}_{\mathbb{R}})^{\frac{1}{2}}_{\mathbb{R}}$.

(38) Suppose $Z \subseteq \mathrm{dom}((-\frac{1}{2})\,((\text{the function cos})\cdot f))$ and for every x such that $x \in Z$ holds $f(x) = 2 \cdot x$. Then

(i) $(-\frac{1}{2})\,((\text{the function cos})\cdot f)$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((-\frac{1}{2})\,((\text{the function cos})\cdot f))'_{\restriction Z}(x) = \sin(2 \cdot x)$.

(39) Suppose that

(i) $Z \subseteq \mathrm{dom}(2\,((\frac{1}{2})_{\mathbb{R}}) \cdot (f - \text{the function cos})))$, and

(ii) for every x such that $x \in Z$ holds $f(x) = 1$ and $(\text{the function sin})(x) > 0$ and $(\text{the function cos})(x) < 1$ and $(\text{the function cos})(x) > -1$.

Then

(iii) $2\,((\frac{1}{2})_{\mathbb{R}}) \cdot (f - \text{the function cos}))$ is differentiable on Z, and

(iv) for every x such that $x \in Z$ holds $(2\,((\frac{1}{2})_{\mathbb{R}}) \cdot (f - \text{the function cos})))'_{\restriction Z}(x) = (1 + (\text{the function cos})(x))_{\mathbb{R}}^{\frac{1}{2}}$.

(40) Suppose that

(i) $Z \subseteq \mathrm{dom}((-2)\,((\frac{1}{2})_{\mathbb{R}}) \cdot (f + \text{the function cos})))$, and

(ii) for every x such that $x \in Z$ holds $f(x) = 1$ and $(\text{the function sin})(x) > 0$ and $(\text{the function cos})(x) < 1$ and $(\text{the function cos})(x) > -1$.

Then

(iii) $(-2)\,((\frac{1}{2})_{\mathbb{R}}) \cdot (f + \text{the function cos}))$ is differentiable on Z, and

(iv) for every x such that $x \in Z$ holds $((-2)\,((\frac{1}{2})_{\mathbb{R}}) \cdot (f + \text{the function cos})))'_{\restriction Z}(x) = (1 - (\text{the function cos})(x))_{\mathbb{R}}^{\frac{1}{2}}$.

(41) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{2}\,(\log\_(e) \cdot f))$ and $f = f_1 + 2\,(\text{the function sin})$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f(x) > 0$. Then

(i) $\frac{1}{2}\,(\log\_(e) \cdot f)$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{2}\,(\log\_(e) \cdot f))'_{\restriction Z}(x) = \frac{(\text{the function cos})(x)}{1+2\cdot(\text{the function sin})(x)}$.

(42) Suppose $Z \subseteq \mathrm{dom}((-\frac{1}{2})\,(\log\_(e) \cdot f))$ and $f = f_1 + 2\,(\text{the function cos})$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f(x) > 0$. Then

(i) $(-\frac{1}{2})\,(\log\_(e) \cdot f)$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((-\frac{1}{2})\,(\log\_(e) \cdot f))'_{\restriction Z}(x) = \frac{(\text{the function sin})(x)}{1+2\cdot(\text{the function cos})(x)}$.

(43) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{4\cdot a}\,((\text{the function sin})\cdot f))$ and for every x such that $x \in Z$ holds $f(x) = 2 \cdot a \cdot x$ and $a \neq 0$. Then

(i) $\frac{1}{4\cdot a}\,((\text{the function sin})\cdot f)$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{4\cdot a}\,((\text{the function sin})\cdot f))'_{\restriction Z}(x) = \frac{1}{2} \cdot \cos(2 \cdot a \cdot x)$.

(44) Suppose $Z \subseteq \mathrm{dom}(f_1 - \frac{1}{4\cdot a}\,((\text{the function sin})\cdot f))$ and for every x such that $x \in Z$ holds $f_1(x) = \frac{x}{2}$ and $f(x) = 2 \cdot a \cdot x$ and $a \neq 0$. Then

(i) $f_1 - \frac{1}{4 \cdot a}$ ((the function sin) $\cdot f$) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(f_1 - \frac{1}{4 \cdot a}$ ((the function sin) $\cdot f))'_{\restriction Z}(x) = (\sin(a \cdot x))^{\mathbf{2}}$.

(45) Suppose $Z \subseteq \mathrm{dom}(f_1 + \frac{1}{4 \cdot a}$ ((the function sin) $\cdot f))$ and for every x such that $x \in Z$ holds $f_1(x) = \frac{x}{2}$ and $f(x) = 2 \cdot a \cdot x$ and $a \neq 0$. Then

(i) $f_1 + \frac{1}{4 \cdot a}$ ((the function sin) $\cdot f$) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(f_1 + \frac{1}{4 \cdot a}$ ((the function sin) $\cdot f))'_{\restriction Z}(x) = (\cos(a \cdot x))^{\mathbf{2}}$.

(46) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{n}$ $((^{n}_{\mathbb{Z}})$ $\cdot$ (the function cos))) and $n > 0$. Then

(i) $\frac{1}{n}$ $((^{n}_{\mathbb{Z}})$ $\cdot$ (the function cos)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{n}$ $((^{n}_{\mathbb{Z}})$ $\cdot$ (the function cos)))$'_{\restriction Z}(x) =$ $-$((the function cos)$(x)^{n-1}_{\mathbb{Z}}$) $\cdot$ (the function sin)(x).

(47) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{3}$ $((^{3}_{\mathbb{Z}})$ $\cdot$ (the function cos))$-$the function cos) and $n > 0$. Then

(i) $\frac{1}{3}$ $((^{3}_{\mathbb{Z}})$ $\cdot$ (the function cos))$-$the function cos is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{3}$ $((^{3}_{\mathbb{Z}})$ $\cdot$ (the function cos))$-$the function cos$)'_{\restriction Z}(x) =$ (the function sin)$(x)^3$.

(48) Suppose $Z \subseteq \mathrm{dom}($(the function sin)$-\frac{1}{3}$ $((^{3}_{\mathbb{Z}})$ $\cdot$ (the function sin))) and $n > 0$. Then

(i) (the function sin)$-\frac{1}{3}$ $((^{3}_{\mathbb{Z}})$ $\cdot$ (the function sin)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin)$-\frac{1}{3}$ $((^{3}_{\mathbb{Z}})$ $\cdot$ (the function sin)))$'_{\restriction Z}(x) =$ (the function cos)$(x)^3$.

(49) Suppose $Z \subseteq \mathrm{dom}$((the function sin) $\cdot \log\_(e)$). Then

(i) (the function sin) $\cdot \log\_(e)$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin) $\cdot \log\_(e))'_{\restriction Z}(x) = \frac{\text{(the function cos)}(\log_e x)}{x}$.

(50) Suppose $Z \subseteq \mathrm{dom}(-$(the function cos) $\cdot \log\_(e)$). Then

(i) $-$(the function cos) $\cdot \log\_(e)$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ($-$(the function cos) $\cdot \log\_(e))'_{\restriction Z}(x) = \frac{\text{(the function sin)}(\log_e x)}{x}$.

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