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# Several Differentiation Formulas of Special Functions. Part VII

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**Summary.** In this article, we prove a series of differentiation identities [2] involving the arctan and arccot functions and specific combinations of special functions including trigonometric and exponential functions.

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The papers [13], [15], [1], [10], [16], [5], [12], [3], [6], [9], [4], [11], [8], [14], and [7] provide the terminology and notation for this paper.

For simplicity, we adopt the following rules: $x$ denotes a real number, $n$ denotes an element of $\mathbb{N}$, $Z$ denotes an open subset of $\mathbb{R}$, and $f$, $g$ denote partial functions from $\mathbb{R}$ to $\mathbb{R}$.

Next we state a number of propositions:

(1) Suppose $Z \subseteq$ dom((the function arctan) $\cdot$(the function sin)) and for every $x$ such that $x \in Z$ holds $-1 < \sin x < 1$. Then

(i) (the function arctan) $\cdot$(the function sin) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function arctan) $\cdot$(the function sin))$'_{\restriction Z}(x) = \frac{\cos x}{1+(\sin x)^2}$.

(2) Suppose $Z \subseteq$ dom((the function arccot) $\cdot$(the function sin)) and for every $x$ such that $x \in Z$ holds $-1 < \sin x < 1$. Then

(i) (the function arccot) $\cdot$(the function sin) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function arccot) $\cdot$(the function sin))$'_{\restriction Z}(x) = -\frac{\cos x}{1+(\sin x)^2}$.

(3) Suppose $Z \subseteq$ dom((the function arctan) $\cdot$(the function cos)) and for every $x$ such that $x \in Z$ holds $-1 < \cos x < 1$. Then




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(i) (the function arctan) $\cdot$(the function cos) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function arctan) $\cdot$(the function cos))$'_{\restriction Z}(x) = -\frac{\sin x}{1+(\cos x)^{\mathbf{2}}}$.

(4) Suppose $Z \subseteq$ dom((the function arccot) $\cdot$(the function cos)) and for every $x$ such that $x \in Z$ holds $-1 < \cos x < 1$. Then

(i) (the function arccot) $\cdot$(the function cos) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function arccot) $\cdot$(the function cos))$'_{\restriction Z}(x) = \frac{\sin x}{1+(\cos x)^{\mathbf{2}}}$.

(5) Suppose $Z \subseteq$ dom((the function arctan) $\cdot$(the function tan)) and for every $x$ such that $x \in Z$ holds $-1 < \tan x < 1$. Then

(i) (the function arctan) $\cdot$(the function tan) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function arctan) $\cdot$(the function tan))$'_{\restriction Z}(x) = 1$.

(6) Suppose $Z \subseteq$ dom((the function arccot) $\cdot$(the function tan)) and for every $x$ such that $x \in Z$ holds $-1 < \tan x < 1$. Then

(i) (the function arccot) $\cdot$(the function tan) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function arccot) $\cdot$(the function tan))$'_{\restriction Z}(x) = -1$.

(7) Suppose $Z \subseteq$ dom((the function arctan) $\cdot$(the function cot)) and for every $x$ such that $x \in Z$ holds $-1 < \cot x < 1$. Then

(i) (the function arctan) $\cdot$(the function cot) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function arctan) $\cdot$(the function cot))$'_{\restriction Z}(x) = -1$.

(8) Suppose $Z \subseteq$ dom((the function arccot) $\cdot$(the function cot)) and for every $x$ such that $x \in Z$ holds $-1 < \cot x < 1$. Then

(i) (the function arccot) $\cdot$(the function cot) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function arccot) $\cdot$(the function cot))$'_{\restriction Z}(x) = 1$.

(9) Suppose $Z \subseteq$ dom((the function arctan) $\cdot$(the function arctan)) and $Z \subseteq ]-1, 1[$ and for every $x$ such that $x \in Z$ holds $-1 < \arctan x < 1$. Then

(i) (the function arctan) $\cdot$(the function arctan) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function arctan) $\cdot$(the function arctan))$'_{\restriction Z}(x) = \frac{1}{(1+x^{\mathbf{2}})\cdot(1+(\arctan x)^{\mathbf{2}})}$.

