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On Rough Subgroup of a Group

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Summary. This article describes a rough subgroup with respect to a normal subgroup of a group, and some properties of the lower and the upper approximations in a group.

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The articles [2], [3], [1], [4], and [5] provide the terminology and notation for this paper.

For simplicity, we adopt the following rules: G denotes a group, A, B denote non empty subsets of G, N, H, H_1, H_2 denote subgroups of G, and x, a, b denote elements of G.

Next we state a number of propositions:

(1) For every normal subgroup N of G and for all elements x_1, x_2 of G holds $x_1 \cdot N \cdot (x_2 \cdot N) = (x_1 \cdot x_2) \cdot N$.

(2) For every group G and for every subgroup N of G and for all elements x, y of G such that $y \in x \cdot N$ holds $x \cdot N = y \cdot N$.

(3) Let N be a subgroup of G, H be a subgroup of G, and x be an element of G. If $x \cdot N$ meets $\overline{H}$, then there exists an element y of G such that $y \in x \cdot N$ and $y \in H$.

(4) For all elements x, y of G and for every normal subgroup N of G such that $y \in N$ holds $x \cdot y \cdot x^{-1} \in N$.

(5) For every subgroup N of G such that for all elements x, y of G such that $y \in N$ holds $x \cdot y \cdot x^{-1} \in N$ holds N is normal.

(6) $x \in H_1 \cdot H_2$ iff there exist a, b such that $x = a \cdot b$ and $a \in H_1$ and $b \in H_2$.

(7) Let G be a group and N_1, N_2 be strict normal subgroups of G. Then there exists a strict subgroup M of G such that the carrier of $M = N_1 \cdot N_2$.

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(8) Let G be a group and N_1, N_2 be strict normal subgroups of G. Then there exists a strict normal subgroup M of G such that the carrier of $M = N_1 \cdot N_2$.

(9) Let G be a group and N, N_1, N_2 be subgroups of G. Suppose the carrier of $N = N_1 \cdot N_2$. Then N_1 is a subgroup of N and N_2 is a subgroup of N.

(10) Let N, N_1, N_2 be normal subgroups of G and a, b be elements of G. If the carrier of $N = N_1 \cdot N_2$, then $a \cdot N_1 \cdot (b \cdot N_2) = (a \cdot b) \cdot N$.

(11) For every normal subgroup N of G and for every x holds $x \cdot N \cdot x^{-1} \subseteq \overline{N}$.

Let G be a group, let A be a subset of G, and let N be a subgroup of G. The functor $N`A$ yielding a subset of G is defined by:

(Def. 1) $N`A = \{x \in G\colon x \cdot N \subseteq A\}$.

The functor $N{\sim}A$ yielding a subset of G is defined as follows:

(Def. 2) $N{\sim}A = \{x \in G\colon x \cdot N$ meets $A\}$.

Next we state a number of propositions:

(12) For every element x of G such that $x \in N`A$ holds $x \cdot N \subseteq A$.

(13) For every element x of G such that $x \cdot N \subseteq A$ holds $x \in N`A$.

(14) For every element x of G such that $x \in N{\sim}A$ holds $x \cdot N$ meets A.

(15) For every element x of G such that $x \cdot N$ meets A holds $x \in N{\sim}A$.

(16) $N`A \subseteq A$.

(17) $A \subseteq N{\sim}A$.

(18) $N`A \subseteq N{\sim}A$.

(19) $N{\sim}A \cup B = (N{\sim}A) \cup (N{\sim}B)$.

(20) $N`A \cap B = (N`A) \cap (N`B)$.

(21) If $A \subseteq B$, then $N`A \subseteq N`B$.

(22) If $A \subseteq B$, then $N{\sim}A \subseteq N{\sim}B$.

(23) $(N`A) \cup (N`B) \subseteq N`(A \cup B)$.

(24) $N{\sim}A \cup B = (N{\sim}A) \cup (N{\sim}B)$.

(25) If N is a subgroup of H, then $H`A \subseteq N`A$.

(26) If N is a subgroup of H, then $N{\sim}A \subseteq H{\sim}A$.

(27) For every group G and for all non empty subsets A, B of G and for every normal subgroup N of G holds $(N`A) \cdot (N`B) \subseteq N`A \cdot B$.

(28) For every element x of G such that $x \in N{\sim}A \cdot B$ holds $x \cdot N$ meets $A \cdot B$.

(29) For every group G and for all non empty subsets A, B of G and for every normal subgroup N of G holds $(N{\sim}A) \cdot (N{\sim}B) = N{\sim}A \cdot B$.

