FORMALIZED MATHEMATICS
Volume 14, Number 1, Pages 37–45
University of Białystok, 2006

Several Differentiation Formulas of Special Functions. Part III

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Summary. In this article, we give several differentiation formulas of special and composite functions including trigonometric function, inverse trigonometric function, polynomial function and logarithmic function.

MML identifier: FDIFF_7, version: 7.7.01 4.60.940

The articles [13], [15], [16], [1], [4], [10], [11], [17], [5], [14], [12], [2], [6], [9], [7], [8], and [3] provide the terminology and notation for this paper.

For simplicity, we follow the rules: x, r, a, b denote real numbers, n denotes a natural number, Z denotes an open subset of $\mathbb{R}$, and f, f_1, f_2, f_3 denote partial functions from $\mathbb{R}$ to $\mathbb{R}$.

One can prove the following propositions:

(1) $x_{\mathbb{Z}}^{2} = x^{\mathbf{2}}$.

(2) If $x > 0$, then $x_{\mathbb{R}}^{\frac{1}{2}} = \sqrt{x}$.

(3) If $x > 0$, then $x_{\mathbb{R}}^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$.

(4) Suppose $Z \subseteq]-1,1[$ and $Z \subseteq \mathrm{dom}(r\,(\text{the function arcsin}))$. Then

(i) $r\,(\text{the function arcsin})$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(r\,(\text{the function arcsin}))'_{\restriction Z}(x) = \frac{r}{\sqrt{1-x^{\mathbf{2}}}}$.

ISSN 1426–2630

(5) Suppose $Z \subseteq]-1, 1[$ and $Z \subseteq \mathrm{dom}(r\,(\text{the function arccos}))$. Then
(i) $r\,(\text{the function arccos})$ is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $(r\,(\text{the function arccos}))'_{\restriction Z}(x) = -\frac{r}{\sqrt{1-x^{\mathbf{2}}}}$.

(6) Suppose f is differentiable in x and $f(x) > -1$ and $f(x) < 1$. Then (the function arcsin) $\cdot f$ is differentiable in x and $((\text{the function arcsin}) \cdot f)'(x) = \frac{f'(x)}{\sqrt{1-f(x)^{\mathbf{2}}}}$.

(7) Suppose f is differentiable in x and $f(x) > -1$ and $f(x) < 1$. Then (the function arccos) $\cdot f$ is differentiable in x and $((\text{the function arccos}) \cdot f)'(x) = -\frac{f'(x)}{\sqrt{1-f(x)^{\mathbf{2}}}}$.

(8) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot (\text{the function arcsin}))$ and $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $(\text{the function arcsin})(x) > 0$. Then
(i) $\log\_(e) \cdot (\text{the function arcsin})$ is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $(\log\_(e) \cdot (\text{the function arcsin}))'_{\restriction Z}(x) = \frac{1}{\sqrt{1-x^{\mathbf{2}}} \cdot (\text{the function arcsin})(x)}$.

(9) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot (\text{the function arccos}))$ and $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $(\text{the function arccos})(x) > 0$. Then
(i) $\log\_(e) \cdot (\text{the function arccos})$ is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $(\log\_(e) \cdot (\text{the function arccos}))'_{\restriction Z}(x) = -\frac{1}{\sqrt{1-x^{\mathbf{2}}} \cdot (\text{the function arccos})(x)}$.

(10) Suppose $Z \subseteq \mathrm{dom}((\binom{n}{\mathbb{Z}}) \cdot (\text{the function arcsin}))$ and $Z \subseteq]-1, 1[$. Then
(i) $(\binom{n}{\mathbb{Z}}) \cdot (\text{the function arcsin})$ is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $((\binom{n}{\mathbb{Z}}) \cdot (\text{the function arcsin}))'_{\restriction Z}(x) = \frac{n \cdot (\text{the function arcsin})(x)^{n-1}_{\mathbb{Z}}}{\sqrt{1-x^{\mathbf{2}}}}$.

