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An Inference System of an Extension of Floyd-Hoare Logic for Partial Predicates

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From now on v, x denote objects, D, V, A denote sets, n denotes a natural number, p, q denote partial predicates of D, and f, g denote BinominativeFunctions of D.

Let us consider D, f, and p. We say that f coincides-with p if and only if

(Def. 1) for every element d of D such that $d \in \mathrm{dom}\, p$ holds $f(d) \in \mathrm{dom}\, p$.

Let us consider g and q. We say that f, g coincide-with p, q if and only if

(Def. 2) for every element d of D such that $d \in \mathrm{rng}\, f$ and $g(d) \in \mathrm{dom}\, q$ holds $d \in \mathrm{dom}\, p$.

Now we state the propositions:

(1) f coincides-with $\bot_{\mathrm{PP}}(D)$.

(2) $\mathrm{PPid}(D)$ coincides-with p.

Let us consider D, p, and q. We say that $p \,||\!= q$ if and only if

(Def. 3) for every element d of D such that $d \in \mathrm{dom}\, p$ and $p(d) = \mathit{true}$ holds $d \in \mathrm{dom}\, q$ and $q(d) = \mathit{true}$.

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Observe that the predicate is reflexive.

In the sequel D denotes a non empty set, d denotes an element of D, f, g denote BinominativeFunctions of D, and p, q, r, s denote partial predicates of D.

Now we state the propositions:

(3) If $p \mid\mid= r$, then $p \wedge q \mid\mid= r$.

(4) $p \wedge q \mid\mid= p$.

(5) If $p \mid\mid= q$ and $r \mid\mid= s$, then $p \wedge r \mid\mid= q \wedge s$.

(6) If $p \vee q \mid\mid= r$, then $p \mid\mid= r$.

(7) Suppose $p \mid\mid= q \vee r$. If $d \in \operatorname{dom} p$ and $p(d) = \mathit{true}$, then $d \in \operatorname{dom} q$ and $q(d) = \mathit{true}$ or $d \in \operatorname{dom} r$ and $r(d) = \mathit{true}$.

(8) $p \vee p \mid\mid= p$.

(9) If $p \mid\mid= q$ and $r \mid\mid= s$, then $p \vee r \mid\mid= q \vee s$.

(10) If $p \vee q \mid\mid= r$, then $p \wedge q \mid\mid= r$.

Let us consider D. The functor SemanticFloydHoareTriples(D) yielding a set is defined by the term

(Def. 4) $\{\langle p, f, q\rangle$, where p, q are partial predicates of D, f is a BinominativeFunction of D : for every element d of D such that $d \in \operatorname{dom} p$ and $p(d) = \mathit{true}$ and $d \in \operatorname{dom} f$ and $f(d) \in \operatorname{dom} q$ holds $q(f(d)) = \mathit{true}\}$.

We introduce the notation SFHTs(D) as a synonym of SemanticFloydHoareTriples(D).

Now we state the propositions:

(11) Suppose $\langle p, f, q\rangle \in$ SFHTs(D). If $d \in \operatorname{dom} p$ and $p(d) = \mathit{true}$ and $d \in \operatorname{dom} f$ and $f(d) \in \operatorname{dom} q$, then $q(f(d)) = \mathit{true}$.

(12) $\langle \emptyset, f, p\rangle \in$ SFHTs(D).

Let us consider D. Observe that SFHTs(D) is non empty.

A SemanticFloydHoareTriple of D is an element of SemanticFloydHoareTriples(D).

A SFHT of D is an element of SFHTs(D). Now we state the propositions:

(13) $\langle p, \mathrm{id}_{\operatorname{dom} p}, p\rangle$ is a SFHT of D.

(14) $\langle p, \mathrm{id}_{\operatorname{field} f}, p\rangle$ is a SFHT of D.

Now we state the proposition:

(15) CONS$_1$ RULE:
If $\langle p, f, q\rangle$ is a SFHT of D and $r \mid\mid= p$, then $\langle r, f, q\rangle$ is a SFHT of D. The theorem is a consequence of (11).

Now we state the proposition:

(16) CONS$_2$ RULE:
Suppose $\langle p, f, q\rangle$ is a SFHT of D and $q \mid\mid= r$ and $\operatorname{dom} r \subseteq \operatorname{dom} q$. Then $\langle p, f, r\rangle$ is a SFHT of D. The theorem is a consequence of (11).

Now we state the propositions:

(17) SKIP RULE:
$\langle p, \mathrm{PPid}(D), p\rangle$ is a SFHT of D.

(18) $\langle \mathrm{false}_{\mathrm{PP}}(D), f, p\rangle$ is a SFHT of D.

Now we state the proposition:

(19) INVERSION RULE:
If p is total, then $\langle \text{PP-inversion}(p), f, q\rangle$ is a SFHT of D. The theorem is a consequence of (18) and (15).

Now we state the proposition:

(20) COMPOSITION RULE:
Suppose $\langle p, f, q\rangle$ is a SFHT of D and $\langle q, g, r\rangle$ is a SFHT of D and f, gcoincide-with q, r. Then $\langle p, \text{PP-composition}(f, g), r\rangle$ is a SFHT of D.

