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The Fashoda Meet Theorem for Continuous Mappings

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The articles [21], [25], [2], [20], [26], [5], [27], [6], [3], [1], [24], [10], [18], [16], [9], [4], [13], [11], [19], [23], [17], [7], [8], [22], [12], [15], and [14] provide the terminology and notation for this paper.

We use the following convention: n is a natural number, p_1, p_2 are points of $\mathcal{E}^n_{\rm T}$, and a, b, c, d are real numbers.

Let us consider a, b, c, d. One can verify that ClosedInsideOfRectangle(a,b,c,d) is convex.

Let us consider a, b, c, d. Observe that Trectangle(a,b,c,d) is convex.

The following propositions are true:

(1) Let e be a positive real number and g be a continuous map from $\mathbb{I}$ into $\mathcal{E}^n_{\rm T}$. Then there exists a finite sequence h of elements of $\mathbb{R}$ such that

(i) $h(1) = 0$,

(ii) $h(\operatorname{len} h) = 1$,

(iii) $5 \leq \operatorname{len} h$,

(iv) $\operatorname{rng} h \subseteq$ the carrier of $\mathbb{I}$,

(v) h is increasing, and

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(vi) for every natural number i and for every subset Q of $\mathbb{I}$ and for every subset W of $\mathcal{E}^n$ such that $1 \leq i$ and $i < \operatorname{len} h$ and $Q = [h_i, h_{i+1}]$ and $W = g^\circ Q$ holds $\varnothing W < e$.

(2) For every subset P of $\mathcal{E}^n_{\rm T}$ such that $P \subseteq \mathcal{L}(p_1, p_2)$ and $p_1 \in P$ and $p_2 \in P$ and P is connected holds $P = \mathcal{L}(p_1, p_2)$.

(3) For every path g from p_1 to p_2 such that $\operatorname{rng} g \subseteq \mathcal{L}(p_1, p_2)$ holds $\operatorname{rng} g = \mathcal{L}(p_1, p_2)$.

(4) Let P, Q be non empty subsets of $\mathcal{E}^2_{\rm T}$, p_1, p_2, q_1, q_2 be points of $\mathcal{E}^2_{\rm T}$, f be a path from p_1 to p_2, and g be a path from q_1 to q_2. Suppose that

(i) $\operatorname{rng} f = P$,

(ii) $\operatorname{rng} g = Q$,

(iii) for every point p of $\mathcal{E}^2_{\rm T}$ such that $p \in P$ holds $(p_1)_{\mathbf{1}} \leq p_{\mathbf{1}}$ and $p_{\mathbf{1}} \leq (p_2)_{\mathbf{1}}$,

(iv) for every point p of $\mathcal{E}^2_{\rm T}$ such that $p \in Q$ holds $(p_1)_{\mathbf{1}} \leq p_{\mathbf{1}}$ and $p_{\mathbf{1}} \leq (p_2)_{\mathbf{1}}$,

(v) for every point p of $\mathcal{E}^2_{\rm T}$ such that $p \in P$ holds $(q_1)_{\mathbf{2}} \leq p_{\mathbf{2}}$ and $p_{\mathbf{2}} \leq (q_2)_{\mathbf{2}}$, and

(vi) for every point p of $\mathcal{E}^2_{\rm T}$ such that $p \in Q$ holds $(q_1)_{\mathbf{2}} \leq p_{\mathbf{2}}$ and $p_{\mathbf{2}} \leq (q_2)_{\mathbf{2}}$.

Then P meets Q.

(5) Let f, g be continuous maps from $\mathbb{I}$ into $\mathcal{E}^2_{\rm T}$ and O, I be points of $\mathbb{I}$. Suppose that $O = 0$ and $I = 1$ and $f(O)_{\mathbf{1}} = a$ and $f(I)_{\mathbf{1}} = b$ and $g(O)_{\mathbf{2}} = c$ and $g(I)_{\mathbf{2}} = d$ and for every point r of $\mathbb{I}$ holds $a \leq f(r)_{\mathbf{1}}$ and $f(r)_{\mathbf{1}} \leq b$ and $a \leq g(r)_{\mathbf{1}}$ and $g(r)_{\mathbf{1}} \leq b$ and $c \leq f(r)_{\mathbf{2}}$ and $f(r)_{\mathbf{2}} \leq d$ and $c \leq g(r)_{\mathbf{2}}$ and $g(r)_{\mathbf{2}} \leq d$. Then $\operatorname{rng} f$ meets $\operatorname{rng} g$.

(6) Let a_1, b_1, c_1, d_1 be points of Trectangle(a, b, c, d), h be a path from a_1 to b_1, v be a path from d_1 to c_1, and A_1, B_1, C_1, D_1 be points of $\mathcal{E}^2_{\rm T}$. Suppose $(A_1)_{\mathbf{1}} = a$ and $(B_1)_{\mathbf{1}} = b$ and $(C_1)_{\mathbf{2}} = c$ and $(D_1)_{\mathbf{2}} = d$ and $a_1 = A_1$ and $b_1 = B_1$ and $c_1 = C_1$ and $d_1 = D_1$. Then there exist points s, t of $\mathbb{I}$ such that $h(s) = v(t)$.

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