(11) Suppose $Z \subseteq \mathrm{dom}((\binom{n}{\mathbb{Z}}) \cdot (\text{the function arccos}))$ and $Z \subseteq]-1, 1[$. Then
(i) $(\binom{n}{\mathbb{Z}}) \cdot (\text{the function arccos})$ is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $((\binom{n}{\mathbb{Z}}) \cdot (\text{the function arccos}))'_{\restriction Z}(x) = -\frac{n \cdot (\text{the function arccos})(x)^{n-1}_{\mathbb{Z}}}{\sqrt{1-x^{\mathbf{2}}}}$.

(12) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{2}\,((\binom{2}{\mathbb{Z}}) \cdot (\text{the function arcsin})))$ and $Z \subseteq]-1, 1[$. Then
(i) $\frac{1}{2}\,((\binom{2}{\mathbb{Z}}) \cdot (\text{the function arcsin}))$ is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $(\frac{1}{2}\,((\binom{2}{\mathbb{Z}}) \cdot (\text{the function arcsin})))'_{\restriction Z}(x) = \frac{(\text{the function arcsin})(x)}{\sqrt{1-x^{\mathbf{2}}}}$.

(13) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{2}\,((\binom{2}{\mathbb{Z}}) \cdot (\text{the function arccos})))$ and $Z \subseteq]-1, 1[$. Then
(i) $\frac{1}{2}\,((\binom{2}{\mathbb{Z}}) \cdot (\text{the function arccos}))$ is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $(\frac{1}{2}\,((\binom{2}{\mathbb{Z}}) \cdot (\text{the function arccos})))'_{\restriction Z}(x) = -\frac{(\text{the function arccos})(x)}{\sqrt{1-x^{\mathbf{2}}}}$.

(14) Suppose $Z \subseteq \mathrm{dom}((\text{the function arcsin}) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$ and $f(x) > -1$ and $f(x) < 1$. Then
 (i) (the function arcsin) $\cdot f$ is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds $((\text{the function arcsin}) \cdot f)'_{\restriction Z}(x) = \frac{a}{\sqrt{1-(a \cdot x+b)^{\mathbf{2}}}}$.

(15) Suppose $Z \subseteq \mathrm{dom}((\text{the function arccos}) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$ and $f(x) > -1$ and $f(x) < 1$. Then
 (i) (the function arccos) $\cdot f$ is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds $((\text{the function arccos}) \cdot f)'_{\restriction Z}(x) = -\frac{a}{\sqrt{1-(a \cdot x+b)^{\mathbf{2}}}}$.

(16) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z\,(\text{the function arcsin}))$ and $Z \subseteq\ ]-1,1[$. Then
 (i) id_Z (the function arcsin) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds $(\mathrm{id}_Z\,(\text{the function arcsin}))'_{\restriction Z}(x) = (\text{the function arcsin})(x) + \frac{x}{\sqrt{1-x^{\mathbf{2}}}}$.

(17) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z\,(\text{the function arccos}))$ and $Z \subseteq\ ]-1,1[$. Then
 (i) id_Z (the function arccos) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds $(\mathrm{id}_Z\,(\text{the function arccos}))'_{\restriction Z}(x) = (\text{the function arccos})(x) - \frac{x}{\sqrt{1-x^{\mathbf{2}}}}$.

(18) Suppose $Z \subseteq \mathrm{dom}(f\,(\text{the function arcsin}))$ and $Z \subseteq\ ]-1,1[$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$. Then
 (i) f (the function arcsin) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds $(f\,(\text{the function arcsin}))'_{\restriction Z}(x) = a \cdot (\text{the function arcsin})(x) + \frac{a \cdot x+b}{\sqrt{1-x^{\mathbf{2}}}}$.

(19) Suppose $Z \subseteq \mathrm{dom}(f\,(\text{the function arccos}))$ and $Z \subseteq\ ]-1,1[$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$. Then
 (i) f (the function arccos) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds $(f\,(\text{the function arccos}))'_{\restriction Z}(x) = a \cdot (\text{the function arccos})(x) - \frac{a \cdot x+b}{\sqrt{1-x^{\mathbf{2}}}}$.

