

Simple-Named Complex-Valued Nominative Data – Definition and Basic Operations

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Summary. In this paper we give a formal definition of the notion of nominative data with simple names and complex values [16? , 19] and formal definitions of the basic operations on such data, including naming, denaming and overlapping, following the work [19].

The notion of nominative data plays an important role in the composition-nominative approach to program formalization [16?] which is a development of composition programming [18]. Both approaches are compared in [?].

The composition-nominative approach considers mathematical models of computer software and data on various levels of abstraction and generality and provides mathematical tools for reasoning about their properties. In particular, nominative data are mathematical models of data which are stored and processed in computer systems. The composition-nominative approach considers different types [? 19] of nominative data, but all of them are based on the name-value relation. One powerful type of nominative data, which is suitable for representing many kinds of data commonly used in programming like lists, multidimensional arrays, trees, tables, etc. is the type of nominative data with simple (abstract) names and complex (structured) values. The set of nominative data of given type together with a number of basic operations on them like naming, denaming and overlapping [19] form an algebra which is called *data algebra*.

In the composition-nominative approach computer programs which process data are modeled as partial functions which map nominative data from the carrier of a given data algebra (input data) to nominative data (output data). Such functions are also called *binominative functions*. Programs which evaluate conditions are modeled as partial predicates on nominative data (nominative predicates). Programming language constructs like sequential execution, branching, cycle, etc. which construct programs from the existing programs are modeled as operations which take binominative functions and predicates and produce binominative functions. Such operations are called *compositions*. A set of binominative functions and a set of predicates together with appropriate compositions form an algebra which is called *program algebra*. This algebra serves as a semantic model of a programming language.

For functions over nominative data a special computability called abstract computability is introduced and complete classes of computable functions are specified [?].

For reasoning about properties of programs modeled as binominative functions a Floyd-Hoare style logic [4, 5] is introduced and applied [14, 15, 10, 13, 11, 12]. One advantage of this approach to reasoning about programs is that it naturally handles programs which process complex data structures (which can be quite straightforwardly represented as nominative data). Also, unlike classical Floyd-Hoare logic, the mentioned logic allows reasoning about assertions which include partial pre- and post-conditions [13].

Besides modeling data processed by programs, nominative data can be also applied to modeling data processed by signal processing systems in the context of the mathematical systems theory [7, 9, 8, 6].

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1. PRELIMINARIES

From now on $a, a_1, a_2, v, v_1, v_2, x$ denote objects, V, A denote sets, m, n denote natural numbers, and S, S_1, S_2 denote finite sequences.

Now we state the propositions:

- (1) Let us consider a finite sequence f . If $n \in \text{dom } f$, then $(\langle x \rangle \wedge f)(n+1) = f(n)$.
- (2) Let us consider a function f . Suppose $\text{dom } f = \mathbb{N}$. Then $f \upharpoonright \text{Seg } n$ is a finite sequence.
- (3) Let us consider finite sequences f, g . Then
 - (i) $\text{dom } f \subseteq \text{dom } g$, or
 - (ii) $\text{dom } g \subseteq \text{dom } f$.

Let f, g be finite sequences. One can check that $f + \cdot g$ is finite sequence-like.

Let f_1, f_2 be functions. Note that $f_2 \cup f_1 \upharpoonright (\text{dom } f_1 \setminus \text{dom } f_2)$ is function-like.

Let f, g be functions and x, y be objects. We say that $f, x = g, y$ if and only if

(Def. 1) $(x \in \text{dom } f \text{ iff } y \in \text{dom } g)$ and if $x \in \text{dom } f$, then $f(x) = g(y)$.

2. DEFINITION OF SIMPLE-NAMED COMPLEX-VALUED NOMINATIVE DATA

Let us consider V and A .

A nominative set of V and A is a partial function from V to A . Let us note that there exists a nominative set of V and A which is finite.

A nominative data with simple names from V and simple values from A is a finite nominative set of V and A . The functor $\text{ND}_{\text{SS}}(V, A)$ yielding a set is defined by the term

(Def. 2) the set of all d where d is a nominative data with simple names from V and simple values from A .

