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A Model of Mizar Concepts – Unification

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Summary. The aim of this paper is to develop a formal theory of Mizar linguistic concepts following the ideas from [6] and [7]. The theory presented is an abstraction from the existing implementation of the Mizar system and is devoted to the formalization of Mizar expressions. The concepts formalized here are: standarized constructor signature, arity-rich signatures, and the unification of Mizar expressions.

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The notation and terminology used in this paper are introduced in the following articles: [20], [21], [12], [22], [10], [14], [13], [17], [18], [15], [1], [8], [11], [2], [3], [4], [19], [16], [5], [9], and [7]. For simplicity the abbreviation $\mathfrak{M}$ = MaxConstrSign is introduced.

1. Preliminary

In this paper i, j denote natural numbers.

Next we state two propositions:

(1) For every pair set x holds $x = \langle x_{\mathbf{1}}, x_{\mathbf{2}} \rangle$.

(2) For every infinite set X there exist sets x_1, x_2 such that $x_1, x_2 \in X$ and $x_1 \neq x_2$.

In this article we present several logical schemes. The scheme *MinimalElement* deals with a finite non empty set $\mathcal{A}$ and a binary predicate $\mathcal{P}$, and states that:

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There exists a set x such that $x \in \mathcal{A}$ and for every set y such that $y \in \mathcal{A}$ holds not $\mathcal{P}[y, x]$

provided the parameters have the following properties:

- For all sets x, y such that x, $y \in \mathcal{A}$ and $\mathcal{P}[x, y]$ holds not $\mathcal{P}[y, x]$, and
- For all sets x, y, z such that x, y, $z \in \mathcal{A}$ and $\mathcal{P}[x, y]$ and $\mathcal{P}[y, z]$ holds $\mathcal{P}[x, z]$.

The scheme *FiniteC* deals with a finite set $\mathcal{A}$ and a unary predicate $\mathcal{P}$, and states that:

$\mathcal{P}[\mathcal{A}]$

provided the following condition is satisfied:

- For every subset A of $\mathcal{A}$ such that for every set B such that $B \subset A$ holds $\mathcal{P}[B]$ holds $\mathcal{P}[A]$.

The scheme *Numeration* deals with a finite set $\mathcal{A}$ and a binary predicate $\mathcal{P}$, and states that:

There exists an one-to-one finite sequence s such that $\operatorname{rng} s = \mathcal{A}$ and for all i, j such that i, $j \in \operatorname{dom} s$ and $\mathcal{P}[s(i), s(j)]$ holds $i < j$

provided the parameters satisfy the following conditions:

- For all sets x, y such that x, $y \in \mathcal{A}$ and $\mathcal{P}[x, y]$ holds not $\mathcal{P}[y, x]$, and
- For all sets x, y, z such that x, y, $z \in \mathcal{A}$ and $\mathcal{P}[x, y]$ and $\mathcal{P}[y, z]$ holds $\mathcal{P}[x, z]$.

One can prove the following two propositions:

(3) For every variable x holds $\operatorname{varcl} \operatorname{vars}(x) = \operatorname{vars}(x)$.

(4) Let $\mathfrak{C}$ be an initialized constructor signature and e be an expression of $\mathfrak{C}$. Then e is compound if and only if it is not true that there exists an element x of Vars such that $e = x_{\mathfrak{C}}$.

2. Standardized Constructor Signature

Let us note that there exists a quasi-locus sequence which is empty.

Let $\mathfrak{C}$ be a constructor signature. We say that $\mathfrak{C}$ is standardized if and only if the condition (Def. 1) is satisfied.

(Def. 1) Let o be an operation symbol of $\mathfrak{C}$. Suppose o is constructor. Then $o \in$ Constructors and $o_{\mathbf{1}} =$ the result sort of o and $\operatorname{Card}((o_{\mathbf{2}})_{\mathbf{1}}) = \operatorname{len} \operatorname{Arity}(o)$.