(20) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{2}\,((\text{the function arcsin}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = 2 \cdot x$ and $f(x) > -1$ and $f(x) < 1$. Then
 (i) $\frac{1}{2}\,((\text{the function arcsin}) \cdot f)$ is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds $(\frac{1}{2}\,((\text{the function arcsin}) \cdot f))'_{\restriction Z}(x) = \frac{1}{\sqrt{1-(2 \cdot x)^{\mathbf{2}}}}$.

(21) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{2}\,((\text{the function arccos}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = 2 \cdot x$ and $f(x) > -1$ and $f(x) < 1$. Then
 (i) $\frac{1}{2}\,((\text{the function arccos}) \cdot f)$ is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds $(\frac{1}{2}\,((\text{the function arccos}) \cdot f))'_{\restriction Z}(x) = -\frac{1}{\sqrt{1-(2 \cdot x)^{\mathbf{2}}}}$.

(22) Suppose $Z \subseteq \mathrm{dom}((\frac{1}{2}_{\mathbb{R}}) \cdot f)$ and $f = f_1 - f_2$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f(x) > 0$. Then $(\frac{1}{2}_{\mathbb{R}}) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $((\frac{1}{2}_{\mathbb{R}}) \cdot f)'_{\restriction Z}(x) = -x \cdot (1 - x^2_{\mathbb{Z}})_{\mathbb{R}}^{-\frac{1}{2}}$.

(23) Suppose that

(i) $Z \subseteq \mathrm{dom}(\mathrm{id}_Z\,(\text{the function arcsin}) + (\frac{1}{2}_{\mathbb{R}}) \cdot f)$,
(ii) $Z \subseteq]-1, 1[$,
(iii) $f = f_1 - f_2$,
(iv) $f_2 = \frac{2}{\mathbb{Z}}$, and
(v) for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f(x) > 0$ and $x \neq 0$.

Then

(vi) $\mathrm{id}_Z\,(\text{the function arcsin}) + (\frac{1}{2}_{\mathbb{R}}) \cdot f$ is differentiable on Z, and
(vii) for every x such that $x \in Z$ holds $(\mathrm{id}_Z\,(\text{the function arcsin}) + (\frac{1}{2}_{\mathbb{R}}) \cdot f)'_{\restriction Z}(x) = (\text{the function arcsin})(x)$.

(24) Suppose that

(i) $Z \subseteq \mathrm{dom}(\mathrm{id}_Z\,(\text{the function arccos}) - (\frac{1}{2}_{\mathbb{R}}) \cdot f)$,
(ii) $Z \subseteq]-1, 1[$,
(iii) $f = f_1 - f_2$,
(iv) $f_2 = \frac{2}{\mathbb{Z}}$, and
(v) for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f(x) > 0$ and $x \neq 0$.

Then

(vi) $\mathrm{id}_Z\,(\text{the function arccos}) - (\frac{1}{2}_{\mathbb{R}}) \cdot f$ is differentiable on Z, and
(vii) for every x such that $x \in Z$ holds $(\mathrm{id}_Z\,(\text{the function arccos}) - (\frac{1}{2}_{\mathbb{R}}) \cdot f)'_{\restriction Z}(x) = (\text{the function arccos})(x)$.

(25) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z\,((\text{the function arcsin}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = \frac{x}{a}$ and $f(x) > -1$ and $f(x) < 1$. Then

(i) $\mathrm{id}_Z\,((\text{the function arcsin}) \cdot f)$ is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $(\mathrm{id}_Z\,((\text{the function arcsin}) \cdot f))'_{\restriction Z}(x) = (\text{the function arcsin})(\frac{x}{a}) + \frac{x}{a \cdot \sqrt{1 - (\frac{x}{a})^{\mathbf{2}}}}$.

(26) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z\,((\text{the function arccos}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = \frac{x}{a}$ and $f(x) > -1$ and $f(x) < 1$. Then

(i) $\mathrm{id}_Z\,((\text{the function arccos}) \cdot f)$ is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $(\mathrm{id}_Z\,((\text{the function arccos}) \cdot f))'_{\restriction Z}(x) = (\text{the function arccos})(\frac{x}{a}) - \frac{x}{a \cdot \sqrt{1 - (\frac{x}{a})^{\mathbf{2}}}}$.