(10) Suppose $Z \subseteq$ dom((the function arccot) $\cdot$(the function arctan)) and $Z \subseteq ]-1, 1[$ and for every $x$ such that $x \in Z$ holds $-1 < \arctan x < 1$. Then

(i) (the function arccot) $\cdot$(the function arctan) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function arccot) $\cdot$(the function arctan))$'_{\restriction Z}(x) = -\frac{1}{(1+x^{\mathbf{2}})\cdot(1+(\arctan x)^{\mathbf{2}})}$.

(11) Suppose $Z \subseteq \mathrm{dom}((\text{the function arctan}) \cdot (\text{the function arccot}))$ and $Z \subseteq ]-1,1[$ and for every $x$ such that $x \in Z$ holds $-1 < \mathrm{arccot}\, x < 1$. Then

(i) (the function arctan) ·(the function arccot) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function arctan) ·(the function arccot))$'_{\restriction Z}(x) = -\frac{1}{(1+x^2)\cdot(1+(\mathrm{arccot}\, x)^2)}$.

(12) Suppose $Z \subseteq \mathrm{dom}((\text{the function arccot}) \cdot (\text{the function arccot}))$ and $Z \subseteq ]-1,1[$ and for every $x$ such that $x \in Z$ holds $-1 < \mathrm{arccot}\, x < 1$. Then

(i) (the function arccot) ·(the function arccot) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function arccot) ·(the function arccot))$'_{\restriction Z}(x) = \frac{1}{(1+x^2)\cdot(1+(\mathrm{arccot}\, x)^2)}$.

(13) Suppose $Z \subseteq \mathrm{dom}((\text{the function sin}) \cdot (\text{the function arctan}))$ and $Z \subseteq ]-1,1[$. Then

(i) (the function sin) ·(the function arctan) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function sin) ·(the function arctan))$'_{\restriction Z}(x) = \frac{\cos \arctan x}{1+x^2}$.

(14) Suppose $Z \subseteq \mathrm{dom}((\text{the function sin}) \cdot (\text{the function arccot}))$ and $Z \subseteq ]-1,1[$. Then

(i) (the function sin) ·(the function arccot) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function sin) ·(the function arccot))$'_{\restriction Z}(x) = -\frac{\cos \mathrm{arccot}\, x}{1+x^2}$.

(15) Suppose $Z \subseteq \mathrm{dom}((\text{the function cos}) \cdot (\text{the function arctan}))$ and $Z \subseteq ]-1,1[$. Then

(i) (the function cos) ·(the function arctan) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function cos) ·(the function arctan))$'_{\restriction Z}(x) = -\frac{\sin \arctan x}{1+x^2}$.

(16) Suppose $Z \subseteq \mathrm{dom}((\text{the function cos}) \cdot (\text{the function arccot}))$ and $Z \subseteq ]-1,1[$. Then

(i) (the function cos) ·(the function arccot) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function cos) ·(the function arccot))$'_{\restriction Z}(x) = \frac{\sin \mathrm{arccot}\, x}{1+x^2}$.

(17) Suppose $Z \subseteq \mathrm{dom}((\text{the function tan}) \cdot (\text{the function arctan}))$ and $Z \subseteq ]-1,1[$. Then

(i) (the function tan) ·(the function arctan) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function tan) ·(the function arctan))$'_{\restriction Z}(x) = \frac{1}{(\cos \arctan x)^2\cdot(1+x^2)}$.

(18) Suppose $Z \subseteq \mathrm{dom}((\text{the function tan}) \cdot (\text{the function arccot}))$ and $Z \subseteq ]-1,1[$. Then

(i) (the function tan) ·(the function arccot) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function tan) ·(the function arccot))$'_{\restriction Z}(x) = -\frac{1}{(\cos \mathrm{arccot}\, x)^2\cdot(1+x^2)}$.

(19) Suppose $Z \subseteq$ dom((the function cot) $\cdot$(the function arctan)) and $Z \subseteq ]-1, 1[$. Then

(i) (the function cot) $\cdot$(the function arctan) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function cot) $\cdot$(the function arctan))$'_{\restriction Z}(x) = -\frac{1}{(\sin \arctan x)^{\mathbf{2}} \cdot (1+x^{\mathbf{2}})}$.

