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Projective Spaces - part IV

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Summary. A continuation of [4]. In the classes of projective spaces, defined in terms of collinearity, introduced in the article [3], we distinguish the subclasses of Desarguesian projective structures. As examples of these objects we consider analytical projective spaces defined over suitable real linear spaces.

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The notation and terminology used here have been introduced in the following papers: [1], [5], [2], [3], and [4]. We adopt the following convention: a, b, c, d denote real numbers, V denotes a non-trivial real linear space, and u, v, w, y, u_1 denote vectors of V. An at least 3 dimensional projective space defined in terms of collinearity is said to be a Desarguesian at least 3 dimensional projective space defined in terms of collinearity if:

(Def.1) Let o, p_1, p_2, p_3, q_1, q_2, q_3, r_1, r_2, r_3 be elements of the points of it . Suppose that

- (i) $o \neq q_1$,
- (ii) $p_1 \neq q_1$,
- (iii) $o \neq q_2$,
- (iv) $p_2 \neq q_2$,
- (v) $o \neq q_3$,
- (vi) $p_3 \neq q_3$,
- (vii) o, p_1 and p_2 are not collinear,
- (viii) o, p_1 and p_3 are not collinear,
- (ix) o, p_2 and p_3 are not collinear,
- (x) p_1, p_2 and r_3 are collinear,
- (xi) q_1, q_2 and r_3 are collinear,
- (xii) p_2, p_3 and r_1 are collinear,

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(xiii) q_2, q_3 and r_1 are collinear,
(xiv) p_1, p_3 and r_2 are collinear,
(xv) q_1, q_3 and r_2 are collinear,
(xvi) o, p_1 and q_1 are collinear,
(xvii) o, p_2 and q_2 are collinear,
(xviii) o, p_3 and q_3 are collinear.

Then r_1, r_2 and r_3 are collinear.

The following propositions are true:

(1) Let C_1 be an at least 3 dimensional projective space defined in terms of collinearity. Then C_1 is a Desarguesian at least 3 dimensional projective space defined in terms of collinearity if and only if for all elements o, p_1, p_2, p_3, q_1, q_2, q_3, r_1, r_2, r_3 of the points of C_1 such that $o \neq q_1$ and $p_1 \neq q_1$ and $o \neq q_2$ and $p_2 \neq q_2$ and $o \neq q_3$ and $p_3 \neq q_3$ and o, p_1 and p_2 are not collinear and o, p_1 and p_3 are not collinear and o, p_2 and p_3 are not collinear and p_1, p_2 and r_3 are collinear and q_1, q_2 and r_3 are collinear and p_2, p_3 and r_1 are collinear and q_2, q_3 and r_1 are collinear and p_1, p_3 and r_2 are collinear and q_1, q_3 and r_2 are collinear and o, p_1 and q_1 are collinear and o, p_2 and q_2 are collinear and o, p_3 and q_3 are collinear holds r_1, r_2 and r_3 are collinear.

(2) If there exist u, v, w, u_1 such that for all a, b, c, d such that $((a \cdot u + b \cdot v) + c \cdot w) + d \cdot u_1 = 0_V$ holds $a = 0$ and $b = 0$ and $c = 0$ and $d = 0$, then the projective space over V is a Desarguesian at least 3 dimensional projective space defined in terms of collinearity.

(3) Let C_1 be a collinearity structure. Then C_1 is a Desarguesian at least 3 dimensional projective space defined in terms of collinearity if and only if the following conditions are satisfied:

(i) for all elements p, q, r of the points of C_1 holds p, q and p are collinear and p, p and q are collinear and p, q and q are collinear,

(ii) for all elements p, q, r, r_1, r_2 of the points of C_1 such that $p \neq q$ and p, q and r are collinear and p, q and r_1 are collinear and p, q and r_2 are collinear holds r, r_1 and r_2 are collinear,

(iii) for every elements p, q of the points of C_1 there exists an element r of the points of C_1 such that $p \neq r$ and $q \neq r$ and p, q and r are collinear,

(iv) for all elements p, p_1, p_2, r, r_1 of the points of C_1 such that p, p_1 and r are collinear and p_1, p_2 and r_1 are collinear there exists an element r_2 of the points of C_1 such that p, p_2 and r_2 are collinear and r, r_1 and r_2 are collinear,