(27) Suppose $Z \subseteq \mathrm{dom}((\frac{1}{2}_{\mathbb{R}}) \cdot f)$ and $f = f_1 - f_2$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $f(x) > 0$. Then $(\frac{1}{2}_{\mathbb{R}}) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $((\frac{1}{2}_{\mathbb{R}}) \cdot f)'_{\restriction Z}(x) = -x \cdot (a^{\mathbf{2}} - x^2_{\mathbb{Z}})_{\mathbb{R}}^{-\frac{1}{2}}$.

(28) Suppose that

(i) $Z \subseteq \mathrm{dom}(\mathrm{id}_Z\,((\text{the function arcsin})\cdot f_3) + (\frac{1}{2}_{\mathbb{R}})\cdot f)$,
(ii) $Z \subseteq \,]-1,1[$,
(iii) $f = f_1 - f_2$,
(iv) $f_2 = \frac{2}{\mathbb{Z}}$, and
(v) for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $f(x) > 0$ and $f_3(x) = \frac{x}{a}$ and $f_3(x) > -1$ and $f_3(x) < 1$ and $x \neq 0$ and $a > 0$.
Then
(vi) $\mathrm{id}_Z\,((\text{the function arcsin})\cdot f_3) + (\frac{1}{2}_{\mathbb{R}})\cdot f$ is differentiable on Z, and
(vii) for every x such that $x \in Z$ holds $(\mathrm{id}_Z\,((\text{the function arcsin})\cdot f_3) + (\frac{1}{2}_{\mathbb{R}})\cdot f)'_{\restriction Z}(x) = (\text{the function arcsin})(\frac{x}{a})$.

(29) Suppose that
(i) $Z \subseteq \mathrm{dom}(\mathrm{id}_Z\,((\text{the function arccos})\cdot f_3) - (\frac{1}{2}_{\mathbb{R}})\cdot f)$,
(ii) $Z \subseteq \,]-1,1[$,
(iii) $f = f_1 - f_2$,
(iv) $f_2 = \frac{2}{\mathbb{Z}}$, and
(v) for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $f(x) > 0$ and $f_3(x) = \frac{x}{a}$ and $f_3(x) > -1$ and $f_3(x) < 1$ and $x \neq 0$ and $a > 0$.
Then
(vi) $\mathrm{id}_Z\,((\text{the function arccos})\cdot f_3) - (\frac{1}{2}_{\mathbb{R}})\cdot f$ is differentiable on Z, and
(vii) for every x such that $x \in Z$ holds $(\mathrm{id}_Z\,((\text{the function arccos})\cdot f_3) - (\frac{1}{2}_{\mathbb{R}})\cdot f)'_{\restriction Z}(x) = (\text{the function arccos})(\frac{x}{a})$.

(30) Suppose $Z \subseteq \mathrm{dom}((-\frac{1}{n})\,((\frac{n}{\mathbb{Z}})\cdot\frac{1}{\text{the function sin}}))$ and $n > 0$ and for every x such that $x \in Z$ holds $(\text{the function sin})(x) \neq 0$. Then
(i) $(-\frac{1}{n})\,((\frac{n}{\mathbb{Z}})\cdot\frac{1}{\text{the function sin}})$ is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $((-\frac{1}{n})\,((\frac{n}{\mathbb{Z}})\cdot\frac{1}{\text{the function sin}}))'_{\restriction Z}(x) = \frac{(\text{the function cos})(x)}{(\text{the function sin})(x)^{n+1}_{\mathbb{Z}}}$.

(31) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{n}\,((\frac{n}{\mathbb{Z}})\cdot\frac{1}{\text{the function cos}}))$ and $n > 0$ and for every x such that $x \in Z$ holds $(\text{the function cos})(x) \neq 0$. Then
(i) $\frac{1}{n}\,((\frac{n}{\mathbb{Z}})\cdot\frac{1}{\text{the function cos}})$ is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $(\frac{1}{n}\,((\frac{n}{\mathbb{Z}})\cdot\frac{1}{\text{the function cos}}))'_{\restriction Z}(x) = \frac{(\text{the function sin})(x)}{(\text{the function cos})(x)^{n+1}_{\mathbb{Z}}}$.