Let us note that $\text{ND}_{\text{SS}}(V, A)$ is non empty.

Now we state the propositions:

- (4) If $x \in \text{ND}_{\text{SS}}(V, A)$, then x is a nominative data with simple names from V and simple values from A .
- (5) $\text{ND}_{\text{SS}}(V, A) \subseteq V \dot{\rightarrow} A$.
- (6) $\emptyset \in \text{ND}_{\text{SS}}(V, A)$.
- (7) Let us consider sets A, B . If $A \subseteq B$, then $\text{ND}_{\text{SS}}(V, A) \subseteq \text{ND}_{\text{SS}}(V, B)$.
- (8) Let us consider finite functions f, g . Suppose $f \approx g$ and $f, g \in \text{ND}_{\text{SS}}(V, A)$. Then $f \cup g \in \text{ND}_{\text{SS}}(V, A)$. The theorem is a consequence of (4).

Let us consider V and A . The functor $\text{FND}_{\text{SC}}(V, A)$ yielding a function is defined by

(Def. 3) $\text{dom } it = \mathbb{N}$ and $it(0) = A$ and for every natural number n , $it(n+1) = \text{ND}_{\text{SS}}(V, A \cup it(n))$.

Now we state the propositions:

- (9) $(\text{FND}_{\text{SC}}(V, A))(1) = \text{ND}_{\text{SS}}(V, A)$.
- (10) $(\text{FND}_{\text{SC}}(V, A))(2) = \text{ND}_{\text{SS}}(V, A \cup \text{ND}_{\text{SS}}(V, A))$. The theorem is a consequence of (9).
- (11) $A \subseteq \bigcup \text{FND}_{\text{SC}}(V, A)$.
- (12) If $1 \leq n$, then $\emptyset \in (\text{FND}_{\text{SC}}(V, A))(n)$. The theorem is a consequence of (6).

Let us consider V , A , and n . One can check that $\text{FND}_{\text{SC}}(V, A) \upharpoonright \text{Seg } n$ is finite sequence-like.

Now we state the proposition:

- (13) If $m \neq 0$ and $m \leq n$, then $(\text{FND}_{\text{SC}}(V, A))(m) \subseteq (\text{FND}_{\text{SC}}(V, A))(n)$.

PROOF: Set $S = \text{FND}_{\text{SC}}(V, A)$. Define $\mathcal{P}[\text{natural number}] \equiv \text{if } m \leq \$_1$, then $S(m) \subseteq S(\$_1)$. For every natural number k such that $\mathcal{P}[k]$ holds $\mathcal{P}[k+1]$ by [1, (8)], (9), (10), (7). For every natural number k , $\mathcal{P}[k]$ from [1, Sch. 2]. \square

Let us consider V and A . Let S be a finite sequence. We say that S is a rank sequence if and only if

- (Def. 4) $S(1) = \text{ND}_{\text{SS}}(V, A)$ and for every natural number n such that $n, n+1 \in \text{dom } S$ holds $S(n+1) = \text{ND}_{\text{SS}}(V, A \cup S(n))$.

Now we state the propositions:

