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Propositional Calculus [1]

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Summary. We develop the classical propositional calculus, following [3].

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The notation and terminology used here are introduced in the articles [1] and [2]. We follow the rules: p, q, r, s are elements of CQC−WFF and X is a subset of CQC−WFF. We now state a number of propositions:

(1) $(p \Rightarrow q) \Rightarrow ((q \Rightarrow r) \Rightarrow (p \Rightarrow r)) \in$ Taut.

(2) If $p \Rightarrow q \in$ Taut, then $(q \Rightarrow r) \Rightarrow (p \Rightarrow r) \in$ Taut.

(3) If $p \Rightarrow q \in$ Taut and $q \Rightarrow r \in$ Taut, then $p \Rightarrow r \in$ Taut.

(4) $p \Rightarrow p \in$ Taut.

(5) $q \Rightarrow (p \Rightarrow q) \in$ Taut.

(6) $((p \Rightarrow q) \Rightarrow r) \Rightarrow (q \Rightarrow r) \in$ Taut.

(7) $q \Rightarrow ((q \Rightarrow p) \Rightarrow p) \in$ Taut.

(8) $(s \Rightarrow (q \Rightarrow p)) \Rightarrow (q \Rightarrow (s \Rightarrow p)) \in$ Taut.

(9) $(q \Rightarrow r) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r)) \in$ Taut.

(10) $(q \Rightarrow (q \Rightarrow r)) \Rightarrow (q \Rightarrow r) \in$ Taut.

(11) $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r)) \in$ Taut.

(12) $\neg$ VERUM $\Rightarrow p \in$ Taut.

(13) If $q \in$ Taut, then $p \Rightarrow q \in$ Taut.

(14) If $p \in$ Taut, then $(p \Rightarrow q) \Rightarrow q \in$ Taut.

(15) If $s \Rightarrow (q \Rightarrow p) \in$ Taut, then $q \Rightarrow (s \Rightarrow p) \in$ Taut.

(16) If $s \Rightarrow (q \Rightarrow p) \in$ Taut and $q \in$ Taut, then $s \Rightarrow p \in$ Taut.

(17) If $s \Rightarrow (q \Rightarrow p) \in$ Taut and $q \in$ Taut and $s \in$ Taut, then $p \in$ Taut.

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(18) If $q \Rightarrow (q \Rightarrow r) \in$ Taut, then $q \Rightarrow r \in$ Taut.

(19) If $p \Rightarrow (q \Rightarrow r) \in$ Taut, then $(p \Rightarrow q) \Rightarrow (p \Rightarrow r) \in$ Taut.

(20) If $p \Rightarrow (q \Rightarrow r) \in$ Taut and $p \Rightarrow q \in$ Taut, then $p \Rightarrow r \in$ Taut.

(21) If $p \Rightarrow (q \Rightarrow r) \in$ Taut and $p \Rightarrow q \in$ Taut and $p \in$ Taut, then $r \in$ Taut.

(22) If $p \Rightarrow (q \Rightarrow r) \in$ Taut and $p \Rightarrow (r \Rightarrow s) \in$ Taut, then $p \Rightarrow (q \Rightarrow s) \in$ Taut.

(23) $p \Rightarrow \mathrm{VERUM} \in$ Taut.

(24) $(\neg p \Rightarrow \neg q) \Rightarrow (q \Rightarrow p) \in$ Taut.

(25) $\neg(\neg p) \Rightarrow p \in$ Taut.

(26) $(p \Rightarrow q) \Rightarrow (\neg q \Rightarrow \neg p) \in$ Taut.

(27) $p \Rightarrow \neg(\neg p) \in$ Taut.

(28) $(\neg(\neg p) \Rightarrow q) \Rightarrow (p \Rightarrow q) \in$ Taut and $(p \Rightarrow q) \Rightarrow (\neg(\neg p) \Rightarrow q) \in$ Taut.