(30) For every element x of G such that $x \in N{\sim}N`(N{\sim}A)$ holds $x \cdot N$ meets $N`(N{\sim}A)$.

(31) For every element x of G such that $x \in N`(N{\sim}A)$ holds $x \cdot N \subseteq N{\sim}A$.

(32) For every element x of G such that $x \in N{\sim}N{\sim}A$ holds $x \cdot N$ meets $N{\sim}A$.

(33) For every element x of G such that $x \in N{\sim}N`A$ holds $x \cdot N$ meets $N`A$.

(34) $N`(N`A) = N`A$.

(35) $N{\sim}A = N{\sim}N{\sim}A$.

(36) $N`(N`A) \subseteq N{\sim}N{\sim}A$.

(37) $N{\sim}N`A \subseteq A$.

(38) $N`(N{\sim}N`A) = N`A$.

(39) If $A \subseteq N`(N{\sim}A)$, then $N{\sim}A \subseteq N{\sim}N`(N{\sim}A)$.

(40) $N{\sim}N`(N{\sim}A) = N{\sim}A$.

(41) For every element x of G such that $x \in N`(N`A)$ holds $x \cdot N \subseteq N`A$.

(42) $N`(N`A) = N{\sim}N`A$.

(43) $N{\sim}N{\sim}A = N`(N{\sim}A)$.

(44) For all subgroups N, N_1, N_2 of G such that $N = N_1 \cap N_2$ holds $N{\sim}A \subseteq (N_1{\sim}A) \cap (N_2{\sim}A)$.

(45) For all subgroups N, N_1, N_2 of G such that $N = N_1 \cap N_2$ holds $(N_1`A) \cap (N_2`A) \subseteq N`A$.

(46) Let N_1, N_2 be strict normal subgroups of G. Then there exists a strict normal subgroup N of G such that the carrier of $N = N_1 \cdot N_2$ and $N`A \subseteq (N_1`A) \cap (N_2`A)$.

(47) Let N_1, N_2 be strict normal subgroups of G. Then there exists a strict normal subgroup N of G such that the carrier of $N = N_1 \cdot N_2$ and $(N_1{\sim}A) \cup (N_2{\sim}A) \subseteq N{\sim}A$.

(48) Let N_1, N_2 be strict normal subgroups of G. Then there exists a strict normal subgroup N of G such that the carrier of $N = N_1 \cdot N_2$ and $N{\sim}A \subseteq ((N_1{\sim}A) \cdot N_2) \cap ((N_2{\sim}A) \cdot N_1)$.

In the sequel N_1, N_2 are subgroups of G.

Let G be a group and let H, N be subgroups of G. The functor $N`H$ yielding a subset of G is defined by:

(Def. 3) $N`H = \{x \in G\colon x \cdot N \subseteq \overline{H}\}$.

The functor $N{\sim}H$ yields a subset of G and is defined as follows:

(Def. 4) $N{\sim}H = \{x \in G\colon x \cdot N \text{ meets } \overline{H}\}$.

We now state a number of propositions:

(49) For every element x of G such that $x \in N`H$ holds $x \cdot N \subseteq \overline{H}$.

(50) For every element x of G such that $x \cdot N \subseteq \overline{H}$ holds $x \in N`H$.

(51) For every element x of G such that $x \in N{\sim}H$ holds $x \cdot N$ meets $\overline{H}$.

(52) For every element x of G such that $x \cdot N$ meets $\overline{H}$ holds $x \in N{\sim}H$.

(53) $N`H \subseteq \overline{H}$.

(54) $\overline{H} \subseteq N{\sim}H$.

(55) $N`H \subseteq N{\sim}H$.

(56) If H_1 is a subgroup of H_2, then $N{\sim}H_1 \subseteq N{\sim}H_2$.

(57) If N_1 is a subgroup of N_2, then $N_1{\sim}H \subseteq N_2{\sim}H$.

(58) If N_1 is a subgroup of N_2, then $N_1{\sim}N_1 \subseteq N_2{\sim}N_2$.

(59) If H_1 is a subgroup of H_2, then $N`H_1 \subseteq N`H_2$.

(60) If N_1 is a subgroup of N_2, then $N_2`H \subseteq N_1`H$.

(61) If N_1 is a subgroup of N_2, then $N_2`N_1 \subseteq N_1`N_2$.

(62) For every normal subgroup N of G holds $(N`H_1) \cdot (N`H_2) \subseteq N`H_1 \cdot H_2$.

(63) For every normal subgroup N of G holds $(N{\sim}H_1) \cdot (N{\sim}H_2) = N{\sim}H_1 \cdot H_2$.