The following proposition is true

(5) Let $\mathfrak{C}$ be a constructor signature. Suppose $\mathfrak{C}$ is standardized. Let o be an operation symbol of $\mathfrak{C}$. Then o is constructor if and only if $o \in$ Constructors .

Let us note that $\mathfrak{M}$ is standardized.

Let us observe that there exists a constructor signature which is initialized, standardized, and strict.

Let $\mathfrak{C}$ be an initialized standardized constructor signature and let c be a constructor operation symbol of $\mathfrak{C}$. The loci of c yielding a quasi-locus sequence is defined by:

(Def. 2) The loci of $c = (c_{\mathbf{2}})_{\mathbf{1}}$.

Let $\mathfrak{C}$ be a constructor signature. One can verify that there exists a subsignature of $\mathfrak{C}$ which is constructor.

Let $\mathfrak{C}$ be an initialized constructor signature. Note that there exists a constructor subsignature of $\mathfrak{C}$ which is initialized.

Let $\mathfrak{C}$ be a standardized constructor signature. One can verify that every constructor subsignature of $\mathfrak{C}$ is standardized.

One can prove the following two propositions:

(6) Let S_1, S_2 be standardized constructor signatures. Suppose the operation symbols of S_1 = the operation symbols of S_2. Then the many sorted signature of S_1 = the many sorted signature of S_2.

(7) For every constructor signature $\mathfrak{C}$ holds $\mathfrak{C}$ is standardized iff $\mathfrak{C}$ is a subsignature of $\mathfrak{M}$.

Let $\mathfrak{C}$ be an initialized constructor signature. Observe that there exists a quasi-term of $\mathfrak{C}$ which is non compound.

Let us mention that every element of Vars is pair.

The following propositions are true:

(8) For every element x of Vars such that vars(x) is natural holds vars(x) = 0.

(9) Vars misses Constructors.

(10) For every element x of Vars holds $x \neq *$ and $x \neq$ **non** .

(11) For every standardized constructor signature $\mathfrak{C}$ holds Vars misses the operation symbols of $\mathfrak{C}$.

(12) Let $\mathfrak{C}$ be an initialized standardized constructor signature and e be an expression of $\mathfrak{C}$. Then

(i) there exists an element x of Vars such that $e = x_{\mathfrak{C}}$ and $e(\emptyset) = \langle x,$ **term** $\rangle$, or

(ii) there exists an operation symbol o of $\mathfrak{C}$ such that $e(\emptyset) = \langle o,$ the carrier of $\mathfrak{C}\rangle$ but $o \in$ Constructors or $o = *$ or $o =$ **non** .

Let $\mathfrak{C}$ be an initialized standardized constructor signature and let e be an expression of $\mathfrak{C}$. Note that $e(\emptyset)$ is pair.

The following propositions are true:

(13) Let $\mathfrak{C}$ be an initialized constructor signature, e be an expression of $\mathfrak{C}$, and o be an operation symbol of $\mathfrak{C}$. Suppose $e(\emptyset) = \langle o,$ the carrier of $\mathfrak{C}\rangle$. Then e is an expression of $\mathfrak{C}$ from the result sort of o.

(14) Let $\mathfrak{C}$ be an initialized standardized constructor signature and e be an expression of $\mathfrak{C}$. Then
 (i) if $e(\emptyset)_{\mathbf{1}} = *$, then e is an expression of $\mathfrak{C}$ from $\mathbf{type}_{\mathfrak{C}}$, and
 (ii) if $e(\emptyset)_{\mathbf{1}} = \mathbf{non}$, then e is an expression of $\mathfrak{C}$ from $\mathbf{adj}_{\mathfrak{C}}$.