(20) Suppose $Z \subseteq$ dom((the function cot) $\cdot$(the function arccot)) and $Z \subseteq ]-1, 1[$. Then

(i) (the function cot) $\cdot$(the function arccot) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function cot) $\cdot$(the function arccot))$'_{\restriction Z}(x) = \frac{1}{(\sin \operatorname{arccot} x)^{\mathbf{2}} \cdot (1+x^{\mathbf{2}})}$.

(21) Suppose $Z \subseteq$ dom((the function sec) $\cdot$(the function arctan)) and $Z \subseteq ]-1, 1[$. Then

(i) (the function sec) $\cdot$(the function arctan) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function sec) $\cdot$(the function arctan))$'_{\restriction Z}(x) = \frac{\sin \arctan x}{(\cos \arctan x)^{\mathbf{2}} \cdot (1+x^{\mathbf{2}})}$.

(22) Suppose $Z \subseteq$ dom((the function sec) $\cdot$(the function arccot)) and $Z \subseteq ]-1, 1[$. Then

(i) (the function sec) $\cdot$(the function arccot) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function sec) $\cdot$(the function arccot))$'_{\restriction Z}(x) = -\frac{\sin \operatorname{arccot} x}{(\cos \operatorname{arccot} x)^{\mathbf{2}} \cdot (1+x^{\mathbf{2}})}$.

(23) Suppose $Z \subseteq$ dom((the function cosec) $\cdot$(the function arctan)) and $Z \subseteq ]-1, 1[$. Then

(i) (the function cosec) $\cdot$(the function arctan) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function cosec) $\cdot$(the function arctan))$'_{\restriction Z}(x) = -\frac{\cos \arctan x}{(\sin \arctan x)^{\mathbf{2}} \cdot (1+x^{\mathbf{2}})}$.

(24) Suppose $Z \subseteq$ dom((the function cosec) $\cdot$(the function arccot)) and $Z \subseteq ]-1, 1[$. Then

(i) (the function cosec) $\cdot$(the function arccot) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function cosec) $\cdot$(the function arccot))$'_{\restriction Z}(x) = \frac{\cos \operatorname{arccot} x}{(\sin \operatorname{arccot} x)^{\mathbf{2}} \cdot (1+x^{\mathbf{2}})}$.

(25) Suppose $Z \subseteq$ dom((the function sin) (the function arctan)) and $Z \subseteq ]-1, 1[$. Then

(i) (the function sin) (the function arctan) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function sin) (the function arctan))$'_{\restriction Z}(x) = \cos x \cdot \arctan x + \frac{\sin x}{1+x^{\mathbf{2}}}$.

(26) Suppose $Z \subseteq$ dom((the function sin) (the function arccot)) and $Z \subseteq ]-1, 1[$. Then

(i) (the function sin) (the function arccot) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function sin) (the function arccot))$'_{\restriction Z}(x) = \cos x \cdot \operatorname{arccot} x - \frac{\sin x}{1+x^{\mathbf{2}}}$.

(27) Suppose $Z \subseteq \mathrm{dom}((\text{the function cos})\ (\text{the function arctan}))$ and $Z \subseteq ]-1,1[$. Then
 (i) (the function cos) (the function arctan) is differentiable on $Z$, and
 (ii) for every $x$ such that $x \in Z$ holds ((the function cos) (the function arctan))$'_{\restriction Z}(x) = -\sin x \cdot \arctan x + \frac{\cos x}{1+x^2}$.

(28) Suppose $Z \subseteq \mathrm{dom}((\text{the function cos})\ (\text{the function arccot}))$ and $Z \subseteq ]-1,1[$. Then
 (i) (the function cos) (the function arccot) is differentiable on $Z$, and
 (ii) for every $x$ such that $x \in Z$ holds ((the function cos) (the function arccot))$'_{\restriction Z}(x) = -\sin x \cdot \mathrm{arccot}\, x - \frac{\cos x}{1+x^2}$.

