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Some Elementary Notions of the Theory of Petri Nets

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Summary. Some fundamental notions of the theory of Petri nets are described in Mizar formalism. A Petri net is defined as a triple of the form ⟨places, transitions, flow⟩ with places and transitions being disjoint sets and flow being a relation included in places × transitions.

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The notation and terminology used here have been introduced in the following articles: [1], and [2]. In the sequel x, y will be arbitrary. We consider nets which are systems

⟨places, transitions, a flow relation⟩,

where the places constitute a set, the transitions constitute a set, and the flow relation is a binary relation. In the sequel N is a net. Let N be a net. We say that N is a Petri net if and only if:

(Def.1) (the places of N)$\cap$ (the transitions of N) $= \emptyset$ and the flow relation of $N \subseteq$ [: the places of N, the transitions of N :] $\cup$ [: the transitions of N, the places of N :].

Let N be a net. The functor Elements(N) yielding a set is defined as follows:

(Def.2) Elements(N) = (the places of N)$\cup$ (the transitions of N).

We now state several propositions:

(1) For every N and for every x such that Elements(N) $\neq \emptyset$ holds x is an element of Elements(N) if and only if $x \in$ Elements(N).

(2) For every N and for every x such that the places of $N \neq \emptyset$ holds x is an element of the places of N if and only if $x \in$ the places of N.

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(3) For every N and for every x such that the transitions of $N \neq \emptyset$ holds x is an element of the transitions of N if and only if $x \in$ the transitions of N.

(4) For every N holds the places of $N \subseteq$ Elements(N).

(5) For every N holds the transitions of $N \subseteq$ Elements(N).

Let N be a net. A set is said to be an element of N if:

(Def.3) it = Elements(N).

Next we state several propositions:

(6) For every N and for every x holds $x \in$ Elements(N) if and only if $x \in$ the places of N or $x \in$ the transitions of N.

(7) For every N and for every x such that Elements(N) $\neq \emptyset$ holds if x is an element of Elements(N), then x is an element of the places of N or x is an element of the transitions of N.

(8) For every N and for every x such that x is an element of the places of N and the places of $N \neq \emptyset$ holds x is an element of Elements(N).

(9) For every N and for every x such that x is an element of the transitions of N and the transitions of $N \neq \emptyset$ holds x is an element of Elements(N).

(10) $\langle \emptyset, \emptyset, \varnothing \rangle$ is a Petri net.

A net is said to be a Petri net if:

(Def.4) it is a Petri net.

We now state several propositions:

(11) For every Petri net N holds it is not true that: $x \in$ the places of N and $x \in$ the transitions of N.

(12) For every Petri net N and for all x, y such that $\langle x, y \rangle \in$ the flow relation of N and $x \in$ the transitions of N holds $y \in$ the places of N.

(13) For every Petri net N and for all x, y such that $\langle x, y \rangle \in$ the flow relation of N and $y \in$ the transitions of N holds $x \in$ the places of N.

(14) For every Petri net N and for all x, y such that $\langle x, y \rangle \in$ the flow relation of N and $x \in$ the places of N holds $y \in$ the transitions of N.

(15) For every Petri net N and for all x, y such that $\langle x, y \rangle \in$ the flow relation of N and $y \in$ the places of N holds $x \in$ the transitions of N.

We now define two new predicates. Let N be a Petri net, and let us consider x, y. We say that x is a pre-element of y in N if and only if:

(Def.5) $\langle y, x \rangle \in$ the flow relation of N and $x \in$ the transitions of N.

We say that x is a post-element of y in N if and only if:

(Def.6) $\langle x, y \rangle \in$ the flow relation of N and $x \in$ the transitions of N.

We now define two new functors. Let N be a net, and let x be an element of Elements(N). The functor Pre(N, x) yielding a set is defined by:

(Def.7) $y \in$ Pre(N, x) if and only if $y \in$ Elements(N) and $\langle y, x \rangle \in$ the flow relation of N.

The functor $\mathrm{Post}(N, x)$ yielding a set is defined by:

(Def.8) $y \in \mathrm{Post}(N, x)$ if and only if $y \in \mathrm{Elements}(N)$ and $\langle x, y \rangle \in$ the flow relation of N.

Next we state several propositions:

(16) For every Petri net N and for every element x of $\mathrm{Elements}(N)$ holds $\mathrm{Pre}(N, x) \subseteq \mathrm{Elements}(N)$.

(17) For every Petri net N and for every element x of $\mathrm{Elements}(N)$ holds $\mathrm{Pre}(N, x) \in 2^{\mathrm{Elements}(N)}$.

(18) For every Petri net N and for every element x of $\mathrm{Elements}(N)$ holds $\mathrm{Post}(N, x) \subseteq \mathrm{Elements}(N)$.

(19) For every Petri net N and for every element x of $\mathrm{Elements}(N)$ holds $\mathrm{Post}(N, x) \in 2^{\mathrm{Elements}(N)}$.

(20) For every Petri net N and for every element y of $\mathrm{Elements}(N)$ such that $y \in$ the transitions of N holds $x \in \mathrm{Pre}(N, y)$ if and only if y is a pre-element of x in N.

(21) For every Petri net N and for every element y of $\mathrm{Elements}(N)$ such that $y \in$ the transitions of N holds $x \in \mathrm{Post}(N, y)$ if and only if y is a post-element of x in N.

Let N be a Petri net, and let x be an element of $\mathrm{Elements}(N)$. Let us assume that $\mathrm{Elements}(N) \neq \emptyset$. The functor $\mathrm{enter}(N, x)$ yielding a set is defined by:

(Def.9) if $x \in$ the places of N, then $\mathrm{enter}(N, x) = \{x\}$ but if $x \in$ the transitions of N, then $\mathrm{enter}(N, x) = \mathrm{Pre}(N, x)$.