(15) Let $\mathfrak{C}$ be an initialized standardized constructor signature and e be an expression of $\mathfrak{C}$. Then
 (i) $e(\emptyset)_{\mathbf{1}} \in$ Vars and $e(\emptyset)_{\mathbf{2}} = \mathbf{term}$ and e is a quasi-term of $\mathfrak{C}$, or
 (ii) $e(\emptyset)_{\mathbf{2}} =$ the carrier of $\mathfrak{C}$ but $e(\emptyset)_{\mathbf{1}} \in$ Constructors and $e(\emptyset)_{\mathbf{1}} \in$ the operation symbols of $\mathfrak{C}$ or $e(\emptyset)_{\mathbf{1}} = *$ or $e(\emptyset)_{\mathbf{1}} = \mathbf{non}$.

(16) Let $\mathfrak{C}$ be an initialized standardized constructor signature and e be an expression of $\mathfrak{C}$. If $e(\emptyset)_{\mathbf{1}} \in$ Constructors, then $e \in$ (the sorts of $\mathrm{Free}_{\mathfrak{C}}(\mathrm{Vars}\,\mathfrak{C}))((e(\emptyset)_{\mathbf{1}})_{\mathbf{1}})$.

(17) Let $\mathfrak{C}$ be an initialized standardized constructor signature and e be an expression of $\mathfrak{C}$. Then $e(\emptyset)_{\mathbf{1}} \notin$ Vars if and only if $e(\emptyset)_{\mathbf{1}}$ is an operation symbol of $\mathfrak{C}$.

(18) Let $\mathfrak{C}$ be an initialized standardized constructor signature and e be an expression of $\mathfrak{C}$. If $e(\emptyset)_{\mathbf{1}} \in$ Vars, then there exists an element x of Vars such that $x = e(\emptyset)_{\mathbf{1}}$ and $e = x_{\mathfrak{C}}$.

(19) Let $\mathfrak{C}$ be an initialized standardized constructor signature and e be an expression of $\mathfrak{C}$. Suppose $e(\emptyset)_{\mathbf{1}} = *$. Then there exists an expression α of $\mathfrak{C}$ from $\mathbf{adj}_{\mathfrak{C}}$ and there exists an expression q of $\mathfrak{C}$ from $\mathbf{type}_{\mathfrak{C}}$ such that $e = \langle *, 3\rangle\text{-tree}(\alpha, q)$.

(20) Let $\mathfrak{C}$ be an initialized standardized constructor signature and e be an expression of $\mathfrak{C}$. If $e(\emptyset)_{\mathbf{1}} = \mathbf{non}$, then there exists an expression α of $\mathfrak{C}$ from $\mathbf{adj}_{\mathfrak{C}}$ such that $e = \langle \mathbf{non}, 3\rangle\text{-tree}(\alpha)$.

(21) Let $\mathfrak{C}$ be an initialized standardized constructor signature and e be an expression of $\mathfrak{C}$. Suppose $e(\emptyset)_{\mathbf{1}} \in$ Constructors. Then there exists an operation symbol o of $\mathfrak{C}$ such that $o = e(\emptyset)_{\mathbf{1}}$ and the result sort of $o = o_{\mathbf{1}}$ and e is an expression of $\mathfrak{C}$ from the result sort of o.

(22) Let $\mathfrak{C}$ be an initialized standardized constructor signature and τ be a quasi-term of $\mathfrak{C}$. Then τ is compound if and only if $\tau(\emptyset)_{\mathbf{1}} \in$ Constructors and $(\tau(\emptyset)_{\mathbf{1}})_{\mathbf{1}} = \mathbf{term}$.

(23) Let $\mathfrak{C}$ be an initialized standardized constructor signature and τ be an expression of $\mathfrak{C}$. Then τ is a non compound quasi-term of $\mathfrak{C}$ if and only if $\tau(\emptyset)_{\mathbf{1}} \in$ Vars.

(24) Let $\mathfrak{C}$ be an initialized standardized constructor signature and τ be an expression of $\mathfrak{C}$. Then τ is a quasi-term of $\mathfrak{C}$ if and only if $\tau(\emptyset)_{\mathbf{1}} \in$ Constructors and $(\tau(\emptyset)_{\mathbf{1}})_{\mathbf{1}} = \mathbf{term}$ or $\tau(\emptyset)_{\mathbf{1}} \in$ Vars.

