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Built-in Concepts

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Summary. This abstract contains the second part of the axiomatics of the Mizar system (the first part is in abstract [1]). The axioms listed here characterize the Mizar built-in concepts that are automatically attached to every Mizar article. We give definitional axioms of the following concepts: element, subset, Cartesian product, domain (non empty subset), subdomain (non empty subset of a domain), set domain (domain consisting of sets). Axioms of strong arithmetics of real numbers are also included.

The notation and terminology used here have been introduced in the axiomatics [1]. For simplicity we adopt the following convention: x, y, z denote objects of the type Any; X, $X1$, $X2$, $X3$, $X4$, Y denote objects of the type set. The following axioms hold:

(1) $(\mathbf{ex}\, x\ \mathbf{st}\ x \in X)\ \mathbf{implies}\ (x\ \mathbf{is}\ \text{Element}\ \mathbf{of}\ X\ \mathbf{iff}\ x \in X)$,

(2) $X\ \mathbf{is}\ \text{Subset}\ \mathbf{of}\ Y\ \mathbf{iff}\ X \subseteq Y$,

(3) $z \in [:X, Y:]\ \mathbf{iff}\ \mathbf{ex}\, x,y\ \mathbf{st}\ x \in X\ \&\ y \in Y\ \&\ z = \langle x, y \rangle$,

(4) $X\ \mathbf{is}\ \text{DOMAIN}\ \mathbf{iff}\ \mathbf{ex}\, x\ \mathbf{st}\ x \in X$,

(5) $[:X1,X2,X3:] = [:[:X1,X2:],X3:]$,

(6) $[:X1,X2,X3,X4:] = [:[:X1,X2,X3:],X4:]$.

In the sequel $D1$, $D2$, $D3$, $D4$ will denote objects of the type DOMAIN. Let us introduce the consecutive axioms:

(7) $\mathbf{for}\ X\ \mathbf{being}\ \text{Element}\ \mathbf{of}\ [:D1,D2:]\ \mathbf{holds}\ X\ \mathbf{is}\ \text{TUPLE}\ \mathbf{of}\ D1,D2$,

(8) $\mathbf{for}\ X\ \mathbf{being}\ \text{Element}\ \mathbf{of}\ [:D1,D2,D3:]\ \mathbf{holds}\ X\ \mathbf{is}\ \text{TUPLE}\ \mathbf{of}\ D1,D2,D3$,

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$$\textbf{for } X \textbf{ being } \text{Element } \textbf{of } [:D1,D2,D3,D4:] \\ \textbf{holds } X \textbf{ is } \text{TUPLE } \textbf{of } D1,D2,D3,D4. \tag{9}$$

In the sequel D has the type DOMAIN. The following axioms hold:

$$D1 \textbf{ is } \text{SUBDOMAIN } \textbf{of } D2 \textbf{ iff } D1 \subseteq D2, \tag{10}$$

$$D \textbf{ is } \text{SET\_DOMAIN}. \tag{11}$$

In the sequel x, y, z denote objects of the type Element **of** REAL. The following axioms are true:

$$x + y = y + x, \tag{12}$$

$$x + (y + z) = (x + y) + z, \tag{13}$$

$$x + 0 = x, \tag{14}$$

$$x \cdot y = y \cdot x, \tag{15}$$

$$x \cdot (y \cdot z) = (x \cdot y) \cdot z, \tag{16}$$

$$x \cdot 1 = x, \tag{17}$$

$$x \cdot (y + z) = x \cdot y + x \cdot z, \tag{18}$$

$$\textbf{ex } y \textbf{ st } x + y = 0, \tag{19}$$

$$x \neq 0 \textbf{ implies ex } y \textbf{ st } x \cdot y = 1, \tag{20}$$

$$x \leq y \ \&\ y \leq x \textbf{ implies } x = y, \tag{21}$$

$$x \leq y \ \&\ y \leq z \textbf{ implies } x \leq z, \tag{22}$$

$$x \leq y \textbf{ or } y \leq x, \tag{23}$$

$$x \leq y \textbf{ implies } x + z \leq y + z, \tag{24}$$

$$x \leq y \ \&\ 0 \leq z \textbf{ implies } x \cdot z \leq y \cdot z, \tag{25}$$

$$\textbf{for } X,Y \textbf{ being } \text{Subset } \textbf{of } \text{REAL } \textbf{st} \\ (\textbf{ex } x \textbf{ st } x \in X)\ \&\ (\textbf{ex } x \textbf{ st } x \in Y)\ \&\ \textbf{for } x,y \textbf{ st } x \in X\ \&\ y \in Y \textbf{ holds } x \leq y \\ \textbf{ex } z \textbf{ st for } x,y \textbf{ st } x \in X\ \&\ y \in Y \textbf{ holds } x \leq z\ \&\ z \leq y, \tag{26}$$

$$x \textbf{ is } \text{Real}, \tag{27}$$

$$x \in \text{NAT} \textbf{ implies } x + 1 \in \text{NAT}, \tag{28}$$

$$\textbf{for } A \textbf{ being set of } \text{Real} \\ \textbf{st } 0 \in A\ \&\ \textbf{for } x \textbf{ st } x \in A \textbf{ holds } x + 1 \in A \textbf{ holds } \text{NAT} \subseteq A, \tag{29}$$

$$x \in \text{NAT} \textbf{ implies } x \textbf{ is } \text{Nat}. \tag{30}$$

References

[1] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1, 1990.

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