(29) $(p \Rightarrow \neg(\neg q)) \Rightarrow (p \Rightarrow q) \in$ Taut and $(p \Rightarrow q) \Rightarrow (p \Rightarrow \neg(\neg q)) \in$ Taut.

(30) $(p \Rightarrow \neg q) \Rightarrow (q \Rightarrow \neg p) \in$ Taut.

(31) $(\neg p \Rightarrow q) \Rightarrow (\neg q \Rightarrow p) \in$ Taut.

We now state a number of propositions:

(32) $(p \Rightarrow \neg p) \Rightarrow \neg p \in$ Taut.

(33) $\neg p \Rightarrow (p \Rightarrow q) \in$ Taut.

(34) $p \Rightarrow q \in$ Taut if and only if $\neg q \Rightarrow \neg p \in$ Taut.

(35) If $\neg p \Rightarrow \neg q \in$ Taut, then $q \Rightarrow p \in$ Taut.

(36) $p \in$ Taut if and only if $\neg(\neg p) \in$ Taut.

(37) $p \Rightarrow q \in$ Taut if and only if $p \Rightarrow \neg(\neg q) \in$ Taut.

(38) $p \Rightarrow q \in$ Taut if and only if $\neg(\neg p) \Rightarrow q \in$ Taut.

(39) If $p \Rightarrow \neg q \in$ Taut, then $q \Rightarrow \neg p \in$ Taut.

(40) If $\neg p \Rightarrow q \in$ Taut, then $\neg q \Rightarrow p \in$ Taut.

(41) $\vdash (p \Rightarrow q) \Rightarrow ((q \Rightarrow r) \Rightarrow (p \Rightarrow r))$.

(42) If $\vdash p \Rightarrow q$, then $\vdash (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$.

(43) If $\vdash p \Rightarrow q$ and $\vdash q \Rightarrow r$, then $\vdash p \Rightarrow r$.

(44) $\vdash p \Rightarrow p$.

(45) $\vdash p \Rightarrow (q \Rightarrow p)$.

(46) If $\vdash p$, then $\vdash q \Rightarrow p$.

(47) $\vdash (s \Rightarrow (q \Rightarrow p)) \Rightarrow (q \Rightarrow (s \Rightarrow p))$.

(48) If $\vdash p \Rightarrow (q \Rightarrow r)$, then $\vdash q \Rightarrow (p \Rightarrow r)$.

(49) If $\vdash p \Rightarrow (q \Rightarrow r)$ and $\vdash q$, then $\vdash p \Rightarrow r$.

(50) $\vdash p \Rightarrow \mathrm{VERUM}$ and $\vdash \neg\,\mathrm{VERUM} \Rightarrow p$.

(51) $\vdash p \Rightarrow ((p \Rightarrow q) \Rightarrow q)$.

(52) $\vdash (q \Rightarrow (q \Rightarrow r)) \Rightarrow (q \Rightarrow r)$.

(53) If $\vdash q \Rightarrow (q \Rightarrow r)$, then $\vdash q \Rightarrow r$.

(54) $\vdash (p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$.

(55) If $\vdash p \Rightarrow (q \Rightarrow r)$, then $\vdash (p \Rightarrow q) \Rightarrow (p \Rightarrow r)$.
(56) If $\vdash p \Rightarrow (q \Rightarrow r)$ and $\vdash p \Rightarrow q$, then $\vdash p \Rightarrow r$.
(57) $\vdash ((p \Rightarrow q) \Rightarrow r) \Rightarrow (q \Rightarrow r)$.
(58) If $\vdash (p \Rightarrow q) \Rightarrow r$, then $\vdash q \Rightarrow r$.
(59) $\vdash (p \Rightarrow q) \Rightarrow ((r \Rightarrow p) \Rightarrow (r \Rightarrow q))$.
(60) If $\vdash p \Rightarrow q$, then $\vdash (r \Rightarrow p) \Rightarrow (r \Rightarrow q)$.
(61) $\vdash (p \Rightarrow q) \Rightarrow (\neg q \Rightarrow \neg p)$.
(62) $\vdash (\neg p \Rightarrow \neg q) \Rightarrow (q \Rightarrow p)$.