(29) Suppose $Z \subseteq \mathrm{dom}((\text{the function tan})\ (\text{the function arctan}))$ and $Z \subseteq ]-1,1[$. Then
 (i) (the function tan) (the function arctan) is differentiable on $Z$, and
 (ii) for every $x$ such that $x \in Z$ holds ((the function tan) (the function arctan))$'_{\restriction Z}(x) = \frac{\arctan x}{(\cos x)^2} + \frac{\tan x}{1+x^2}$.

(30) Suppose $Z \subseteq \mathrm{dom}((\text{the function tan})\ (\text{the function arccot}))$ and $Z \subseteq ]-1,1[$. Then
 (i) (the function tan) (the function arccot) is differentiable on $Z$, and
 (ii) for every $x$ such that $x \in Z$ holds ((the function tan) (the function arccot))$'_{\restriction Z}(x) = \frac{\mathrm{arccot}\, x}{(\cos x)^2} - \frac{\tan x}{1+x^2}$.

(31) Suppose $Z \subseteq \mathrm{dom}((\text{the function cot})\ (\text{the function arctan}))$ and $Z \subseteq ]-1,1[$. Then
 (i) (the function cot) (the function arctan) is differentiable on $Z$, and
 (ii) for every $x$ such that $x \in Z$ holds ((the function cot) (the function arctan))$'_{\restriction Z}(x) = -\frac{\arctan x}{(\sin x)^2} + \frac{\cot x}{1+x^2}$.

(32) Suppose $Z \subseteq \mathrm{dom}((\text{the function cot})\ (\text{the function arccot}))$ and $Z \subseteq ]-1,1[$. Then
 (i) (the function cot) (the function arccot) is differentiable on $Z$, and
 (ii) for every $x$ such that $x \in Z$ holds ((the function cot) (the function arccot))$'_{\restriction Z}(x) = -\frac{\mathrm{arccot}\, x}{(\sin x)^2} - \frac{\cot x}{1+x^2}$.

(33) Suppose $Z \subseteq \mathrm{dom}((\text{the function sec})\ (\text{the function arctan}))$ and $Z \subseteq ]-1,1[$. Then
 (i) (the function sec) (the function arctan) is differentiable on $Z$, and
 (ii) for every $x$ such that $x \in Z$ holds ((the function sec) (the function arctan))$'_{\restriction Z}(x) = \frac{\sin x \cdot \arctan x}{(\cos x)^2} + \frac{1}{\cos x \cdot (1+x^2)}$.

(34) Suppose $Z \subseteq \mathrm{dom}((\text{the function sec})\ (\text{the function arccot}))$ and $Z \subseteq ]-1,1[$. Then
 (i) (the function sec) (the function arccot) is differentiable on $Z$, and
 (ii) for every $x$ such that $x \in Z$ holds ((the function sec) (the function arccot))$'_{\restriction Z}(x) = \frac{\sin x \cdot \mathrm{arccot}\, x}{(\cos x)^2} - \frac{1}{\cos x \cdot (1+x^2)}$.

(35) Suppose $Z \subseteq \mathrm{dom}((\text{the function cosec})\ (\text{the function arctan}))$ and $Z \subseteq ]-1,1[$. Then

(i) (the function cosec) (the function arctan) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function cosec) (the function arctan))$'_{\restriction Z}(x) = -\frac{\cos x \cdot \arctan x}{(\sin x)^{\mathbf{2}}} + \frac{1}{\sin x \cdot (1+x^{\mathbf{2}})}$.

(36) Suppose $Z \subseteq \mathrm{dom}((\text{the function cosec})\ (\text{the function arccot}))$ and $Z \subseteq ]-1,1[$. Then

(i) (the function cosec) (the function arccot) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function cosec) (the function arccot))$'_{\restriction Z}(x) = -\frac{\cos x \cdot \mathrm{arccot}\, x}{(\sin x)^{\mathbf{2}}} - \frac{1}{\sin x \cdot (1+x^{\mathbf{2}})}$.

(37) Suppose $Z \subseteq ]-1,1[$. Then

(i) (the function arctan)+(the function arccot) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function arctan)+(the function arccot))$'_{\restriction Z}(x) = 0$.

