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Binary Operations on Finite Sequences

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Summary. We generalize the semigroup operation on finite sequences introduced in [6] for binary operations that have a unity or for non-empty finite sequences.

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The papers [9], [4], [5], [2], [3], [8], [6], [7], and [1] provide the notation and terminology for this paper. For simplicity we adopt the following convention: D denotes a non-empty set, d, d_1, d_2, d_3 denote elements of D, F, G, H denote finite sequences of elements of D, f denotes a function from $\mathbb{N}$ into D, g denotes a binary operation on D, k, n, l denote natural numbers, and P denotes a permutation of $\mathrm{Seg}(\mathrm{len}\, F)$. Let us consider D, n, d. Then $n \longmapsto d$ is a finite sequence of elements of D.

Let us consider D, F, g. Let us assume that g has a unity or $\mathrm{len}\, F \geq 1$. The functor $g \odot F$ yields an element of D and is defined by:

(Def.1) $g \odot F = \mathbf{1}_g$ if g has a unity and $\mathrm{len}\, F = 0$, there exists f such that $f(1) = F(1)$ and for every n such that $0 \neq n$ and $n < \mathrm{len}\, F$ holds $f(n+1) = g(f(n), F(n+1))$ and $g \odot F = f(\mathrm{len}\, F)$, otherwise.

One can prove the following propositions:

(1) If g has a unity and $\mathrm{len}\, F = 0$, then $g \odot F = \mathbf{1}_g$.

(2) Suppose $\mathrm{len}\, F \geq 1$. Then there exists f such that $f(1) = F(1)$ and for every n such that $0 \neq n$ and $n < \mathrm{len}\, F$ holds $f(n+1) = g(f(n), F(n+1))$ and $g \odot F = f(\mathrm{len}\, F)$.

(3) Suppose $\mathrm{len}\, F \geq 1$ and there exists f such that $f(1) = F(1)$ and for every n such that $0 \neq n$ and $n < \mathrm{len}\, F$ holds $f(n+1) = g(f(n), F(n+1))$ and $d = f(\mathrm{len}\, F)$. Then $d = g \odot F$.

(4) If g has a unity or $\mathrm{len}\, F \geq 1$ but g is associative and g is commutative, then $g \odot F = g \circledast F$.

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(5) If g has a unity or $\operatorname{len} F \geq 1$, then $g \odot F \frown \langle d \rangle = g(g \odot F,\ d)$.

(6) If g is associative but g has a unity or $\operatorname{len} F \geq 1$ and $\operatorname{len} G \geq 1$, then $g \odot F \frown G = g(g \odot F,\ g \odot G)$.

(7) If g is associative but g has a unity or $\operatorname{len} F \geq 1$, then $g \odot \langle d \rangle \frown F = g(d,\ g \odot F)$.

(8) If g is commutative and g is associative but g has a unity or $\operatorname{len} F \geq 1$ and $G = F \cdot P$, then $g \odot F = g \odot G$.

(9) If g has a unity or $\operatorname{len} F \geq 1$ but g is associative and g is commutative and F is one-to-one and G is one-to-one and $\operatorname{rng} F = \operatorname{rng} G$, then $g \odot F = g \odot G$.

(10) Suppose g is associative and g is commutative but g has a unity or $\operatorname{len} F \geq 1$ and $\operatorname{len} F = \operatorname{len} G$ and $\operatorname{len} F = \operatorname{len} H$ and for every k such that $k \in \operatorname{Seg}(\operatorname{len} F)$ holds $F(k) = g(G(k),\ H(k))$. Then $g \odot F = g(g \odot G,\ g \odot H)$.

(11) If g has a unity, then $g \odot \varepsilon_D = \mathbf{1}_g$.

(12) $g \odot \langle d \rangle = d$.

(13) $g \odot \langle d_1, d_2 \rangle = g(d_1,\ d_2)$.

(14) If g is commutative, then $g \odot \langle d_1, d_2 \rangle = g \odot \langle d_2, d_1 \rangle$.

(15) $g \odot \langle d_1, d_2, d_3 \rangle = g(g(d_1,\ d_2),\ d_3)$.

(16) If g is commutative, then $g \odot \langle d_1, d_2, d_3 \rangle = g \odot \langle d_2, d_1, d_3 \rangle$.

(17) $g \odot (1 \longmapsto d) = d$.

(18) $g \odot (2 \longmapsto d) = g(d,\ d)$.

(19) If g is associative but g has a unity or $k \neq 0$ and $l \neq 0$, then $g \odot (k+l \longmapsto d) = g(g \odot (k \longmapsto d),\ g \odot (l \longmapsto d))$.

(20) If g is associative but g has a unity or $k \neq 0$ and $l \neq 0$, then $g \odot (k \cdot l \longmapsto d) = g \odot (l \longmapsto g \odot (k \longmapsto d))$.

(21) If $\operatorname{len} F = 1$, then $g \odot F = F(1)$.

(22) If $\operatorname{len} F = 2$, then $g \odot F = g(F(1),\ F(2))$.

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