(v) there exist elements p, p_1, q, q_1 of the points of C_1 such that for no element r of the points of C_1 holds p, p_1 and r are collinear and q, q_1 and r are collinear,

(vi) for all elements o, p_1, p_2, p_3, q_1, q_2, q_3, r_1, r_2, r_3 of the points of C_1 such that $o \neq q_1$ and $p_1 \neq q_1$ and $o \neq q_2$ and $p_2 \neq q_2$ and $o \neq q_3$ and $p_3 \neq q_3$ and o, p_1 and p_2 are not collinear and o, p_1 and p_3 are not collinear and o, p_2 and p_3 are not collinear and p_1, p_2 and r_3 are collinear and q_1, q_2

and r_3 are collinear and p_2, p_3 and r_1 are collinear and q_2, q_3 and r_1 are collinear and p_1, p_3 and r_2 are collinear and q_1, q_3 and r_2 are collinear and o, p_1 and q_1 are collinear and o, p_2 and q_2 are collinear and o, p_3 and q_3 are collinear holds r_1, r_2 and r_3 are collinear.

(4) For every C_1 being a collinearity structure holds C_1 is a Desarguesian at least 3 dimensional projective space defined in terms of collinearity if and only if C_1 is a Desarguesian projective space defined in terms of collinearity and there exist elements p, p_1, q, q_1 of the points of C_1 such that for no element r of the points of C_1 holds p, p_1 and r are collinear and q, q_1 and r are collinear.

A Fanoian at least 3 dimensional projective space defined in terms of collinearity is called a Fano-Desarguesian at least 3 dimensional projective space defined in terms of collinearity if:

(Def.2) Let o, p_1, p_2, p_3, q_1, q_2, q_3, r_1, r_2, r_3 be elements of the points of it . Suppose that

(i) $o \neq q_1$,
(ii) $p_1 \neq q_1$,
(iii) $o \neq q_2$,
(iv) $p_2 \neq q_2$,
(v) $o \neq q_3$,
(vi) $p_3 \neq q_3$,
(vii) o, p_1 and p_2 are not collinear,
(viii) o, p_1 and p_3 are not collinear,
(ix) o, p_2 and p_3 are not collinear,
(x) p_1, p_2 and r_3 are collinear,
(xi) q_1, q_2 and r_3 are collinear,
(xii) p_2, p_3 and r_1 are collinear,
(xiii) q_2, q_3 and r_1 are collinear,
(xiv) p_1, p_3 and r_2 are collinear,
(xv) q_1, q_3 and r_2 are collinear,
(xvi) o, p_1 and q_1 are collinear,
(xvii) o, p_2 and q_2 are collinear,
(xviii) o, p_3 and q_3 are collinear.

Then r_1, r_2 and r_3 are collinear.

We now state several propositions:

(5) Let C_1 be a Fanoian at least 3 dimensional projective space defined in terms of collinearity. Then C_1 is a Fano-Desarguesian at least 3 dimensional projective space defined in terms of collinearity if and only if for all elements o, p_1, p_2, p_3, q_1, q_2, q_3, r_1, r_2, r_3 of the points of C_1 such that $o \neq q_1$ and $p_1 \neq q_1$ and $o \neq q_2$ and $p_2 \neq q_2$ and $o \neq q_3$ and $p_3 \neq q_3$ and o, p_1 and p_2 are not collinear and o, p_1 and p_3 are not collinear and o, p_2 and p_3 are not collinear and p_1, p_2 and r_3 are collinear and q_1, q_2 and r_3 are collinear and p_2, p_3 and r_1 are collinear and q_2, q_3 and r_1 are collinear and p_1, p_3 and r_2 are collinear and q_1, q_3 and r_2 are collinear and o, p_1

and q_1 are collinear and o, p_2 and q_2 are collinear and o, p_3 and q_3 are collinear holds r_1, r_2 and r_3 are collinear.

(6) If there exist u, v, w, u_1 such that for all a, b, c, d such that $((a \cdot u + b \cdot v) + c \cdot w) + d \cdot u_1 = 0_V$ holds $a = 0$ and $b = 0$ and $c = 0$ and $d = 0$, then the projective space over V is a Fano-Desarguesian at least 3 dimensional projective space defined in terms of collinearity.