PROOF: Set $F = \text{PP-composition}(f, g)$. For every d such that $d \in \mathrm{dom}\, p$ and $p(d) = \mathit{true}$ and $d \in \mathrm{dom}\, F$ and $F(d) \in \mathrm{dom}\, r$ holds $r(F(d)) = \mathit{true}$ by [3, (12)], [7, (25)], [4, (4)], [3, (11)]. □

Now we state the propositions:

(21) IF RULE:
Suppose $\langle r \wedge p, f, q\rangle$ is a SFHT of D and $\langle \neg r \wedge p, g, q\rangle$ is a SFHT of D. Then $\langle p, \text{PP-IF}(r, f, g), q\rangle$ is a SFHT of D.

PROOF: Set $F = \text{PP-IF}(r, f, g)$. For every d such that $d \in \mathrm{dom}\, p$ and $p(d) = \mathit{true}$ and $d \in \mathrm{dom}\, F$ and $F(d) \in \mathrm{dom}\, q$ holds $q(F(d)) = \mathit{true}$ by [6, (16), (18)], (11). □

(22) Suppose f coincides-with p and $\langle p, f, p\rangle$ is a SFHT of D. Then $\langle p, f^n, p\rangle$ is a SFHT of D.

PROOF: Define $\mathcal{P}[\text{natural number}] \equiv \langle p, f^{\$_1}, p\rangle$ is a SFHT of D. $\mathcal{P}[0]$. For every natural number k such that $\mathcal{P}[k]$ holds $\mathcal{P}[k+1]$ by [2, (86), (69)], [3, (12), (11)]. For every natural number k, $\mathcal{P}[k]$ from [1, Sch. 2]. □

Now we state the proposition:

(23) WHILE RULE:
Suppose f coincides-with p and $\mathrm{dom}\, p \subseteq \mathrm{dom}\, f$ and $\langle r \wedge p, f, p\rangle$ is a SFHT of D. Then $\langle p, \text{PP-while}(r, f), \neg r \wedge p\rangle$ is a SFHT of D.

PROOF: Set $F = \text{PP-while}(r, f)$. Set $q = \neg r \wedge p$. For every d such that $d \in \mathrm{dom}\, p$ and $p(d) = \mathit{true}$ and $d \in \mathrm{dom}\, F$ and $F(d) \in \mathrm{dom}\, q$ holds $q(F(d)) = \mathit{true}$ by [2, (86)], [3, (11), (12)], [6, (17), (18)]. □

Now we state the proposition:

(24) UNCONDITIONAL COMPOSITION RULE (USEQ):
Suppose $\langle p, f, q\rangle$ is a SFHT of D and $\langle q, g, r\rangle$ is a SFHT of D and $\langle \text{PP-inversion}(q),$

$g, s\rangle$ is a SFHT of D. Then $\langle p, \text{PP-composition}(f, g), r \vee s\rangle$ is a SFHT of D.

PROOF: Set $F = \text{PP-composition}(f, g)$. For every d such that $d \in \operatorname{dom} p$ and $p(d) = \mathit{true}$ and $d \in \operatorname{dom} F$ and $F(d) \in \operatorname{dom}(r \vee s)$ holds $(r \vee s)(F(d)) = \mathit{true}$ by [6, (3), (13)], [3, (12), (11)]. □

Now we state the proposition:

(25) UNCONDITIONAL WHILE RULE (UWH):
Suppose $\langle r \wedge p, f, p\rangle$ is a SFHT of D and $\langle r \wedge \text{PP-inversion}(p), f, p\rangle$ is a SFHT of D. Then $\langle p, \text{PP-while}(r, f), \neg r \wedge p\rangle$ is a SFHT of D.

PROOF: Set $F = \text{PP-while}(r, f)$. Set $q = \neg r \wedge p$. For every d such that $d \in \operatorname{dom} p$ and $p(d) = \mathit{true}$ and $d \in \operatorname{dom} F$ and $F(d) \in \operatorname{dom} q$ holds $q(F(d)) = \mathit{true}$ by [2, (86)], [3, (11), (12)], [6, (17), (18)]. □

Now we state the proposition:

(26) DP RULE:
Suppose $\langle p, f, r\rangle$ is a SFHT of D and $\langle q, f, r\rangle$ is a SFHT of D. Then $\langle p \vee q, f, r\rangle$ is a SFHT of D.

PROOF: Set $P = p \vee q$. For every d such that $d \in \operatorname{dom} P$ and $P(d) = \mathit{true}$ and $d \in \operatorname{dom} f$ and $f(d) \in \operatorname{dom} r$ holds $r(f(d)) = \mathit{true}$ by [6, (10)], (11). □

In the sequel p, q denote SCPartialNominativePredicates of V, A, f, g denote SCBinominativeFunctions of V, A, E denotes a (V,A)-FPrg-yielding finite sequence, e denotes an element of $\prod E$, and d denotes a nominative data with simple names from V and complex values from A.