(32) Suppose $Z \subseteq \mathrm{dom}((\text{the function sin})\cdot \log\_(e))$ and for every x such that $x \in Z$ holds $x > 0$. Then
(i) $(\text{the function sin})\cdot \log\_(e)$ is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $((\text{the function sin})\cdot \log\_(e))'_{\restriction Z}(x) = \frac{(\text{the function cos})((\log\_(e))(x))}{x}$.

(33) Suppose $Z \subseteq \mathrm{dom}((\text{the function cos})\cdot \log\_(e))$ and for every x such that $x \in Z$ holds $x > 0$. Then

(i) (the function cos) $\cdot$ log_(e) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function cos) $\cdot$ log_$(e))'_{\restriction Z}(x) = -\frac{\text{(the function sin)((log_}(e))(x))}{x}$.

(34) Suppose $Z \subseteq$ dom((the function sin) $\cdot$(the function exp)). Then

(i) (the function sin) $\cdot$(the function exp) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin) $\cdot$(the function exp))$'_{\restriction Z}(x) =$ (the function exp)$(x) \cdot$ (the function cos)((the function exp)(x)).

(35) Suppose $Z \subseteq$ dom((the function cos) $\cdot$(the function exp)). Then

(i) (the function cos) $\cdot$(the function exp) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function cos) $\cdot$(the function exp))$'_{\restriction Z}(x) =$
$-$(the function exp)$(x) \cdot$ (the function sin)((the function exp)(x)).

(36) Suppose $Z \subseteq$ dom((the function exp) $\cdot$(the function cos)). Then

(i) (the function exp) $\cdot$(the function cos) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function exp) $\cdot$(the function cos))$'_{\restriction Z}(x) =$
$-$(the function exp)((the function cos)(x)) $\cdot$ (the function sin)(x).

(37) Suppose $Z \subseteq$ dom((the function exp) $\cdot$(the function sin)). Then

(i) (the function exp) $\cdot$(the function sin) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function exp) $\cdot$(the function sin))$'_{\restriction Z}(x) =$ (the function exp)((the function sin)(x)) $\cdot$ (the function cos)(x).

(38) Suppose $Z \subseteq$ dom((the function sin)+(the function cos)). Then

(i) (the function sin)+(the function cos) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin)+(the function cos))$'_{\restriction Z}(x) =$ (the function cos)$(x) -$ (the function sin)(x).

(39) Suppose $Z \subseteq$ dom((the function sin)$-$(the function cos)). Then

(i) (the function sin)$-$(the function cos) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin)$-$(the function cos))$'_{\restriction Z}(x) =$ (the function cos)$(x) +$ (the function sin)(x).

(40) Suppose $Z \subseteq$ dom((the function exp) ((the function sin)$-$(the function cos))). Then

(i) (the function exp) ((the function sin)$-$(the function cos)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function exp) ((the function sin)$-$(the function cos)))$'_{\restriction Z}(x) = 2 \cdot$ (the function exp)$(x) \cdot$ (the function sin)(x).

(41) Suppose $Z \subseteq$ dom((the function exp) ((the function sin)+(the function cos))). Then

(i) (the function exp) ((the function sin)+(the function cos)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function exp) ((the function sin)+(the function cos)))$'_{\restriction Z}(x) = 2 \cdot$ (the function exp)$(x) \cdot$ (the function cos)(x).

(42) Suppose $Z \subseteq \mathrm{dom}(\frac{\text{(the function sin)+(the function cos)}}{\text{the function exp}})$. Then

(i) $\frac{\text{(the function sin)+(the function cos)}}{\text{the function exp}}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{\text{(the function sin)+(the function cos)}}{\text{the function exp}})'_{\restriction Z}(x) = -\frac{2 \cdot \text{(the function sin)}(x)}{\text{(the function exp)}(x)}$.

(43) Suppose $Z \subseteq \mathrm{dom}(\frac{\text{(the function sin)}-\text{(the function cos)}}{\text{the function exp}})$. Then

(i) $\frac{\text{(the function sin)}-\text{(the function cos)}}{\text{the function exp}}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{\text{(the function sin)}-\text{(the function cos)}}{\text{the function exp}})'_{\restriction Z}(x) = \frac{2 \cdot \text{(the function cos)}(x)}{\text{(the function exp)}(x)}$.