(25) Let $\mathfrak{C}$ be an initialized standardized constructor signature and α be an expression of $\mathfrak{C}$. Then α is a positive quasi-adjective of $\mathfrak{C}$ if and only if

$\alpha(\emptyset)_{\mathbf{1}} \in$ Constructors and $(\alpha(\emptyset)_{\mathbf{1}})_{\mathbf{1}} = \mathbf{adj}$.

(26) Let $\mathfrak{C}$ be an initialized standardized constructor signature and α be a quasi-adjective of $\mathfrak{C}$. Then α is negative if and only if $\alpha(\emptyset)_{\mathbf{1}} = \mathbf{non}$.

(27) Let $\mathfrak{C}$ be an initialized standardized constructor signature and τ be an expression of $\mathfrak{C}$. Then τ is a pure expression of $\mathfrak{C}$ from $\mathbf{type}_{\mathfrak{C}}$ if and only if $\tau(\emptyset)_{\mathbf{1}} \in$ Constructors and $(\tau(\emptyset)_{\mathbf{1}})_{\mathbf{1}} = \mathbf{type}$.

3. Expressions

In the sequel i is a natural number, x is a variable, and ℓ is a quasi-locus sequence.

An expression is an expression of $\mathfrak{M}$. A valuation is a valuation of $\mathfrak{M}$. A quasi-adjective is a quasi-adjective of $\mathfrak{M}$. The subset QuasiAdjs of $\mathrm{Free}_{\mathfrak{M}}(\mathrm{Vars}\,\mathfrak{M})$ is defined as follows:

(Def. 3) QuasiAdjs = QuasiAdjs $\mathfrak{M}$.

A quasi-term is a quasi-term of $\mathfrak{M}$. The subset QuasiTerms of $\mathrm{Free}_{\mathfrak{M}}(\mathrm{Vars}\,\mathfrak{M})$ is defined as follows:

(Def. 4) QuasiTerms = QuasiTerms $\mathfrak{M}$.

A quasi-type is a quasi-type of $\mathfrak{M}$. The functor QuasiTypes is defined as follows:

(Def. 5) QuasiTypes = QuasiTypes $\mathfrak{M}$.

One can verify the following observations:

* QuasiAdjs is non empty,
* QuasiTerms is non empty, and
* QuasiTypes is non empty.

Modes is a non empty subset of Constructors. Then Attrs is a non empty subset of Constructors. Then Funcs is a non empty subset of Constructors.

In the sequel $\mathfrak{C}$ denotes an initialized constructor signature.

The element set-constr of Modes is defined by:

(Def. 6) set-constr $= \langle \mathbf{type}, \langle \emptyset, 0 \rangle \rangle$.

One can prove the following propositions:

(28) The kind of set-constr $= \mathbf{type}$ and the loci of set-constr $= \emptyset$ and the index of set-constr $= 0$.

(29) Constructors $= \{\mathbf{type}, \mathbf{adj}, \mathbf{term}\} \times (\mathrm{QuasiLoci} \times \mathbb{N})$.

(30) $\langle \mathrm{rng}\,\ell, i \rangle \in$ Vars and $\ell \frown \langle \langle \mathrm{rng}\,\ell, i \rangle \rangle$ is a quasi-locus sequence.

(31) There exists ℓ such that $\mathrm{len}\,\ell = i$.

(32) For every finite subset X of Vars there exists ℓ such that $\mathrm{rng}\,\ell = \mathrm{varcl}\,X$.

(33) Let X, o be sets and p be a decorated tree yielding finite sequence. Given $\mathfrak{C}$ such that $X = \bigcup$(the sorts of $\mathrm{Free}_{\mathfrak{C}}(\mathrm{Vars}\,\mathfrak{C})$). If o-tree$(p) \in X$, then p is a finite sequence of elements of X.

Let us consider $\mathfrak{C}$ and let e be an expression of $\mathfrak{C}$. An expression of $\mathfrak{C}$ is called a subexpression of e if:

(Def. 7) It $\in$ Subtrees(e).

