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Partial Sum and Partial Product of Some Series

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Summary. This article contains partial sum and partial product of some series which are often used.

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The notation and terminology used in this paper have been introduced in the following articles: [2], [1], [3], [4], [5], [6], and [7].

We use the following convention: n is a natural number, a, b, c, d are real numbers, and s is a sequence of real numbers.

We now state a number of propositions:

(1) $(a+b+c)^2 = a^2+b^2+c^2+2\cdot a\cdot b+2\cdot a\cdot c+2\cdot b\cdot c$.

(2) $(a+b)^3 = a^3+3\cdot a^2\cdot b+3\cdot b^2\cdot a+b^3$.

(3) $((a-b)+c)^2 = (((a^2+b^2+c^2)-2\cdot a\cdot b)+2\cdot a\cdot c)-2\cdot b\cdot c$.

(4) $(a-b-c)^2 = ((a^2+b^2+c^2)-2\cdot a\cdot b-2\cdot a\cdot c)+2\cdot b\cdot c$.

(5) $(a-b)^3 = ((a^3-3\cdot a^2\cdot b)+3\cdot b^2\cdot a)-b^3$.

(6) $(a+b)^4 = a^4+4\cdot a^3\cdot b+6\cdot a^2\cdot b^2+4\cdot b^3\cdot a+b^4$.

(7) $(a+b+c+d)^2 = a^2+b^2+c^2+d^2+(2\cdot a\cdot b+2\cdot a\cdot c+2\cdot a\cdot d)+(2\cdot b\cdot c+2\cdot b\cdot d)+2\cdot c\cdot d$.

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(8) $(a+b+c)^3 = a^3+b^3+c^3+(3\cdot a^2\cdot b+3\cdot a^2\cdot c)+(3\cdot b^2\cdot a+3\cdot b^2\cdot c)+(3\cdot c^2\cdot a+3\cdot c^2\cdot b)+6\cdot a\cdot b\cdot c$.

(9) If $a \neq 0$, then $((\frac{1}{a})^{n+1}+a^{n+1})^2 = (\frac{1}{a})^{2\cdot n+2}+a^{2\cdot n+2}+2$.

(10) If $a \neq 1$ and for every n holds $s(n) = a^n$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{1-a^{n+1}}{1-a}$.

(11) If $a \neq 1$ and $a \neq 0$ and for every n holds $s(n) = (\frac{1}{a})^n$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{(\frac{1}{a})^n-a}{1-a}$.

(12) If for every n holds $s(n) = 10^n+2\cdot n+1$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = (\frac{10^{n+1}}{9}-\frac{1}{9})+(n+1)^2$.

(13) If for every n holds $s(n) = (2\cdot n-1)+(\frac{1}{2})^n$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = (n^2+1)-(\frac{1}{2})^n$.

(14) If for every n holds $s(n) = n\cdot(\frac{1}{2})^n$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = 2-(2+n)\cdot(\frac{1}{2})^n$.

(15) If for every n holds $s(n) = ((\frac{1}{2})^n+2^n)^2$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = -\frac{(\frac{1}{4})^n}{3}+\frac{4^{n+1}}{3}+2\cdot n+3$.

(16) If for every n holds $s(n) = ((\frac{1}{3})^n+3^n)^2$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = -\frac{(\frac{1}{9})^n}{8}+\frac{9^{n+1}}{8}+2\cdot n+3$.

(17) If for every n holds $s(n) = n\cdot 2^n$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = (n\cdot 2^{n+1}-2^{n+1})+2$.

(18) If for every n holds $s(n) = (2\cdot n+1)\cdot 3^n$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = n\cdot 3^{n+1}+1$.

(19) If $a \neq 1$ and for every n holds $s(n) = n\cdot a^n$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{a\cdot(1-a^n)}{(1-a)^2}-\frac{n\cdot a^{n+1}}{1-a}$.

(20) If for every n holds $s(n) = \frac{1}{(\mathrm{root}_2(n+1))+(\mathrm{root}_2(n))}$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = \mathrm{root}_2(n+1)$.

(21) If for every n holds $s(n) = 2^n+(\frac{1}{2})^n$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = (2^{n+1}-(\frac{1}{2})^n)+1$.

(22) If for every n holds $s(n) = n!\cdot n+\frac{n}{(n+1)!}$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = (n+1)!-\frac{1}{(n+1)!}$.

(23) Suppose $a \neq 1$ and for every n such that $n \geq 1$ holds $s(n) = (\frac{a}{a-1})^n$ and $s(0) = 0$. Let given n. If $n \geq 1$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = a\cdot((\frac{a}{a-1})^n-1)$.

(24) If for every n such that $n \geq 1$ holds $s(n) = 2^n\cdot\frac{3\cdot n-1}{4}$ and $s(0) = 0$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) = 2^n\cdot\frac{3\cdot n-4}{2}+2$.

(25) If for every n holds $s(n) = \frac{n+1}{n+2}$, then (the partial product of s)$(n) = \frac{1}{n+2}$.

(26) If for every n holds $s(n) = \frac{1}{n+1}$, then (the partial product of s)$(n) = \frac{1}{(n+1)!}$.

(27) Suppose that for every n such that $n \geq 1$ holds $s(n) = n$ and $s(0) = 1$. Let given n. If $n \geq 1$, then (the partial product of $s)(n) = n!$.

(28) Suppose that for every n such that $n \geq 1$ holds $s(n) = \frac{a}{n}$ and $s(0) = 1$. Let given n. If $n \geq 1$, then (the partial product of $s)(n) = \frac{a^n}{n!}$.

(29) Suppose that for every n such that $n \geq 1$ holds $s(n) = a$ and $s(0) = 1$. Let given n. If $n \geq 1$, then (the partial product of $s)(n) = a^n$.

(30) Suppose that for every n such that $n \geq 2$ holds $s(n) = 1 - \frac{1}{n^2}$ and $s(0) = 1$ and $s(1) = 1$. Let given n. If $n \geq 2$, then (the partial product of $s)(n) = \frac{n+1}{2 \cdot n}$.

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