(38) Suppose $Z \subseteq ]-1,1[$. Then

(i) (the function arctan)−(the function arccot) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function arctan)−(the function arccot))$'_{\restriction Z}(x) = \frac{2}{1+x^{\mathbf{2}}}$.

(39) Suppose $Z \subseteq ]-1,1[$. Then

(i) (the function sin) ((the function arctan)+(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function sin) ((the function arctan)+(the function arccot)))$'_{\restriction Z}(x) = \cos x \cdot (\arctan x + \mathrm{arccot}\, x)$.

(40) Suppose $Z \subseteq ]-1,1[$. Then

(i) (the function sin) ((the function arctan)−(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function sin) ((the function arctan)−(the function arccot)))$'_{\restriction Z}(x) = \cos x \cdot (\arctan x - \mathrm{arccot}\, x) + \frac{2 \cdot \sin x}{1+x^{\mathbf{2}}}$.

(41) Suppose $Z \subseteq ]-1,1[$. Then

(i) (the function cos) ((the function arctan)+(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function cos) ((the function arctan)+(the function arccot)))$'_{\restriction Z}(x) = -\sin x \cdot (\arctan x + \mathrm{arccot}\, x)$.

(42) Suppose $Z \subseteq ]-1,1[$. Then

(i) (the function cos) ((the function arctan)−(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function cos) ((the function arctan)−(the function arccot)))$'_{\restriction Z}(x) = -\sin x \cdot (\arctan x - \mathrm{arccot}\, x) + \frac{2 \cdot \cos x}{1+x^{\mathbf{2}}}$.

(43) Suppose $Z \subseteq \mathrm{dom}\,(\text{the function tan})$ and $Z \subseteq ]-1,1[$. Then

(i) (the function tan) ((the function arctan)+(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function tan) ((the function arctan)+(the function arccot)))$'_{\restriction Z}(x) = \frac{\arctan x + \operatorname{arccot} x}{(\cos x)^2}$.

(44) Suppose $Z \subseteq \operatorname{dom}$ (the function tan) and $Z \subseteq ]-1,1[$. Then

(i) (the function tan) ((the function arctan)−(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function tan) ((the function arctan)−(the function arccot)))$'_{\restriction Z}(x) = \frac{\arctan x - \operatorname{arccot} x}{(\cos x)^2} + \frac{2 \cdot \tan x}{1+x^2}$.

(45) Suppose $Z \subseteq \operatorname{dom}$ (the function cot) and $Z \subseteq ]-1,1[$. Then

(i) (the function cot) ((the function arctan)+(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function cot) ((the function arctan)+(the function arccot)))$'_{\restriction Z}(x) = -\frac{\arctan x + \operatorname{arccot} x}{(\sin x)^2}$.

(46) Suppose $Z \subseteq \operatorname{dom}$ (the function cot) and $Z \subseteq ]-1,1[$. Then

(i) (the function cot) ((the function arctan)−(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function cot) ((the function arctan)−(the function arccot)))$'_{\restriction Z}(x) = -\frac{\arctan x - \operatorname{arccot} x}{(\sin x)^2} + \frac{2 \cdot \cot x}{1+x^2}$.

(47) Suppose $Z \subseteq \operatorname{dom}$ (the function sec) and $Z \subseteq ]-1,1[$. Then

(i) (the function sec) ((the function arctan)+(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function sec) ((the function arctan)+(the function arccot)))$'_{\restriction Z}(x) = \frac{(\arctan x + \operatorname{arccot} x) \cdot \sin x}{(\cos x)^2}$.

(48) Suppose $Z \subseteq \operatorname{dom}$ (the function sec) and $Z \subseteq ]-1,1[$. Then

(i) (the function sec) ((the function arctan)−(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function sec) ((the function arctan)−(the function arccot)))$'_{\restriction Z}(x) = \frac{(\arctan x - \operatorname{arccot} x) \cdot \sin x}{(\cos x)^2} + \frac{2 \cdot \sec x}{1+x^2}$.