The functor constrs e is defined by:

(Def. 8) constrs $e = \pi_1(\operatorname{rng} e) \cap \{o : o$ ranges over constructor operation symbols of $\mathfrak{C}\}$.

The functor main-constr e is defined by:

(Def. 9) $\text{main-constr}\, e = \begin{cases} e(\emptyset)_{\mathbf{1}}, & \text{if } e \text{ is compound}, \\ \emptyset, & \text{otherwise.} \end{cases}$

The functor args e yields a finite sequence of elements of $\mathrm{Free}_{\mathfrak{C}}(\mathrm{Vars}\,\mathfrak{C})$ and is defined by:

(Def. 10) $e = e(\emptyset)\text{-tree}(\mathrm{args}\, e)$.

Next we state three propositions:

(34) For every $\mathfrak{C}$ holds every expression e of $\mathfrak{C}$ is a subexpression of e.

(35) $\text{main-constr}(x_{\mathfrak{C}}) = \emptyset$.

(36) Let c be a constructor operation symbol of $\mathfrak{C}$ and p be a finite sequence of elements of QuasiTerms $\mathfrak{C}$. If $\operatorname{len} p = \operatorname{len} \mathrm{Arity}(c)$, then $\text{main-constr}(c^{\to}(p)) = c$.

Let us consider $\mathfrak{C}$ and let e be an expression of $\mathfrak{C}$. We say that e is constructor if and only if:

(Def. 11) e is compound and main-constr e is a constructor operation symbol of $\mathfrak{C}$.

Let us consider $\mathfrak{C}$. Observe that every expression of $\mathfrak{C}$ which is constructor is also compound.

Let us consider $\mathfrak{C}$. Observe that there exists an expression of $\mathfrak{C}$ which is constructor.

Let us consider $\mathfrak{C}$ and let e be a constructor expression of $\mathfrak{C}$. One can verify that there exists a subexpression of e which is constructor.

Let S be a non void signature, let X be a non empty yielding many sorted set indexed by S, and let τ be an element of $\mathrm{Free}_S(X)$. Observe that $\operatorname{rng}\tau$ is relation-like.

One can prove the following proposition

(37) For every constructor expression e of $\mathfrak{C}$ holds main-constr $e \in$ constrs e.

4. Arity

For simplicity, we follow the rules: α is a quasi-adjective, τ, τ_1, τ_2 are quasi-terms, ϑ is a quasi-type, and c is an element of Constructors.

Let $\mathfrak{C}$ be a non void signature. We say that $\mathfrak{C}$ is arity-rich if and only if the condition (Def. 12) is satisfied.

(Def. 12) Let n be a natural number and s be a sort symbol of $\mathfrak{C}$. Then $\{o; o$ ranges over operation symbols of $\mathfrak{C}$: the result sort of $o = s \ \wedge\ \operatorname{len}\operatorname{Arity}(o) = n\}$ is infinite.

Let o be an operation symbol of $\mathfrak{C}$. We say that o is nullary if and only if:

(Def. 13) $\operatorname{Arity}(o) = \emptyset$.

We say that o is unary if and only if:

(Def. 14) $\operatorname{len}\operatorname{Arity}(o) = 1$.

We say that o is binary if and only if:

(Def. 15) $\operatorname{len}\operatorname{Arity}(o) = 2$.

The following proposition is true

(38) Let $\mathfrak{C}$ be a non void signature and o be an operation symbol of $\mathfrak{C}$. Then
(i) if o is nullary, then o is not unary,
(ii) if o is nullary, then o is not binary, and
(iii) if o is unary, then o is not binary.

Let $\mathfrak{C}$ be a constructor signature. Observe that $\mathbf{non}_{\mathfrak{C}}$ is unary and $*_{\mathfrak{C}}$ is binary.

Let $\mathfrak{C}$ be a constructor signature. Note that every operation symbol of $\mathfrak{C}$ which is nullary is also constructor.