- (14) If S is a rank sequence, then $S \neq \emptyset$.
- (15) If S is a rank sequence and $S_1 \subseteq S$ and $S_1 \neq \emptyset$, then S_1 is a rank sequence.
- (16) If S is a rank sequence and $n \in \text{dom } S$, then $S \upharpoonright n$ is a rank sequence. The theorem is a consequence of (15).
- (17) If S is a rank sequence, then $S \hat{\ } \langle \text{ND}_{\text{SS}}(V, A \cup S(\text{len } S)) \rangle$ is a rank sequence.
- (18) If $1 \leq n$, then $\text{FND}_{\text{SC}}(V, A) \upharpoonright \text{Seg } n$ is a rank sequence. The theorem is a consequence of (9).
- (19) If S is a rank sequence and $n \in \text{dom } S$, then $S(n) = (\text{FND}_{\text{SC}}(V, A))(n)$.
PROOF: Set $F = \text{FND}_{\text{SC}}(V, A)$. Define $\mathcal{P}[\text{natural number}] \equiv \text{if } \$_1 \in \text{dom } S$, then $S(\$_1) = F(\$_1)$. For every n such that $\mathcal{P}[n]$ holds $\mathcal{P}[n+1]$ by (9), [1, (14)], [22, (25)], [1, (11)]. For every n , $\mathcal{P}[n]$ from [1, Sch. 2]. \square
- (20) If S is a rank sequence, then $S = \text{FND}_{\text{SC}}(V, A) \upharpoonright \text{dom } S$. The theorem is a consequence of (19).
- (21) If S_1 is a rank sequence and S_2 is a rank sequence, then $S_1 \approx S_2$.
PROOF: Define $\mathcal{P}[\text{natural number}] \equiv \text{if } \$_1 \in \text{dom } S_1 \cap \text{dom } S_2$, then $S_1(\$_1) = S_2(\$_1)$. $\mathcal{P}[0]$ by [22, (24)]. For every n such that $\mathcal{P}[n]$ holds $\mathcal{P}[n+1]$ by [1, (14), (11)], [22, (25)]. For every n , $\mathcal{P}[n]$ from [1, Sch. 2]. \square
- (22) If S_1 is a rank sequence and S_2 is a rank sequence, then $S_1 \subseteq S_2$ or $S_2 \subseteq S_1$. The theorem is a consequence of (20) and (3).
- (23) If S_1 is a rank sequence and S_2 is a rank sequence, then $S_1 + \cdot S_2 = S_1$ or $S_1 + \cdot S_2 = S_2$. The theorem is a consequence of (21) and (3).
- (24) If S_1 is a rank sequence and S_2 is a rank sequence, then $S_1 + \cdot S_2$ is a rank sequence.

- (25) If S is a rank sequence and $m \leq n$ and $n \in \text{dom } S$, then $S(m) \subseteq S(n)$. The theorem is a consequence of (19) and (13).
- (26) Let us consider a finite sequence F . Suppose F is a rank sequence. Then there exists a finite sequence S such that
- (i) $\text{len } S = 1 + \text{len } F$, and
 - (ii) S is a rank sequence, and
 - (iii) for every natural number n such that $n \in \text{dom } S$ holds $S(n) = \text{ND}_{\text{SS}}(V, A \cup (\langle A \rangle \wedge F)(n))$.

PROOF: Set $G = \langle A \rangle \wedge F$. Define $\mathcal{F}(\text{object}) = \text{ND}_{\text{SS}}(V, A \cup G(\$_1))$. Consider S being a finite sequence such that $\text{len } S = \text{len } G$ and for every natural number n such that $n \in \text{dom } S$ holds $S(n) = \mathcal{F}(n)$ from [2, Sch. 2]. For every natural number n such that $n \in \text{dom } F$ holds $G(n+1) = F(n)$. S is a rank sequence by [22, (25)], (1), [21, (5)], [17, (20)]. \square

- (27) $\langle \text{ND}_{\text{SS}}(V, A) \rangle$ is a rank sequence.
- (28) $\langle \text{ND}_{\text{SS}}(V, A), \text{ND}_{\text{SS}}(V, A \cup \text{ND}_{\text{SS}}(V, A)) \rangle$ is a rank sequence. The theorem is a consequence of (27) and (17).
- (29) $\langle \text{ND}_{\text{SS}}(V, A), \text{ND}_{\text{SS}}(V, A \cup \text{ND}_{\text{SS}}(V, A)), \text{ND}_{\text{SS}}(V, A \cup \text{ND}_{\text{SS}}(V, A \cup \text{ND}_{\text{SS}}(V, A))) \rangle$ is a rank sequence. The theorem is a consequence of (17) and (28).

Let us consider V and A .

A non-atomic nominative data of V and A is a function and is defined by

(Def. 5) there exists a finite sequence S such that S is a rank sequence and $it \in \bigcup S$.

