

Mizar

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Mizar system has been designed and implemented in the Section of Application of Mathematics at the Plock Scientific Society. Its author is the author of this paper and the system was implemented by Piotr Rudnicki (ICS PAS), Czesław Byliński, Roman Matuszewski, Stanisław Żukowski (WU).

Plock Scientific Society is the oldest scientific society in Poland. It is a regional society but, as one can see, the works of the society are not limited to history, culture and geography of Mazovia.

People try to construct computer programs which could prove mathematical theorems. Systems already working cannot replace (up to the present?) mathematicians. Our group posed somewhat easier question: to work out a system that would find errors in proofs written by a man.

To check whether the system does check proofs we have rewritten a part of Professor Wanda Szmielew's paper "Oriented and Non-Oriented Linear Orders". In particular we had to prove that a linear relation contained in an antisymmetric one must be equal to the given one. Here is the printout:

for every... such that... holds...
predicate... denotes...

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ENVIRON GIVEN U BEING NONEMPTYSET;
- TYPE RELATION DENOTES SUBSET OF [U,U];
- LET X,Y DENOTE ELEMENT OF U;
- BEGIN
- FOR R,S BEING RELATION PRED R <= S DENOTES
  FOR X,Y HOLDS [X,Y] IN R IMPLIES [X,Y] IN S;
DEFINITION LET R BE RELATION;
  PRED R IS CONNECTED DENOTES
    FOR X,Y HOLDS [X,Y] IN R OR [Y,X] IN R;
  PRED R IS ANTISYMMETRIC DENOTES
    FOR X,Y ST [X,Y] IN R & [Y,X] IN R HOLDS X=Y
END;
FOR R,S BEING RELATION
  ST R IS CONNECTED & R <= S & S IS ANTISYMMETRIC
  HOLDS S <= R
PROOF LET R,S BE RELATION SUCH THAT
  Z1: FOR X,Y HOLDS [X,Y] IN R OR [Y,X] IN R AND
  Z2: FOR X,Y ST [X,Y] IN R HOLDS [X,Y] IN S AND
  Z3: FOR X,Y ST [X,Y] IN S & [Y,X] IN S HOLDS X=Y;
LET X,Y BE ELEMENT OF U SUCH THAT A: [X,Y] IN S;
NOW ASSUME B: [Y,X] IN R; THEN [Y,X] IN S BY Z2;
THEN X=Y BY A,Z3; HENCE [X,Y] IN R BY B END;
HENCE [X,Y] IN R BY Z1 END END
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THANKS,O.K

No error occurs — computer thanks for the collaboration. But if there were any...

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ENVIRON TYPE POINT; TYPE RELATION;
- BEGIN
- FOR R BEING RELATION ST R IS CONNECTED & ;
**** *59 *37,31
  FOR R,R1 BEING RELATION HOLDS R<>R1;
**** *82
  END
SORRY
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Computer is much more vigorous now (*59 — a nondeclared predicate, *37 — the sentence is not closed, *31 — the sentence is syntactically incorrect, *82 — it is not obvious).

To give a computer a text for checking, a suitable language is required. This, used in the examples above, is called Mizar. Just like the system which uses it.

The Mizar system only verifies proofs. But even this may be useful. For students (not only those of mathematics) who want to verify or to improve their competence on constructing proofs. For programmers, in long and involved proofs that their programs really work as they are expected to. And a mathematician might also be glad to ascertain that the small gaps he often makes in proofs do not contain any important mistake.

It is not possible to describe the whole language in a short note like this. The participants of the Congress are kindly invited to visit the consulting centre of Plock Scientific Society. At the same time we promise the readers of Delta to give a short course of Mizar, starting just from this September. Everyone who sends to us a text in this language will get an answer. Not from us, from the computer. It only depends on him how kind this answer will be.