The following proposition is true

(39) Let $\mathfrak{C}$ be a constructor signature. Then $\mathfrak{C}$ is initialized if and only if there exists an operation symbol m of $\mathbf{type}_{\mathfrak{C}}$ and there exists an operation symbol α of $\mathbf{adj}_{\mathfrak{C}}$ such that m is nullary and α is nullary.

Let $\mathfrak{C}$ be an initialized constructor signature. One can verify that there exists an operation symbol of $\mathbf{type}_{\mathfrak{C}}$ which is nullary and constructor and there exists an operation symbol of $\mathbf{adj}_{\mathfrak{C}}$ which is nullary and constructor.

Let $\mathfrak{C}$ be an initialized constructor signature. Observe that there exists an operation symbol of $\mathfrak{C}$ which is nullary and constructor.

One can check that every non void signature which is arity-rich has also an operation for each sort and every constructor signature which is arity-rich is also initialized.

One can check that $\mathfrak{M}$ is arity-rich.

Let us mention that there exists a constructor signature which is arity-rich and initialized.

Let $\mathfrak{C}$ be an arity-rich constructor signature and let s be a sort symbol of $\mathfrak{C}$. One can verify the following observations:

* there exists an operation symbol of s which is nullary and constructor,
* there exists an operation symbol of s which is unary and constructor, and
* there exists an operation symbol of s which is binary and constructor.

Let $\mathfrak{C}$ be an arity-rich constructor signature. One can check that there exists an operation symbol of $\mathfrak{C}$ which is unary and constructor and there exists an operation symbol of $\mathfrak{C}$ which is binary and constructor.

The following proposition is true

(40) Let o be a nullary operation symbol of $\mathfrak{C}$. Then $\langle o$, the carrier of $\mathfrak{C}\rangle$-tree$(\emptyset)$ is an expression of $\mathfrak{C}$ from the result sort of o.

Let $\mathfrak{C}$ be an initialized constructor signature and let m be a nullary constructor operation symbol of $\mathbf{type}_{\mathfrak{C}}$. Then m_{t} is a pure expression of $\mathfrak{C}$ from $\mathbf{type}_{\mathfrak{C}}$.

Let c be an element of Constructors. The functor ${}^{@}c$ yielding a constructor operation symbol of $\mathfrak{M}$ is defined by:

(Def. 16) ${}^{@}c = c$.

Let m be an element of Modes. Then ${}^{@}m$ is a constructor operation symbol of $\mathbf{type}_{\mathfrak{M}}$.

Let us note that ${}^{@}$set-constr is nullary.

We now state the proposition

(41) Arity(${}^{@}$set-constr) $= \emptyset$.

The quasi-type set-type is defined by:

(Def. 17) set-type $= \emptyset_{\mathrm{QuasiAdjs}\,\mathfrak{M}} * ({}^{@}\text{set-constr})_{\mathrm{t}}$.

The following proposition is true

(42) adjs set-type $= \emptyset$ and the base of set-type $= ({}^{@}\text{set-constr})_{\mathrm{t}}$.

Let ℓ be a finite sequence of elements of Vars. The functor args ℓ yields a finite sequence of elements of QuasiTerms $\mathfrak{M}$ and is defined as follows:

(Def. 18) $\mathrm{len\,args}\,\ell = \mathrm{len}\,\ell$ and for every i such that $i \in \mathrm{dom}\,\ell$ holds $(\mathrm{args}\,\ell)(i) = (\ell_i)_{\mathfrak{M}}$.

Let us consider c. The base expression of c yields an expression and is defined as follows:

(Def. 19) The base expression of $c = ({}^{@}c)^{\neg}$(args (the loci of c)).

Next we state several propositions:

(43) For every operation symbol o of $\mathfrak{M}$ holds o is constructor iff $o \in$ Constructors.

(44) For every nullary operation symbol m of $\mathfrak{M}$ holds main-constr$(m_{\mathrm{t}}) = m$.