Let us note that .

From now on D, D_1, D_2 denote non-atomic nominative data of V and A .

Now we state the propositions:

- (30) \emptyset is a non-atomic nominative data of V and A . The theorem is a consequence of (27).
- (31) $D \in \bigcup \text{FND}_{\text{SC}}(V, A)$.
- (32) Let us consider a set d . If $d \subseteq D$, then d is a non-atomic nominative data of V and A . The theorem is a consequence of (4).
- (33) There exists a natural number n such that D is a nominative data with simple names from V and simple values from $A \cup (\text{FND}_{\text{SC}}(V, A))(n)$. The theorem is a consequence of (19) and (4).

Let us consider V and A . Note that every non-atomic nominative data of V and A is finite.

Now we state the propositions:

- (34) If $D_1 \approx D_2$ and S is a rank sequence and $D_1, D_2 \in S(m)$, then $D_1 \cup D_2 \in S(m)$. The theorem is a consequence of (4) and (8).
- (35) If $D_1 \approx D_2$ and S_2 is a rank sequence and $S_1 \subseteq S_2$ and $D_1 \in \bigcup S_1$ and $D_2 \in \bigcup S_2$, then $D_1 \cup D_2 \in \bigcup S_2$. The theorem is a consequence of (25) and (34).
- (36) If $D_1 \approx D_2$, then $D_1 \cup D_2$ is a non-atomic nominative data of V and A . The theorem is a consequence of (22) and (35).
- (37) If $D_1 \approx D_2$, then $D_1 + D_2$ is a non-atomic nominative data of V and A . The theorem is a consequence of (36).

Let us consider V and A .

A nominative data with simple names from V and complex values from A

is a set and is defined by

(Def. 6) $it \in A$ or it is a non-atomic nominative data of V and A .

Let us note that .

The functor **$ND_{SC}(V, A)$** yielding a set is defined by the term

(Def. 7) the set of all D where D is a nominative data with simple names from V and complex values from A .

Let us observe that $ND_{SC}(V, A)$ is non empty.

Now we state the propositions:

- (38) $\emptyset \in ND_{SC}(V, A)$. The theorem is a consequence of (30).
- (39) If $x \in ND_{SC}(V, A)$, then x is a nominative data with simple names from V and complex values from A .
- (40) $ND_{SC}(V, A) = \bigcup FND_{SC}(V, A)$. The theorem is a consequence of (39), (11), (31), (4), and (18).
- (41) $D \in ND_{SC}(V, A)$.
- (42) If $D \notin A$, then $D \in ND_{SC}(V, A) \setminus A$. The theorem is a consequence of (41).
- (43) If $x \in ND_{SC}(V, A) \setminus A$, then x is a non-atomic nominative data of V and A .
- (44) Let us consider a nominative data D with simple names from V and complex values from A . Then $D \in \bigcup FND_{SC}(V, A)$. The theorem is a consequence of (11) and (31).

3. EXAMPLES OF SIMPLE-NAMED COMPLEX-VALUED NOMINATIVE DATA

Let us consider v and a . The functor **$ND\text{-ex-1}(v, a)$** yielding a set is defined by the term

(Def. 8) $v \mapsto a$.

Observe that $\text{ND-ex-1}(v, a)$ is function-like and relation-like.

Now we state the propositions:

(45) If $v \in V$ and $a \in A$, then $\text{ND-ex-1}(v, a) \in \text{ND}_{\text{SS}}(V, A)$.

(46) If $v \in V$ and $a \in A$, then $\text{ND-ex-1}(v, a)$ is a non-atomic nominative data of V and A . The theorem is a consequence of (27) and (45).

Let V, A be non empty sets, v be an element of V , and a be an element of A . Observe that the functor $\text{ND-ex-1}(v, a)$ yields a non-atomic nominative data of V and A . Now we state the proposition:

(47) If $v \in V$ and $a \in A$, then $\text{ND-ex-1}(v, a)$ is a nominative data with simple names from V and complex values from A . The theorem is a consequence of (46).