We now state three propositions:

(22) For every Petri net N and for every element x of $\mathrm{Elements}(N)$ such that $\mathrm{Elements}(N) \neq \emptyset$ holds $\mathrm{enter}(N, x) = \{x\}$ or $\mathrm{enter}(N, x) = \mathrm{Pre}(N, x)$.

(23) For every Petri net N and for every element x of $\mathrm{Elements}(N)$ such that $\mathrm{Elements}(N) \neq \emptyset$ holds $\mathrm{enter}(N, x) \subseteq \mathrm{Elements}(N)$.

(24) For every Petri net N and for every element x of $\mathrm{Elements}(N)$ such that $\mathrm{Elements}(N) \neq \emptyset$ holds $\mathrm{enter}(N, x) \in 2^{\mathrm{Elements}(N)}$.

Let N be a Petri net, and let x be an element of $\mathrm{Elements}(N)$. Let us assume that $\mathrm{Elements}(N) \neq \emptyset$. The functor $\mathrm{exit}(N, x)$ yields a set and is defined by:

(Def.10) if $x \in$ the places of N, then $\mathrm{exit}(N, x) = \{x\}$ but if $x \in$ the transitions of N, then $\mathrm{exit}(N, x) = \mathrm{Post}(N, x)$.

We now state three propositions:

(25) For every Petri net N and for every element x of $\mathrm{Elements}(N)$ such that $\mathrm{Elements}(N) \neq \emptyset$ holds $\mathrm{exit}(N, x) = \{x\}$ or $\mathrm{exit}(N, x) = \mathrm{Post}(N, x)$.

(26) For every Petri net N and for every element x of $\mathrm{Elements}(N)$ such that $\mathrm{Elements}(N) \neq \emptyset$ holds $\mathrm{exit}(N, x) \subseteq \mathrm{Elements}(N)$.

(27) For every Petri net N and for every element x of $\mathrm{Elements}(N)$ such that $\mathrm{Elements}(N) \neq \emptyset$ holds $\mathrm{exit}(N, x) \in 2^{\mathrm{Elements}(N)}$.

Let N be a Petri net, and let x be an element of Elements(N). Let us assume that Elements(N) $\neq \emptyset$. The functor field(N, x) yielding a set is defined as follows:

(Def.11) field$(N, x) = $ enter$(N, x) \cup$ exit(N, x).

We now define two new functors. Let N be a net, and let x be an element of the transitions of N. The functor Prec(N, x) yielding a set is defined by:

(Def.12) $y \in$ Prec(N, x) if and only if $y \in$ the places of N and $\langle y, x \rangle \in$ the flow relation of N.

The functor Postc(N, x) yielding a set is defined as follows:

(Def.13) $y \in$ Postc(N, x) if and only if $y \in$ the places of N and $\langle x, y \rangle \in$ the flow relation of N.

We now define two new functors. Let N be a Petri net, and let X be a set. Let us assume that $X \subseteq$ Elements(N). The functor Entr(N, X) yields a set and is defined by:

(Def.14) $x \in$ Entr(N, X) if and only if $x \in 2^{\text{Elements}(N)}$ and there exists an element y of Elements(N) such that $y \in X$ and $x = $ enter(N, y).

The functor Ext(N, X) yielding a set is defined by:

(Def.15) $x \in$ Ext(N, X) if and only if $x \in 2^{\text{Elements}(N)}$ and there exists an element y of Elements(N) such that $y \in X$ and $x = $ exit(N, y).

Next we state two propositions:

(28) For every Petri net N and for every element x of Elements(N) and for every set X such that Elements(N) $\neq \emptyset$ and $X \subseteq$ Elements(N) and $x \in X$ holds enter$(N, x) \in$ Entr(N, X).

(29) For every Petri net N and for every element x of Elements(N) and for every set X such that Elements(N) $\neq \emptyset$ and $X \subseteq$ Elements(N) and $x \in X$ holds exit$(N, x) \in$ Ext(N, X).

We now define two new functors. Let N be a Petri net, and let X be a set. Let us assume that $X \subseteq$ Elements(N). The functor Input(N, X) yields a set and is defined by:

(Def.16) Input$(N, X) = \bigcup$ Entr(N, X).

The functor Output(N, X) yielding a set is defined by:

(Def.17) Output$(N, X) = \bigcup$ Ext(N, X).

The following four propositions are true:

(30) For every Petri net N and for every x and for every set X such that Elements(N) $\neq \emptyset$ and $X \subseteq$ Elements(N) holds $x \in$ Input(N, X) if and only if there exists an element y of Elements(N) such that $y \in X$ and $x \in$ enter(N, y).

(31) For every Petri net N and for every x and for every set X such that Elements(N) $\neq \emptyset$ and $X \subseteq$ Elements(N) holds $x \in$ Output(N, X) if and only if there exists an element y of Elements(N) such that $y \in X$ and $x \in$ exit(N, y).

(32) Let N be a Petri net. Then for every subset X of Elements(N) and for every element x of Elements(N) such that Elements(N) $\neq \emptyset$ holds $x \in$ Input(N, X) if and only if $x \in X$ and $x \in$ the places of N or there exists an element y of Elements(N) such that $y \in X$ and $y \in$ the transitions of N and y is a pre-element of x in N.

(33) Let N be a Petri net. Then for every subset X of Elements(N) and for every element x of Elements(N) such that Elements(N) $\neq \emptyset$ holds $x \in$ Output(N, X) if and only if $x \in X$ and $x \in$ the places of N or there exists an element y of Elements(N) such that $y \in X$ and $y \in$ the transitions of N and y is a post-element of x in N.

References

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