(45) For every unary constructor operation symbol m of $\mathfrak{M}$ and for every τ holds main-constr$(m(\tau)) = m$.

(46) For every α holds main-constr$(\mathbf{non}_{\mathfrak{M}}(\alpha)) = \mathbf{non}$.

(47) For every binary constructor operation symbol m of $\mathfrak{M}$ and for all τ_1, τ_2 holds main-constr$(m(\tau_1, \tau_2)) = m$.

(48) For every expression q of $\mathfrak{M}$ from $\mathbf{type}_{\mathfrak{M}}$ and for every α holds main-constr$(*_{\mathfrak{M}}(\alpha, q)) = *$.

Let ϑ be a quasi-type. The functor $\operatorname{constrs}\vartheta$ is defined by:

(Def. 20) $\operatorname{constrs}\vartheta = \operatorname{constrs}(\text{the base of } \vartheta) \cup \bigcup\{\operatorname{constrs}\alpha : \alpha \in \operatorname{adjs}\vartheta\}$.

The following two propositions are true:

(49) For every pure expression q of $\mathfrak{M}$ from $\mathbf{type}_{\mathfrak{M}}$ and for every finite subset A of QuasiAdjs $\mathfrak{M}$ holds $\operatorname{constrs}(A * q) = \operatorname{constrs} q \cup \bigcup\{\operatorname{constrs}\alpha : \alpha \in A\}$.

(50) $\operatorname{constrs}(\alpha * \vartheta) = \operatorname{constrs}\alpha \cup \operatorname{constrs}\vartheta$.

5. Unification

Let $\mathfrak{C}$ be an initialized constructor signature and let τ, p be expressions of $\mathfrak{C}$. We say that τ matches p if and only if:

(Def. 21) There exists a valuation f of $\mathfrak{C}$ such that $\tau = p[f]$.

Let us note that the predicate τ matches p is reflexive.

The following proposition is true

(51) For all expressions τ_1, τ_2, τ_3 of $\mathfrak{C}$ such that τ_1 matches τ_2 and τ_2 matches τ_3 holds τ_1 matches τ_3.

Let $\mathfrak{C}$ be an initialized constructor signature and let A, B be subsets of QuasiAdjs $\mathfrak{C}$. We say that A matches B if and only if:

(Def. 22) There exists a valuation f of $\mathfrak{C}$ such that $B[f] \subseteq A$.

Let us note that the predicate A matches B is reflexive.

The following proposition is true

(52) For all subsets A_1, A_2, A_3 of QuasiAdjs $\mathfrak{C}$ such that A_1 matches A_2 and A_2 matches A_3 holds A_1 matches A_3.

Let $\mathfrak{C}$ be an initialized constructor signature and let ϑ, P be quasi-types of $\mathfrak{C}$. We say that ϑ matches P if and only if:

(Def. 23) There exists a valuation f of $\mathfrak{C}$ such that $(\operatorname{adjs} P)[f] \subseteq \operatorname{adjs}\vartheta$ and (the base of $P)[f] =$ the base of ϑ.

Let us note that the predicate ϑ matches P is reflexive.

One can prove the following proposition

(53) For all quasi-types ϑ_1, ϑ_2, ϑ_3 of $\mathfrak{C}$ such that ϑ_1 matches ϑ_2 and ϑ_2 matches ϑ_3 holds ϑ_1 matches ϑ_3.

Let $\mathfrak{C}$ be an initialized constructor signature, let τ_1, τ_2 be expressions of $\mathfrak{C}$, and let f be a valuation of $\mathfrak{C}$. We say that f unifies τ_1 with τ_2 if and only if:

(Def. 24) $\tau_1[f] = \tau_2[f]$.

The following proposition is true

(54) Let τ_1, τ_2 be expressions of $\mathfrak{C}$ and f be a valuation of $\mathfrak{C}$. If f unifies τ_1 with τ_2, then f unifies τ_2 with τ_1.