(49) Suppose $Z \subseteq \operatorname{dom}$ (the function cosec) and $Z \subseteq ]-1,1[$. Then

(i) (the function cosec) ((the function arctan)+(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function cosec) ((the function arctan)+(the function arccot)))$'_{\restriction Z}(x) = -\frac{(\arctan x + \operatorname{arccot} x) \cdot \cos x}{(\sin x)^2}$.

(50) Suppose $Z \subseteq \operatorname{dom}$ (the function cosec) and $Z \subseteq ]-1,1[$. Then

(i) (the function cosec) ((the function arctan)−(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function cosec) ((the function arctan)−(the function arccot)))$'_{\restriction Z}(x) = -\frac{(\arctan x - \operatorname{arccot} x) \cdot \cos x}{(\sin x)^2} + \frac{2 \cdot \operatorname{cosec} x}{1+x^2}$.

(51) Suppose $Z \subseteq ]-1,1[$. Then

(i) (the function exp) ((the function arctan)+(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function exp) ((the function arctan)+(the function arccot)))$'_{\restriction Z}(x) = \exp x \cdot (\arctan x + \operatorname{arccot} x)$.

(52) Suppose $Z \subseteq ]-1, 1[$. Then

(i) (the function exp) ((the function arctan)−(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function exp) ((the function arctan)−(the function arccot)))$'_{\restriction Z}(x) = \exp x \cdot (\arctan x - \operatorname{arccot} x) + \frac{2 \cdot \exp x}{1+x^{\mathbf{2}}}$.

(53) Suppose $Z \subseteq ]-1, 1[$. Then

(i) $\frac{\text{(the function arctan)+(the function arccot)}}{\text{the function exp}}$ is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds $(\frac{\text{(the function arctan)+(the function arccot)}}{\text{the function exp}})'_{\restriction Z}(x) = -\frac{\arctan x + \operatorname{arccot} x}{\exp x}$.

(54) Suppose $Z \subseteq ]-1, 1[$. Then

(i) $\frac{\text{(the function arctan)−(the function arccot)}}{\text{the function exp}}$ is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds $(\frac{\text{(the function arctan)−(the function arccot)}}{\text{the function exp}})'_{\restriction Z}(x) = \frac{(\frac{2}{1+x^{\mathbf{2}}} - \arctan x) + \operatorname{arccot} x}{\exp x}$.

(55) Suppose $Z \subseteq \operatorname{dom}$((the function exp) ·((the function arctan)+(the function arccot))) and $Z \subseteq ]-1, 1[$. Then

(i) (the function exp) ·((the function arctan)+(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function exp) ·((the function arctan)+(the function arccot)))$'_{\restriction Z}(x) = 0$.

(56) Suppose $Z \subseteq \operatorname{dom}$((the function exp) ·((the function arctan)−(the function arccot))) and $Z \subseteq ]-1, 1[$. Then

(i) (the function exp) ·((the function arctan)−(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function exp) ·((the function arctan)−(the function arccot)))$'_{\restriction Z}(x) = \frac{2 \cdot \exp(\arctan x - \operatorname{arccot} x)}{1+x^{\mathbf{2}}}$.

(57) Suppose $Z \subseteq \operatorname{dom}$((the function sin) ·((the function arctan)+(the function arccot))) and $Z \subseteq ]-1, 1[$. Then

(i) (the function sin) ·((the function arctan)+(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function sin) ·((the function arctan)+(the function arccot)))$'_{\restriction Z}(x) = 0$.

(58) Suppose $Z \subseteq \operatorname{dom}$((the function sin) ·((the function arctan)−(the function arccot))) and $Z \subseteq ]-1, 1[$. Then

(i) (the function sin) ·((the function arctan)−(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function sin) $\cdot$((the function arctan)$-$(the function arccot)))$'_{\restriction Z}(x) = \frac{2\cdot\cos(\arctan x - \operatorname{arccot} x)}{1+x^{2}}$.

(59) Suppose $Z \subseteq$ dom((the function cos) $\cdot$((the function arctan)+(the function arccot))) and $Z \subseteq ]-1,1[$. Then

(i) (the function cos) $\cdot$((the function arctan)+(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function cos) $\cdot$((the function arctan)+(the function arccot)))$'_{\restriction Z}(x) = 0$.