The following propositions are true:

(63) $\vdash \neg p \Rightarrow \neg q$ if and only if $\vdash q \Rightarrow p$.
(64) $\vdash p \Rightarrow \neg(\neg p)$.
(65) $\vdash \neg(\neg p) \Rightarrow p$.
(66) $\vdash \neg(\neg p)$ if and only if $\vdash p$.
(67) $\vdash (\neg(\neg p) \Rightarrow q) \Rightarrow (p \Rightarrow q)$.
(68) $\vdash \neg(\neg p) \Rightarrow q$ if and only if $\vdash p \Rightarrow q$.
(69) $\vdash (p \Rightarrow \neg(\neg q)) \Rightarrow (p \Rightarrow q)$.
(70) $\vdash p \Rightarrow \neg(\neg q)$ if and only if $\vdash p \Rightarrow q$.
(71) $\vdash (p \Rightarrow \neg q) \Rightarrow (q \Rightarrow \neg p)$.
(72) If $\vdash p \Rightarrow \neg q$, then $\vdash q \Rightarrow \neg p$.
(73) $\vdash (\neg p \Rightarrow q) \Rightarrow (\neg q \Rightarrow p)$.
(74) If $\vdash \neg p \Rightarrow q$, then $\vdash \neg q \Rightarrow p$.
(75) If $X \vdash p \Rightarrow q$, then $X \vdash (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$.
(76) If $X \vdash p \Rightarrow q$ and $X \vdash q \Rightarrow r$, then $X \vdash p \Rightarrow r$.
(77) $X \vdash p \Rightarrow p$.
(78) If $X \vdash p$, then $X \vdash q \Rightarrow p$.
(79) If $X \vdash p$, then $X \vdash (p \Rightarrow q) \Rightarrow q$.
(80) If $X \vdash p \Rightarrow (q \Rightarrow r)$, then $X \vdash q \Rightarrow (p \Rightarrow r)$.
(81) If $X \vdash p \Rightarrow (q \Rightarrow r)$ and $X \vdash q$, then $X \vdash p \Rightarrow r$.
(82) If $X \vdash p \Rightarrow (p \Rightarrow q)$, then $X \vdash p \Rightarrow q$.
(83) If $X \vdash (p \Rightarrow q) \Rightarrow r$, then $X \vdash q \Rightarrow r$.
(84) If $X \vdash p \Rightarrow (q \Rightarrow r)$, then $X \vdash (p \Rightarrow q) \Rightarrow (p \Rightarrow r)$.
(85) If $X \vdash p \Rightarrow (q \Rightarrow r)$ and $X \vdash p \Rightarrow q$, then $X \vdash p \Rightarrow r$.
(86) $X \vdash \neg p \Rightarrow \neg q$ if and only if $X \vdash q \Rightarrow p$.
(87) $X \vdash \neg(\neg p)$ if and only if $X \vdash p$.
(88) $X \vdash p \Rightarrow \neg(\neg q)$ if and only if $X \vdash p \Rightarrow q$.
(89) $X \vdash \neg(\neg p) \Rightarrow q$ if and only if $X \vdash p \Rightarrow q$.
(90) If $X \vdash p \Rightarrow \neg q$, then $X \vdash q \Rightarrow \neg p$.
(91) If $X \vdash \neg p \Rightarrow q$, then $X \vdash \neg q \Rightarrow p$.
(92) If $X \vdash p \Rightarrow \neg q$ and $X \vdash q$, then $X \vdash \neg p$.

(93) If $X \vdash \neg p \Rightarrow q$ and $X \vdash \neg q$, then $X \vdash p$.

References

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