(64) For all subgroups N, N_1, N_2 of G such that $N = N_1 \cap N_2$ holds $N{\sim}H \subseteq (N_1{\sim}H) \cap (N_2{\sim}H)$.

(65) For all subgroups N, N_1, N_2 of G such that $N = N_1 \cap N_2$ holds $(N_1`H) \cap (N_2`H) \subseteq N`H$.

(66) Let N_1, N_2 be strict normal subgroups of G. Then there exists a strict normal subgroup N of G such that the carrier of $N = N_1 \cdot N_2$ and $N`H \subseteq (N_1`H) \cap (N_2`H)$.

(67) Let N_1, N_2 be strict normal subgroups of G. Then there exists a strict normal subgroup N of G such that the carrier of $N = N_1 \cdot N_2$ and $(N_1{\sim}H) \cup (N_2{\sim}H) \subseteq N{\sim}H$.

(68) Let N_1, N_2 be strict normal subgroups of G. Then there exists a strict normal subgroup N of G such that the carrier of $N = N_1 \cdot N_2$ and $(N_1`H) \cdot (N_2`H) \subseteq N`H$.

(69) Let N_1, N_2 be strict normal subgroups of G. Then there exists a strict normal subgroup N of G such that the carrier of $N = N_1 \cdot N_2$ and $(N_1{\sim}H) \cdot (N_2{\sim}H) \subseteq N{\sim}H$.

(70) Let N_1, N_2 be strict normal subgroups of G. Then there exists a strict normal subgroup N of G such that the carrier of $N = N_1 \cdot N_2$ and $N{\sim}H \subseteq ((N_1{\sim}H) \cdot N_2) \cap ((N_2{\sim}H) \cdot N_1)$.

(71) Let H be a subgroup of G and N be a normal subgroup of G. Then there exists a strict subgroup M of G such that the carrier of $M = N{\sim}H$.

(72) Let H be a subgroup of G and N be a normal subgroup of G. Suppose N is a subgroup of H. Then there exists a strict subgroup M of G such that the carrier of $M = N`H$.

(73) For all normal subgroups H, N of G there exists a strict normal subgroup M of G such that the carrier of $M = N{\sim}H$.

(74) Let H, N be normal subgroups of G. Suppose N is a subgroup of H. Then there exists a strict normal subgroup M of G such that the carrier of $M = N`H$.

(75) Let N, N_1 be normal subgroups of G. Suppose N_1 is a subgroup of N. Then there exist strict normal subgroups N_2, N_3 of G such that the carrier of $N_2 = N_1 \sim N$ and the carrier of $N_3 = N_1 ‘ N$ and $N_2 ‘ N \subseteq N_3 ‘ N$.

(76) Let N, N_1 be normal subgroups of G. Suppose N_1 is a subgroup of N. Then there exist strict normal subgroups N_2, N_3 of G such that the carrier of $N_2 = N_1 \sim N$ and the carrier of $N_3 = N_1 ‘ N$ and $N_3 \sim N \subseteq N_2 \sim N$.

(77) Let N, N_1 be normal subgroups of G. Suppose N_1 is a subgroup of N. Then there exist strict normal subgroups N_2, N_3 of G such that the carrier of $N_2 = N_1 \sim N$ and the carrier of $N_3 = N_1 ‘ N$ and $N_2 ‘ N \subseteq N_3 \sim N$.

(78) Let N, N_1 be normal subgroups of G. Suppose N_1 is a subgroup of N. Then there exist strict normal subgroups N_2, N_3 of G such that the carrier of $N_2 = N_1 \sim N$ and the carrier of $N_3 = N_1 ‘ N$ and $N_3 ‘ N \subseteq N_2 \sim N$.

(79) Let N, N_1, N_2 be normal subgroups of G. Suppose N_1 is a subgroup of N_2. Then there exist strict normal subgroups N_3, N_4 of G such that the carrier of $N_3 = N \sim N_1$ and the carrier of $N_4 = N \sim N_2$ and $N_3 \sim N_1 \subseteq N_4 \sim N_1$.

(80) Let N, N_1 be normal subgroups of G. Then there exists a strict normal subgroup N_2 of G such that the carrier of $N_2 = N ‘ N$ and $N ‘ N_1 \subseteq N_2 ‘ N_1$.

(81) Let N, N_1 be normal subgroups of G. Then there exists a strict normal subgroup N_2 of G such that the carrier of $N_2 = N \sim N$ and $N \sim N_1 \subseteq N_2 \sim N_1$.

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