Now we state the proposition:

(27) Suppose for every nominative data d with simple names from V and complex values from A such that $d \in \operatorname{dom} p$ and $p(d) = \mathit{true}$ and $d \in \operatorname{dom} f$ and $f(d) \in \operatorname{dom} q$ holds $q(f(d)) = \mathit{true}$. Then $\langle p, f, q\rangle$ is a SFHT of $\text{ND}_{\text{SC}}(V, A)$.

PROOF: For every element d of $\text{ND}_{\text{SC}}(V, A)$ such that $d \in \operatorname{dom} p$ and $p(d) = \mathit{true}$ and $d \in \operatorname{dom} f$ and $f(d) \in \operatorname{dom} q$ holds $q(f(d)) = \mathit{true}$ by [5, (39)]. □

Now we state the proposition:

(28) ASSIGNMENT RULE:
$\langle \text{SC-Psuperpos}(p, f, v), \text{SC-assignment}(f, v), p\rangle$ is a SFHT of $\text{ND}_{\text{SC}}(V, A)$.

PROOF: Set $P = \text{SC-Psuperpos}(p, f, v)$. Set $F = \text{SC-assignment}(f, v)$. For every d such that $d \in \operatorname{dom} P$ and $P(d) = \mathit{true}$ and $d \in \operatorname{dom} F$ and $F(d) \in \operatorname{dom} p$ holds $p(F(d)) = \mathit{true}$ by [**?** , (34)]. □

Now we state the proposition:

(29) SFID$_1$ rule:
$\langle$SC-Psuperpos(p, f, v), SC-Fsuperpos(PPid(ND$_{SC}(V, A)$)$, f, v), p\rangle$ is a SFHT of ND$_{SC}(V, A)$.
Proof: Set I = PPid(ND$_{SC}(V, A)$). Set P = SC-Psuperpos(p, f, v). Set F = SC-Fsuperpos(I, f, v). For every d such that $d \in \mathrm{dom}\, P$ and $P(d) = true$ and $d \in \mathrm{dom}\, F$ and $F(d) \in \mathrm{dom}\, p$ holds $p(F(d)) = true$ by [3, (18)], [? , (37), (34)]. □

Now we state the proposition:

(30) SFID rule:
Suppose $\prod E \neq \emptyset$. Then $\langle$SC-Psuperpos(p, e, E), SC-Fsuperpos(PPid(ND$_{SC}(V, A)$)$, e,$ $p\rangle$ is a SFHT of ND$_{SC}(V, A)$.
Proof: Set I = PPid(ND$_{SC}(V, A)$). Set P = SC-Psuperpos(p, e, E). Set F = SC-Fsuperpos(I, e, E). For every d such that $d \in \mathrm{dom}\, P$ and $P(d) = true$ and $d \in \mathrm{dom}\, F$ and $F(d) \in \mathrm{dom}\, p$ holds $p(F(d)) = true$ by [3, (18)], [? , (36), (33)]. □

Now we state the proposition:

(31) SF$_1$ rule:
Suppose $\langle p$, PP-composition(SC-Fsuperpos(PPid(ND$_{SC}(V, A)$)$, g, v), f), q\rangle$ is a SFHT of ND$_{SC}(V, A)$. Then $\langle p$, SC-Fsuperpos$(f, g, v), q\rangle$ is a SFHT of ND$_{SC}(V, A)$.
Proof: Set I = PPid(ND$_{SC}(V, A)$). Set F = SC-Fsuperpos(f, g, v). Set G = SC-Fsuperpos(I, g, v). Set C = PP-composition(G, f). For every d such that $d \in \mathrm{dom}\, p$ and $p(d) = true$ and $d \in \mathrm{dom}\, C$ and $C(d) \in \mathrm{dom}\, q$ holds $q(C(d)) = true$. For every d such that $d \in \mathrm{dom}\, p$ and $p(d) = true$ and $d \in \mathrm{dom}\, F$ and $F(d) \in \mathrm{dom}\, q$ holds $q(F(d)) = true$ by [? , (37)], [3, (18), (13), (11)]. □

Now we state the proposition:

(32) SF rule:
Suppose $\prod E \neq \emptyset$ and $\langle p$, PP-composition(SC-Fsuperpos(PPid(ND$_{SC}(V, A)$)$, e, E), f$ $q\rangle$ is a SFHT of ND$_{SC}(V, A)$. Then $\langle p$, SC-Fsuperpos$(f, e, E), q\rangle$ is a SFHT of ND$_{SC}(V, A)$.
Proof: Set I = PPid(ND$_{SC}(V, A)$). Set F = SC-Fsuperpos(f, e, E). Set G = SC-Fsuperpos(I, e, E). Set C = PP-composition(G, f). For every d such that $d \in \mathrm{dom}\, p$ and $p(d) = true$ and $d \in \mathrm{dom}\, C$ and $C(d) \in \mathrm{dom}\, q$ holds $q(C(d)) = true$. For every d such that $d \in \mathrm{dom}\, p$ and $p(d) = true$ and $d \in \mathrm{dom}\, F$ and $F(d) \in \mathrm{dom}\, q$ holds $q(F(d)) = true$ by [? , (36)], [3, (18), (13), (11)]. □

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