(7) Let C_1 be a collinearity structure. Then C_1 is a Fano-Desarguesian at least 3 dimensional projective space defined in terms of collinearity if and only if the following conditions are satisfied:

(i) for all elements p, q, r of the points of C_1 holds p, q and p are collinear and p, p and q are collinear and p, q and q are collinear,

(ii) for all elements p, q, r, r_1, r_2 of the points of C_1 such that $p \neq q$ and p, q and r are collinear and p, q and r_1 are collinear and p, q and r_2 are collinear holds r, r_1 and r_2 are collinear,

(iii) for every elements p, q of the points of C_1 there exists an element r of the points of C_1 such that $p \neq r$ and $q \neq r$ and p, q and r are collinear,

(iv) for all elements p, p_1, p_2, r, r_1 of the points of C_1 such that p, p_1 and r are collinear and p_1, p_2 and r_1 are collinear there exists an element r_2 of the points of C_1 such that p, p_2 and r_2 are collinear and r, r_1 and r_2 are collinear,

(v) for all elements p_1, r_2, q, r_1, q_1, p, r of the points of C_1 such that p_1, r_2 and q are collinear and r_1, q_1 and q are collinear and p_1, r_1 and p are collinear and r_2, q_1 and p are collinear and p_1, q_1 and r are collinear and r_2, r_1 and r are collinear and p, q and r are collinear holds p_1, r_2 and q_1 are collinear or p_1, r_2 and r_1 are collinear or p_1, r_1 and q_1 are collinear or r_2, r_1 and q_1 are collinear,

(vi) there exist elements p, p_1, q, q_1 of the points of C_1 such that for no element r of the points of C_1 holds p, p_1 and r are collinear and q, q_1 and r are collinear,

(vii) for all elements o, p_1, p_2, p_3, q_1, q_2, q_3, r_1, r_2, r_3 of the points of C_1 such that $o \neq q_1$ and $p_1 \neq q_1$ and $o \neq q_2$ and $p_2 \neq q_2$ and $o \neq q_3$ and $p_3 \neq q_3$ and o, p_1 and p_2 are not collinear and o, p_1 and p_3 are not collinear and o, p_2 and p_3 are not collinear and p_1, p_2 and r_3 are collinear and q_1, q_2 and r_3 are collinear and p_2, p_3 and r_1 are collinear and q_2, q_3 and r_1 are collinear and p_1, p_3 and r_2 are collinear and q_1, q_3 and r_2 are collinear and o, p_1 and q_1 are collinear and o, p_2 and q_2 are collinear and o, p_3 and q_3 are collinear holds r_1, r_2 and r_3 are collinear.

(8) Let C_1 be a collinearity structure. Then C_1 is a Fano-Desarguesian at least 3 dimensional projective space defined in terms of collinearity if and only if the following conditions are satisfied:

(i) C_1 is a Desarguesian at least 3 dimensional projective space defined in terms of collinearity,

(ii) for all elements p_1, r_2, q, r_1, q_1, p, r of the points of C_1 such that p_1, r_2 and q are collinear and r_1, q_1 and q are collinear and p_1, r_1 and p are collinear and r_2, q_1 and p are collinear and p_1, q_1 and r are collinear and r_2, r_1 and r are collinear and p, q and r are collinear holds p_1, r_2 and q_1 are collinear or p_1, r_2 and r_1 are collinear or p_1, r_1 and q_1 are collinear or r_2, r_1 and q_1 are collinear.

(9) For every C_1 being a collinearity structure holds
C_1
is a Fano-Desarguesian at least 3 dimensional projective space defined in terms of collinearity if and only if C_1 is a Fano-Desarguesian projective space defined in terms of collinearity and there exist elements p, p_1, q, q_1 of the points of C_1 such that for no element r of the points of C_1 holds p, p_1 and r are collinear and q, q_1 and r are collinear.