Let $\mathfrak{C}$ be an initialized constructor signature and let τ_1, τ_2 be expressions of $\mathfrak{C}$. We say that τ_1 and τ_2 are unifiable if and only if:

(Def. 25) There exists a valuation f of $\mathfrak{C}$ such that f unifies τ_1 with τ_2.

Let us notice that the predicate τ_1 and τ_2 are unifiable is reflexive and symmetric.

Let $\mathfrak{C}$ be an initialized constructor signature and let τ_1, τ_2 be expressions of $\mathfrak{C}$. We say that τ_1 and τ_2 are weakly-unifiable if and only if:

(Def. 26) There exists an irrelevant one-to-one valuation g of $\mathfrak{C}$ such that $\operatorname{Var} \tau_2 \subseteq \operatorname{dom} g$ and τ_1 and $\tau_2[g]$ are unifiable.

Let us note that the predicate τ_1 and τ_2 are weakly-unifiable is reflexive.

We now state the proposition

(55) For all expressions τ_1, τ_2 of $\mathfrak{C}$ such that τ_1 and τ_2 are unifiable holds τ_1 and τ_2 are weakly-unifiable.

Let $\mathfrak{C}$ be an initialized constructor signature and let τ, τ_1, τ_2 be expressions of $\mathfrak{C}$. We say that τ is a unification of τ_1 and τ_2 if and only if:

(Def. 27) There exists a valuation f of $\mathfrak{C}$ such that f unifies τ_1 with τ_2 and $\tau = \tau_1[f]$.

We now state two propositions:

(56) For all expressions τ_1, τ_2, τ of $\mathfrak{C}$ such that τ is a unification of τ_1 and τ_2 holds τ is a unification of τ_2 and τ_1.

(57) For all expressions τ_1, τ_2, τ of $\mathfrak{C}$ such that τ is a unification of τ_1 and τ_2 holds τ matches τ_1 and τ matches τ_2.

Let $\mathfrak{C}$ be an initialized constructor signature and let τ, τ_1, τ_2 be expressions of $\mathfrak{C}$. We say that τ is a general-unification of τ_1 and τ_2 if and only if the conditions (Def. 28) are satisfied.

(Def. 28)(i) τ is a unification of τ_1 and τ_2, and

(ii) for every expression u of $\mathfrak{C}$ such that u is a unification of τ_1 and τ_2 holds u matches τ.

6. Type Distribution

The following three propositions are true:

(58) Let n be a natural number and s be a sort symbol of $\mathfrak{M}$. Then there exists a constructor operation symbol m of s such that $\operatorname{len} \operatorname{Arity}(m) = n$.

(59) Let given ℓ, s be a sort symbol of $\mathfrak{M}$, and m be a constructor operation symbol of s. If $\operatorname{len} \operatorname{Arity}(m) = \operatorname{len} \ell$, then $\operatorname{Var}(m^{\to}(\operatorname{args} \ell)) = \operatorname{rng} \ell$.

(60) Let X be a finite subset of Vars. Suppose $\operatorname{varcl} X = X$. Let s be a sort symbol of $\mathfrak{M}$. Then there exists a constructor operation symbol m of s and there exists a finite sequence p of elements of QuasiTerms $\mathfrak{M}$ such that $\operatorname{len} p = \operatorname{len} \operatorname{Arity}(m)$ and $\operatorname{vars}(m^{\to}(p)) = X$.

Let d be a partial function from Vars to QuasiTypes. We say that d is even if and only if:

(Def. 29) For all x, ϑ such that $x \in \operatorname{dom} d$ and $\vartheta = d(x)$ holds $\operatorname{vars}(\vartheta) = \operatorname{vars}(x)$.

Let ℓ be a quasi-locus sequence. A partial function from Vars to QuasiTypes is said to be a type-distribution for ℓ if:

(Def. 30) $\operatorname{dom} \mathrm{it} = \operatorname{rng} \ell$ and it is even.

We now state the proposition

(61) For every empty quasi-locus sequence ℓ holds $\emptyset$ is a type-distribution for ℓ.

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