Let us consider v, v_1 , and a_1 . The functor $\text{ND-ex-2}(v, v_1, a_1)$ yielding a set is defined by the term

(Def. 9) $v \mapsto (v_1 \mapsto a_1)$.

Note that $\text{ND-ex-2}(v, v_1, a_1)$ is function-like and relation-like.

Now we state the propositions:

(48) If $\{v, v_1\} \subseteq V$ and $a_1 \in A$, then $\text{ND-ex-2}(v, v_1, a_1) \in \text{ND}_{\text{SS}}(V, A \cup \text{ND}_{\text{SS}}(V, A))$.

(49) If $\{v, v_1\} \subseteq V$ and $a_1 \in A$, then $\text{ND-ex-2}(v, v_1, a_1)$ is a non-atomic nominative data of V and A . The theorem is a consequence of (28) and (48).

Let V, A be non empty sets, v, v_1 be elements of V , and a be an element of A . Let us note that the functor $\text{ND-ex-2}(v, v_1, a)$ yields a non-atomic nominative data of V and A . Now we state the proposition:

(50) Suppose $\{v, v_1\} \subseteq V$ and $a_1 \in A$. Then $\text{ND-ex-2}(v, v_1, a_1)$ is a nominative data with simple names from V and complex values from A . The theorem is a consequence of (49).

Let us consider v, v_1, a , and a_1 . The functor $\text{ND-ex-3}(v, v_1, a, a_1)$ yielding a set is defined by the term

(Def. 10) $[v \mapsto a, v_1 \mapsto a_1]$.

Let us note that $\text{ND-ex-3}(v, v_1, a, a_1)$ is function-like and relation-like.

Now we state the propositions:

(51) If $\{v, v_1\} \subseteq V$ and $\{a, a_1\} \subseteq A$, then $\text{ND-ex-3}(v, v_1, a, a_1) \in \text{ND}_{\text{SS}}(V, A)$. The theorem is a consequence of (45) and (8).

(52) Suppose $\{v, v_1\} \subseteq V$ and $\{a, a_1\} \subseteq A$. Then $\text{ND-ex-3}(v, v_1, a, a_1)$ is a non-atomic nominative data of V and A . The theorem is a consequence

of (27) and (51).

Let V, A be non empty sets, v, v_1 be elements of V , and a, a_1 be elements of A . Let us observe that the functor $\text{ND-ex-3}(v, v_1, a, a_1)$ yields a non-atomic nominative data of V and A . Now we state the proposition:

- (53) Suppose $\{v, v_1\} \subseteq V$ and $\{a, a_1\} \subseteq A$. Then $\text{ND-ex-3}(v, v_1, a, a_1)$ is a nominative data with simple names from V and complex values from A . The theorem is a consequence of (52).

Let us consider v, v_1, v_2, a , and a_1 . The functor $\text{ND-ex-4}(v, v_1, v_2, a, a_1)$ yielding a set is defined by the term

(Def. 11) $[v \mapsto a, v_1 \mapsto v_2 \dot{\mapsto} a_1]$.

Let us note that $\text{ND-ex-4}(v, v_1, v_2, a, a_1)$ is function-like and relation-like.

Now we state the propositions:

- (54) Suppose $\{v, v_1, v_2\} \subseteq V$ and $\{a, a_1\} \subseteq A$. Then $\text{ND-ex-4}(v, v_1, v_2, a, a_1) \in \text{ND}_{\text{SS}}(V, A \cup \text{ND}_{\text{SS}}(V, A))$.

PROOF: Set $g = \text{ND-ex-4}(v, v_1, v_2, a, a_1)$. $\text{rng } g \subseteq A \cup \text{ND}_{\text{SS}}(V, A)$ by [20, (13)], [3, (31)], [20, (8)], [23, (4)]. \square

- (55) Suppose $\{v, v_1, v_2\} \subseteq V$ and $\{a, a_1\} \subseteq A$. Then $\text{ND-ex-4}(v, v_1, v_2, a, a_1)$ is a non-atomic nominative data of V and A . The theorem is a consequence of (54) and (28).