A 3 dimensional projective space defined in terms of collinearity is called a Desarguesian 3 dimensional projective space defined in terms of collinearity if:

(Def.3) Let o, p_1, p_2, p_3, q_1, q_2, q_3, r_1, r_2, r_3 be elements of the points of it . Suppose that

- (i) $o \neq q_1$,
- (ii) $p_1 \neq q_1$,
- (iii) $o \neq q_2$,
- (iv) $p_2 \neq q_2$,
- (v) $o \neq q_3$,
- (vi) $p_3 \neq q_3$,
- (vii) o, p_1 and p_2 are not collinear,
- (viii) o, p_1 and p_3 are not collinear,
- (ix) o, p_2 and p_3 are not collinear,
- (x) p_1, p_2 and r_3 are collinear,
- (xi) q_1, q_2 and r_3 are collinear,
- (xii) p_2, p_3 and r_1 are collinear,
- (xiii) q_2, q_3 and r_1 are collinear,
- (xiv) p_1, p_3 and r_2 are collinear,
- (xv) q_1, q_3 and r_2 are collinear,
- (xvi) o, p_1 and q_1 are collinear,
- (xvii) o, p_2 and q_2 are collinear,
- (xviii) o, p_3 and q_3 are collinear.

Then r_1, r_2 and r_3 are collinear.

We now state four propositions:

(10) Let C_1 be a 3 dimensional projective space defined in terms of collinearity. Then C_1 is a Desarguesian 3 dimensional projective space defined in terms of collinearity if and only if for all elements o, p_1, p_2, p_3, q_1, q_2, q_3, r_1, r_2, r_3 of the points of C_1 such that $o \neq q_1$ and $p_1 \neq q_1$ and $o \neq q_2$ and $p_2 \neq q_2$ and $o \neq q_3$ and $p_3 \neq q_3$ and o, p_1 and p_2 are not collinear and o, p_1 and p_3 are not collinear and o, p_2 and p_3 are not collinear and p_1, p_2 and r_3 are collinear and q_1, q_2 and r_3 are collinear and p_2, p_3 and r_1 are collinear and q_2, q_3 and r_1 are collinear and p_1, p_3 and r_2 are collinear and q_1, q_3 and r_2 are collinear and o, p_1 and q_1 are collinear and o, p_2

and q_2 are collinear and o, p_3 and q_3 are collinear holds r_1, r_2 and r_3 are collinear.

(11) Suppose that

(i) there exist u, v, w, u_1 such that for all a, b, c, d such that $((a \cdot u + b \cdot v) + c \cdot w) + d \cdot u_1 = 0_V$ holds $a = 0$ and $b = 0$ and $c = 0$ and $d = 0$ and for every y there exist a, b, c, d such that $y = ((a \cdot u + b \cdot v) + c \cdot w) + d \cdot u_1$. Then the projective space over V is a Desarguesian 3 dimensional projective space defined in terms of collinearity.

(12) Let C_1 be a collinearity structure. Then C_1 is a Desarguesian 3 dimensional projective space defined in terms of collinearity if and only if the following conditions are satisfied:

(i) for all elements p, q, r of the points of C_1 holds p, q and p are collinear and p, p and q are collinear and p, q and q are collinear,

(ii) for all elements p, q, r, r_1, r_2 of the points of C_1 such that $p \neq q$ and p, q and r are collinear and p, q and r_1 are collinear and p, q and r_2 are collinear holds r, r_1 and r_2 are collinear,

(iii) for every elements p, q of the points of C_1 there exists an element r of the points of C_1 such that $p \neq r$ and $q \neq r$ and p, q and r are collinear,

(iv) for all elements p, p_1, p_2, r, r_1 of the points of C_1 such that p, p_1 and r are collinear and p_1, p_2 and r_1 are collinear there exists an element r_2 of the points of C_1 such that p, p_2 and r_2 are collinear and r, r_1 and r_2 are collinear,

(v) there exist elements p, p_1, q, q_1 of the points of C_1 such that for no element r of the points of C_1 holds p, p_1 and r are collinear and q, q_1 and r are collinear,

(vi) for every elements p, p_1, q, q_1, r_2 of the points of C_1 there exist elements r, r_1 of the points of C_1 such that p, q and r are collinear and p_1, q_1 and r_1 are collinear and r_2, r and r_1 are collinear,