(60) Suppose $Z \subseteq$ dom((the function cos) $\cdot$((the function arctan)$-$(the function arccot))) and $Z \subseteq ]-1,1[$. Then

(i) (the function cos) $\cdot$((the function arctan)$-$(the function arccot)) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function cos) $\cdot$((the function arctan)$-$(the function arccot)))$'_{\restriction Z}(x) = -\frac{2\cdot\sin(\arctan x - \operatorname{arccot} x)}{1+x^{2}}$.

(61) Suppose $Z \subseteq ]-1,1[$. Then

(i) (the function arctan) (the function arccot) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ((the function arctan) (the function arccot))$'_{\restriction Z}(x) = \frac{\operatorname{arccot} x - \arctan x}{1+x^{2}}$.

(62) Suppose that

(i) $Z \subseteq$ dom(((the function arctan) $\cdot\frac{1}{f}$) ((the function arccot) $\cdot\frac{1}{f}$)), and

(ii) for every $x$ such that $x \in Z$ holds $f(x) = x$ and $-1 < (\frac{1}{f})(x) < 1$.

Then

(iii) ((the function arctan) $\cdot\frac{1}{f}$) ((the function arccot) $\cdot\frac{1}{f}$) is differentiable on $Z$, and

(iv) for every $x$ such that $x \in Z$ holds (((the function arctan) $\cdot\frac{1}{f}$) ((the function arccot) $\cdot\frac{1}{f}$))$'_{\restriction Z}(x) = \frac{\arctan(\frac{1}{x}) - \operatorname{arccot}(\frac{1}{x})}{1+x^{2}}$.

(63) Suppose $Z \subseteq$ dom($\mathrm{id}_Z$ ((the function arctan) $\cdot\frac{1}{f}$)) and for every $x$ such that $x \in Z$ holds $f(x) = x$ and $-1 < (\frac{1}{f})(x) < 1$. Then

(i) $\mathrm{id}_Z$ ((the function arctan) $\cdot\frac{1}{f}$) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ($\mathrm{id}_Z$ ((the function arctan) $\cdot\frac{1}{f}$))$'_{\restriction Z}(x) = \arctan(\frac{1}{x}) - \frac{x}{1+x^{2}}$.

(64) Suppose $Z \subseteq$ dom($\mathrm{id}_Z$ ((the function arccot) $\cdot\frac{1}{f}$)) and for every $x$ such that $x \in Z$ holds $f(x) = x$ and $-1 < (\frac{1}{f})(x) < 1$. Then

(i) $\mathrm{id}_Z$ ((the function arccot) $\cdot\frac{1}{f}$) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds ($\mathrm{id}_Z$ ((the function arccot) $\cdot\frac{1}{f}$))$'_{\restriction Z}(x) = \operatorname{arccot}(\frac{1}{x}) + \frac{x}{1+x^{2}}$.

(65) Suppose $Z \subseteq$ dom($g$ ((the function arctan) $\cdot\frac{1}{f}$)) and $g = \square^{2}$ and for every $x$ such that $x \in Z$ holds $f(x) = x$ and $-1 < (\frac{1}{f})(x) < 1$. Then

(i) $g$ ((the function arctan) $\cdot\frac{1}{f}$) is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds $(g\,((\text{the function arctan}) \cdot \frac{1}{f}))'_{\restriction Z}(x) = 2 \cdot x \cdot \arctan(\frac{1}{x}) - \frac{x^{2}}{1+x^{2}}$.

(66) Suppose $Z \subseteq \operatorname{dom}(g\,((\text{the function arccot}) \cdot \frac{1}{f}))$ and $g = \square^{2}$ and for every $x$ such that $x \in Z$ holds $f(x) = x$ and $-1 < (\frac{1}{f})(x) < 1$. Then

(i) $g\,((\text{the function arccot}) \cdot \frac{1}{f})$ is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds $(g\,((\text{the function arccot}) \cdot \frac{1}{f}))'_{\restriction Z}(x) = 2 \cdot x \cdot \operatorname{arccot}(\frac{1}{x}) + \frac{x^{2}}{1+x^{2}}$.