Let V, A be non empty sets, v, v_1, v_2 be elements of V , and a, a_1 be elements of A . One can check that the functor $\text{ND-ex-4}(v, v_1, v_2, a, a_1)$ yields a non-atomic nominative data of V and A . Now we state the propositions:

- (56) Suppose $\{v, v_1, v_2\} \subseteq V$ and $\{a, a_1\} \subseteq A$. Then $\text{ND-ex-4}(v, v_1, v_2, a, a_1)$ is a nominative data with simple names from V and complex values from A . The theorem is a consequence of (55).

- (57) $\langle x \rangle$ is a non-atomic nominative data of $\{1\}$ and $\{x\}$.

PROOF: $\langle x \rangle \in \text{ND}_{\text{SS}}(\{1\}, \{x\})$ by [23, (3)]. \square

4. OPERATIONS ON SIMPLE-NAMED COMPLEX-VALUED NOMINATIVE DATA

Let us consider V, A, v , and D . Assume $v \in \text{dom } D$. The functor $v \Rightarrow_a D$ yielding a nominative data with simple names from V and complex values from A is defined by the term

(Def. 12) $D(v)$.

Let v, D be objects. Assume D is a nominative data with simple names from V and complex values from A . Assume $v \in V$. The functor $\Rightarrow v(D)$ yielding a non-atomic nominative data of V and A is defined by the term

(Def. 13) $v \dot{\mapsto} D$.

Let a be an object and f be a V -valued finite sequence. Assume $\text{len } f > 0$. The functor $\text{namingSeq}(V, A, f, a)$ yielding a finite sequence is defined by

(Def. 14) $\text{len } it = \text{len } f$ and $it(1) \Rightarrow (f(\text{len } f))(a)$ and for every natural number n such that $1 \leq n < \text{len } it$ holds $it(n+1) \Rightarrow (f(\text{len } f - n))(it(n))$.

Now we state the proposition:

(58) Let us consider a V -valued finite sequence f . Suppose $1 \leq n \leq \text{len } f$. Then $(\text{namingSeq}(V, A, f, a))(n)$ is a non-atomic nominative data of V and A .

Let us consider V and A . Let f be a V -valued finite sequence and a be an object. The functor $\Rightarrow f(a)$ yielding a set is defined by the term

(Def. 15) $(\text{namingSeq}(V, A, f, a))(\text{len } \text{namingSeq}(V, A, f, a))$.

Now we state the propositions:

(59) Let us consider a V -valued finite sequence f . Suppose $\text{len } f > 0$. Then $\Rightarrow f(a)$ is a non-atomic nominative data of V and A . The theorem is a consequence of (58).

(60) Let us consider a non empty set V , and an element v of V . Then $\Rightarrow \langle v \rangle(a) \Rightarrow v(a)$.

(61) Let us consider a non empty set V , and elements v_1, v_2 of V . Suppose a is a nominative data with simple names from V and complex values from A . Then $\Rightarrow \langle v_1, v_2 \rangle(a) = v_1 \mapsto (v_2 \mapsto a)$. The theorem is a consequence of (58).

(62) Let us consider a nominative data D with simple names from V and complex values from A . If $v \in V$, then $v \Rightarrow_a D = v(D) = D$.

(63) If $v \in \text{dom } D$, then $\Rightarrow v(v \Rightarrow_a D) = v \mapsto D(v)$. The theorem is a consequence of (33).

Let us consider V and A . Let d_1, d_2 be objects. Assume d_1 is a nominative data with simple names from V and complex values from A and d_2 is a nominative data with simple names from V and complex values from A . The functor $d_1 \nabla_a d_2$ yielding a nominative data with simple names from V and complex values from A is defined by

(Def. 16) (i) there exist functions f_1, f_2 such that $f_1 = d_1$ and $f_2 = d_2$ and $it = f_2 \cup f_1 \upharpoonright (\text{dom } f_1 \setminus \text{dom } f_2)$, **if** $d_1 \notin A$ and $d_2 \notin A$,

(ii) $it = d_2$, **otherwise**.