(vii) for all elements o, p_1, p_2, p_3, q_1, q_2, q_3, r_1, r_2, r_3 of the points of C_1 such that $o \neq q_1$ and $p_1 \neq q_1$ and $o \neq q_2$ and $p_2 \neq q_2$ and $o \neq q_3$ and $p_3 \neq q_3$ and o, p_1 and p_2 are not collinear and o, p_1 and p_3 are not collinear and o, p_2 and p_3 are not collinear and p_1, p_2 and r_3 are collinear and q_1, q_2 and r_3 are collinear and p_2, p_3 and r_1 are collinear and q_2, q_3 and r_1 are collinear and p_1, p_3 and r_2 are collinear and q_1, q_3 and r_2 are collinear and o, p_1 and q_1 are collinear and o, p_2 and q_2 are collinear and o, p_3 and q_3 are collinear holds r_1, r_2 and r_3 are collinear.

(13) For every C_1 being a collinearity structure holds C_1 is a Desarguesian 3 dimensional projective space defined in terms of collinearity if and only if C_1 is a Desarguesian at least 3 dimensional projective space defined in terms of collinearity and for every elements p, p_1, q, q_1, r_2 of the points of C_1 there exist elements r, r_1 of the points of C_1 such that p, q and r are collinear and p_1, q_1 and r_1 are collinear and r_2, r and r_1 are collinear.

A Fanoian 3 dimensional projective space defined in terms of collinearity is called a Fano-Desarguesian 3 dimensional projective space defined in terms of collinearity if:

(Def.4) Let o, p_1, p_2, p_3, q_1, q_2, q_3, r_1, r_2, r_3 be elements of the points of it . Suppose that

(i) $o \neq q_1$,
(ii) $p_1 \neq q_1$,
(iii) $o \neq q_2$,
(iv) $p_2 \neq q_2$,
(v) $o \neq q_3$,
(vi) $p_3 \neq q_3$,
(vii) o, p_1 and p_2 are not collinear,
(viii) o, p_1 and p_3 are not collinear,
(ix) o, p_2 and p_3 are not collinear,
(x) p_1, p_2 and r_3 are collinear,
(xi) q_1, q_2 and r_3 are collinear,
(xii) p_2, p_3 and r_1 are collinear,
(xiii) q_2, q_3 and r_1 are collinear,
(xiv) p_1, p_3 and r_2 are collinear,
(xv) q_1, q_3 and r_2 are collinear,
(xvi) o, p_1 and q_1 are collinear,
(xvii) o, p_2 and q_2 are collinear,
(xviii) o, p_3 and q_3 are collinear.

Then r_1, r_2 and r_3 are collinear.

We now state several propositions:

(14) Let C_1 be a Fanoian 3 dimensional projective space defined in terms of collinearity. Then C_1 is a Fano-Desarguesian 3 dimensional projective space defined in terms of collinearity if and only if for all elements o, p_1, p_2, p_3, q_1, q_2, q_3, r_1, r_2, r_3 of the points of C_1 such that $o \neq q_1$ and $p_1 \neq q_1$ and $o \neq q_2$ and $p_2 \neq q_2$ and $o \neq q_3$ and $p_3 \neq q_3$ and o, p_1 and p_2 are not collinear and o, p_1 and p_3 are not collinear and o, p_2 and p_3 are not collinear and p_1, p_2 and r_3 are collinear and q_1, q_2 and r_3 are collinear and p_2, p_3 and r_1 are collinear and q_2, q_3 and r_1 are collinear and p_1, p_3 and r_2 are collinear and q_1, q_3 and r_2 are collinear and o, p_1 and q_1 are collinear and o, p_2 and q_2 are collinear and o, p_3 and q_3 are collinear holds r_1, r_2 and r_3 are collinear.

(15) Suppose that

(i) there exist u, v, w, u_1 such that for all a, b, c, d such that $((a \cdot u + b \cdot v) + c \cdot w) + d \cdot u_1 = 0_V$ holds $a = 0$ and $b = 0$ and $c = 0$ and $d = 0$ and for every y there exist a, b, c, d such that $y = ((a \cdot u + b \cdot v) + c \cdot w) + d \cdot u_1$.

Then the projective space over V is a Fano-Desarguesian 3 dimensional projective space defined in terms of collinearity.