(67) Suppose $Z \subseteq\, ]-1, 1[$ and for every $x$ such that $x \in Z$ holds (the function arctan)$(x) \neq 0$. Then

(i) $\frac{1}{\text{the function arctan}}$ is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds $(\frac{1}{\text{the function arctan}})'_{\restriction Z}(x) = -\frac{1}{(\arctan x)^{2} \cdot (1+x^{2})}$.

(68) Suppose $Z \subseteq\, ]-1, 1[$. Then

(i) $\frac{1}{\text{the function arccot}}$ is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds $(\frac{1}{\text{the function arccot}})'_{\restriction Z}(x) = \frac{1}{(\operatorname{arccot} x)^{2} \cdot (1+x^{2})}$.

One can prove the following propositions:

(69) Suppose $Z \subseteq \operatorname{dom}(\frac{1}{n\,(\text{the function arctan})^{n}})$ and $Z \subseteq\, ]-1, 1[$ and $n > 0$ and for every $x$ such that $x \in Z$ holds $\arctan x \neq 0$. Then

(i) $\frac{1}{n\,(\text{the function arctan})^{n}}$ is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds $(\frac{1}{n\,(\text{the function arctan})^{n}})'_{\restriction Z}(x) = -\frac{1}{((\arctan x)^{n+1}) \cdot (1+x^{2})}$.

(70) Suppose $Z \subseteq \operatorname{dom}(\frac{1}{n\,(\text{the function arccot})^{n}})$ and $Z \subseteq\, ]-1, 1[$ and $n > 0$. Then

(i) $\frac{1}{n\,(\text{the function arccot})^{n}}$ is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds $(\frac{1}{n\,(\text{the function arccot})^{n}})'_{\restriction Z}(x) = \frac{1}{((\operatorname{arccot} x)^{n+1}) \cdot (1+x^{2})}$.

(71) Suppose $Z \subseteq \operatorname{dom}(2\,(\text{the function arctan})^{\frac{1}{2}})$ and $Z \subseteq\, ]-1, 1[$ and for every $x$ such that $x \in Z$ holds $\arctan x > 0$. Then

(i) $2\,(\text{the function arctan})^{\frac{1}{2}}$ is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds $(2\,(\text{the function arctan})^{\frac{1}{2}})'_{\restriction Z}(x) = \frac{(\arctan x)^{-\frac{1}{2}}}{1+x^{2}}$.

(72) Suppose $Z \subseteq \operatorname{dom}(2\,(\text{the function arccot})^{\frac{1}{2}})$ and $Z \subseteq\, ]-1, 1[$. Then

(i) $2\,(\text{the function arccot})^{\frac{1}{2}}$ is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds $(2\,(\text{the function arccot})^{\frac{1}{2}})'_{\restriction Z}(x) = -\frac{(\operatorname{arccot} x)^{-\frac{1}{2}}}{1+x^{2}}$.

(73) Suppose $Z \subseteq \mathrm{dom}(\frac{2}{3}\,(\text{the function arctan})^{\frac{3}{2}})$ and $Z \subseteq\ ]-1,1[$ and for every $x$ such that $x \in Z$ holds $\arctan x > 0$. Then

(i) $\frac{2}{3}\,(\text{the function arctan})^{\frac{3}{2}}$ is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds $(\frac{2}{3}\,(\text{the function arctan})^{\frac{3}{2}})'_{\restriction Z}(x) = \frac{(\arctan x)^{\frac{1}{2}}}{1+x^{\mathbf{2}}}$.

(74) Suppose $Z \subseteq \mathrm{dom}(\frac{2}{3}\,(\text{the function arccot})^{\frac{3}{2}})$ and $Z \subseteq\ ]-1,1[$. Then

(i) $\frac{2}{3}\,(\text{the function arccot})^{\frac{3}{2}}$ is differentiable on $Z$, and

(ii) for every $x$ such that $x \in Z$ holds $(\frac{2}{3}\,(\text{the function arccot})^{\frac{3}{2}})'_{\restriction Z}(x) = -\frac{(\operatorname{arccot} x)^{\frac{1}{2}}}{1+x^{\mathbf{2}}}$.

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