Let d_1, d_2, v be objects. The functor $d_1 \nabla_a^v d_2$ yielding a nominative data with simple names from V and complex values from A is defined by the term

(Def. 17) $d_1 \nabla_a (\Rightarrow v(d_2))$.

Now we state the propositions:

- (64) If $D_1 \notin A$ and $D_2 \notin A$, then $D_1 \nabla_a D_2 = D_2 \cup D_1 \upharpoonright (\text{dom } D_1 \setminus \text{dom } D_2)$.
- (65) If $D_1 \notin A$ and $D_2 \notin A$ and $\text{dom } D_1 \subseteq \text{dom } D_2$, then $D_1 \nabla_a D_2 = D_2$. The theorem is a consequence of (64).
- (66) If $D \notin A$, then $D \nabla_a D = D$. The theorem is a consequence of (65).
- (67) Suppose $v \in V$ and $v \mapsto a_1 \notin A$ and $v \mapsto a_2 \notin A$ and a_1 is a nominative data with simple names from V and complex values from A and a_2 is a nominative data with simple names from V and complex values from A . Then $(v \mapsto a_1) \nabla_a (v \mapsto a_2) = v \mapsto a_2$. The theorem is a consequence of (65).
- (68) Suppose $\{v_1, v_2\} \subseteq V$ and $v_1 \neq v_2$ and $v_1 \mapsto a_1 \notin A$ and $v_2 \mapsto a_2 \notin A$ and a_1 is a nominative data with simple names from V and complex values from A and a_2 is a nominative data with simple names from V and complex values from A . Then $(v_1 \mapsto a_1) \nabla_a (v_2 \mapsto a_2) = [v_2 \mapsto a_2, v_1 \mapsto a_1]$. The theorem is a consequence of (64).
- (69) Suppose $\{v, v_1, v_2\} \subseteq V$ and $v \neq v_1$ and $a_2 \in A$ and $v_1 \mapsto a_1 \notin A$ and $v \mapsto (v_2 \mapsto a_2) \notin A$ and a_1 is a nominative data with simple names from V and complex values from A . Then $(v_1 \mapsto a_1) \nabla_a^v (v_2 \mapsto a_2) = [v \mapsto v_2 \mapsto a_2, v_1 \mapsto a_1]$. The theorem is a consequence of (47) and (68).

Let us consider V , A , and v . The functor $v \Rightarrow_a$ yielding a partial function from $\text{ND}_{\text{SC}}(V, A)$ to $\text{ND}_{\text{SC}}(V, A)$ is defined by

- (Def. 18) $\text{dom } it = \{d, \text{ where } d \text{ is a non-atomic nominative data of } V \text{ and } A : v \in \text{dom } d\}$ and for every non-atomic nominative data D of V and A such that $D \in \text{dom } it$ holds $it(D) = v \Rightarrow_a D$.

The functor $\Rightarrow v$ yielding a function from $\text{ND}_{\text{SC}}(V, A)$ into $\text{ND}_{\text{SC}}(V, A)$ is defined by

- (Def. 19) for every nominative data D with simple names from V and complex values from A , $it(D) = \Rightarrow v(D)$.

The functor ∇_a^v yielding a partial function from $\text{ND}_{\text{SC}}(V, A) \times \text{ND}_{\text{SC}}(V, A)$ to $\text{ND}_{\text{SC}}(V, A)$ is defined by

- (Def. 20) $\text{dom } it = (\text{ND}_{\text{SC}}(V, A) \setminus A) \times \text{ND}_{\text{SC}}(V, A)$ and for every non-atomic nominative data d_1 of V and A and for every object d_2 such that $d_1 \notin A$ and $d_2 \in \text{ND}_{\text{SC}}(V, A)$ holds $it(\langle d_1, d_2 \rangle) = d_1 \nabla_a^v d_2$.

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