(44) Suppose $Z \subseteq$ dom((the function exp) (the function sin)). Then

(i) (the function exp) (the function sin) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function exp) (the function sin))$'_{\restriction Z}(x) =$ (the function exp)$(x) \cdot$ ((the function sin)(x) + (the function cos)(x)).

(45) Suppose $Z \subseteq$ dom((the function exp) (the function cos)). Then

(i) (the function exp) (the function cos) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function exp) (the function cos))$'_{\restriction Z}(x) =$ (the function exp)$(x) \cdot$ ((the function cos)(x) − (the function sin)(x)).

(46) Suppose (the function cos)$(x) \neq 0$. Then

(i) $\frac{\text{the function sin}}{\text{the function cos}}$ is differentiable in x, and

(ii) $(\frac{\text{the function sin}}{\text{the function cos}})'(x) = \frac{1}{\text{(the function cos)}(x)^{\mathbf{2}}}$.

(47) Suppose (the function sin)$(x) \neq 0$. Then

(i) $\frac{\text{the function cos}}{\text{the function sin}}$ is differentiable in x, and

(ii) $(\frac{\text{the function cos}}{\text{the function sin}})'(x) = -\frac{1}{\text{(the function sin)}(x)^{\mathbf{2}}}$.

(48) Suppose $Z \subseteq \mathrm{dom}((\frac{2}{\mathbb{Z}}) \cdot \frac{\text{the function sin}}{\text{the function cos}})$ and for every x such that $x \in Z$ holds (the function cos)$(x) \neq 0$. Then

(i) $(\frac{2}{\mathbb{Z}}) \cdot \frac{\text{the function sin}}{\text{the function cos}}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\frac{2}{\mathbb{Z}}) \cdot \frac{\text{the function sin}}{\text{the function cos}})'_{\restriction Z}(x) = \frac{2 \cdot \text{(the function sin)}(x)}{\text{(the function cos)}(x)^{3}_{\mathbb{Z}}}$.

(49) Suppose $Z \subseteq \mathrm{dom}((\frac{2}{\mathbb{Z}}) \cdot \frac{\text{the function cos}}{\text{the function sin}})$ and for every x such that $x \in Z$ holds (the function sin)$(x) \neq 0$. Then

(i) $(\frac{2}{\mathbb{Z}}) \cdot \frac{\text{the function cos}}{\text{the function sin}}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\frac{2}{\mathbb{Z}}) \cdot \frac{\text{the function cos}}{\text{the function sin}})'_{\restriction Z}(x) = -\frac{2 \cdot (\text{the function cos})(x)}{(\text{the function sin})(x)^3_{\mathbb{Z}}}$.

(50) Suppose that

(i) $Z \subseteq \mathrm{dom}(\frac{\text{the function sin}}{\text{the function cos}} \cdot f)$, and

(ii) for every x such that $x \in Z$ holds $f(x) = \frac{x}{2}$ and (the function cos)$(f(x)) \neq 0$.

Then

(iii) $\frac{\text{the function sin}}{\text{the function cos}} \cdot f$ is differentiable on Z, and

(iv) for every x such that $x \in Z$ holds $(\frac{\text{the function sin}}{\text{the function cos}} \cdot f)'_{\restriction Z}(x) = \frac{1}{1+(\text{the function cos})(x)}$.

(51) Suppose that

(i) $Z \subseteq \mathrm{dom}(\frac{\text{the function cos}}{\text{the function sin}} \cdot f)$, and

(ii) for every x such that $x \in Z$ holds $f(x) = \frac{x}{2}$ and (the function sin)$(f(x)) \neq 0$.

Then

(iii) $\frac{\text{the function cos}}{\text{the function sin}} \cdot f$ is differentiable on Z, and

(iv) for every x such that $x \in Z$ holds $(\frac{\text{the function cos}}{\text{the function sin}} \cdot f)'_{\restriction Z}(x) = -\frac{1}{1-(\text{the function cos})(x)}$.

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Received March 22, 2006