(16) Let C_1 be a collinearity structure. Then C_1 is a Fano-Desarguesian 3 dimensional projective space defined in terms of collinearity if and only if the following conditions are satisfied:

(i) for all elements p, q, r of the points of C_1 holds p, q and p are collinear and p, p and q are collinear and p, q and q are collinear,

(ii) for all elements p, q, r, r_1, r_2 of the points of C_1 such that $p \neq q$ and p, q and r are collinear and p, q and r_1 are collinear and p, q and r_2 are collinear holds r, r_1 and r_2 are collinear,

(iii) for every elements p, q of the points of C_1 there exists an element r of the points of C_1 such that $p \neq r$ and $q \neq r$ and p, q and r are collinear,

(iv) for all elements p, p_1, p_2, r, r_1 of the points of C_1 such that p, p_1 and r are collinear and p_1, p_2 and r_1 are collinear there exists an element r_2 of the points of C_1 such that p, p_2 and r_2 are collinear and r, r_1 and r_2 are collinear,

(v) for all elements p_1, r_2, q, r_1, q_1, p, r of the points of C_1 such that p_1, r_2 and q are collinear and r_1, q_1 and q are collinear and p_1, r_1 and p are collinear and r_2, q_1 and p are collinear and p_1, q_1 and r are collinear and r_2, r_1 and r are collinear and p, q and r are collinear holds p_1, r_2 and q_1 are collinear or p_1, r_2 and r_1 are collinear or p_1, r_1 and q_1 are collinear or r_2, r_1 and q_1 are collinear,

(vi) there exist elements p, p_1, q, q_1 of the points of C_1 such that for no element r of the points of C_1 holds p, p_1 and r are collinear and q, q_1 and r are collinear,

(vii) for every elements p, p_1, q, q_1, r_2 of the points of C_1 there exist elements r, r_1 of the points of C_1 such that p, q and r are collinear and p_1, q_1 and r_1 are collinear and r_2, r and r_1 are collinear,

(viii) for all elements o, p_1, p_2, p_3, q_1, q_2, q_3, r_1, r_2, r_3 of the points of C_1 such that $o \neq q_1$ and $p_1 \neq q_1$ and $o \neq q_2$ and $p_2 \neq q_2$ and $o \neq q_3$ and $p_3 \neq q_3$ and o, p_1 and p_2 are not collinear and o, p_1 and p_3 are not collinear and o, p_2 and p_3 are not collinear and p_1, p_2 and r_3 are collinear and q_1, q_2 and r_3 are collinear and p_2, p_3 and r_1 are collinear and q_2, q_3 and r_1 are collinear and p_1, p_3 and r_2 are collinear and q_1, q_3 and r_2 are collinear and o, p_1 and q_1 are collinear and o, p_2 and q_2 are collinear and o, p_3 and q_3 are collinear holds r_1, r_2 and r_3 are collinear.

(17) Let C_1 be a collinearity structure. Then C_1 is a Fano-Desarguesian 3 dimensional projective space defined in terms of collinearity if and only if the following conditions are satisfied:

(i) C_1 is a Desarguesian 3 dimensional projective space defined in terms of collinearity,

(ii) for all elements p_1, r_2, q, r_1, q_1, p, r of the points of C_1 such that p_1, r_2 and q are collinear and r_1, q_1 and q are collinear and p_1, r_1 and p are collinear and r_2, q_1 and p are collinear and p_1, q_1 and r are collinear and r_2, r_1 and r are collinear and p, q and r are collinear holds p_1, r_2 and q_1 are collinear or p_1, r_2 and r_1 are collinear or p_1, r_1 and q_1 are collinear or r_2, r_1 and q_1 are collinear.

(18) For every C_1 being a collinearity structure holds
C_1

is a Fano-Desarguesian 3 dimensional projective space defined in terms of collinearity if and only if C_1 is a Fano-Desarguesian at least 3 dimensional projective space defined in terms of collinearity and for every elements p, p_1, q, q_1, r_2 of the points of C_1 there exist elements r, r_1 of the points of C_1 such that p, q and r are collinear and p_1, q_1 and r_1 are collinear and r_2, r and r_1 are collinear.

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