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Infimum and Supremum of the Set of Real Numbers. Measure Theory

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Summary. We introduce some properties of the least upper bound and the greatest lower bound of the subdomain of $\overline{\mathbb{R}}$ numbers, where $\overline{\mathbb{R}}$ denotes the enlarged set of real numbers, $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$. The paper contains definitions of majorant and minorant elements, bounded from above, bounded from below and bounded sets, sup and inf of set, for nonempty subset of $\overline{\mathbb{R}}$. We prove theorems describing the basic relationships among those definitions. The work is the first part of the series of articles concerning the Lebesgue measure theory.

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The terminology and notation used here have been introduced in the following articles: [3], [1], and [2]. The constant $+\infty$ is defined by:

(Def.1) $+\infty = \mathbb{R}$.

The following propositions are true:

(1) $+\infty = \mathbb{R}$.

(2) $+\infty \notin \mathbb{R}$.

A positive infinite number is defined as follows:

(Def.2) it $= +\infty$.

One can prove the following proposition

(4)[1] $+\infty$ is a positive infinite number.

The constant $-\infty$ is defined as follows:

(Def.3) $-\infty = \{\mathbb{R}\}$.

The following propositions are true:

(5) $-\infty = \{\mathbb{R}\}$.

(6) $-\infty \notin \mathbb{R}$.

[1]The proposition (3) was either repeated or obvious.

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A negative infinite number is defined as follows:

(Def.4) it $= -\infty$.

One can prove the following proposition

(8)[2] $-\infty$ is a negative infinite number.

A *Real number* is defined as follows:

(Def.5) it $\in \mathbb{R} \cup \{-\infty, +\infty\}$.

One can prove the following propositions:

(10)[3] For every real number x holds x is a *Real number*.

(11) For an arbitrary x such that $x = -\infty$ or $x = +\infty$ holds x is a *Real number*.

Let us note that it makes sense to consider the following constant. Then $+\infty$ is a *Real number*.

Let us note that it makes sense to consider the following constant. Then $-\infty$ is a *Real number*.

Next we state the proposition

(14)[4] $-\infty \neq +\infty$.

Let x, y be *Real numbers*. The predicate $x \leq y$ is defined by:

(Def.6) there exist real numbers p, q such that $p = x$ and $q = y$ and $p \leq q$ or there exists a positive infinite number q such that $q = y$ or there exists a negative infinite number p such that $p = x$.

Next we state several propositions:

(16)[5] For all *Real numbers* x, y such that x is a real number and y is a real number holds $x \leq y$ if and only if there exist real numbers p, q such that $p = x$ and $q = y$ and $p \leq q$.

(17) For every *Real number* x such that $x \in \mathbb{R}$ holds $x \not\leq -\infty$.

(18) For every *Real number* x such that $x \in \mathbb{R}$ holds $+\infty \not\leq x$.

(19) $+\infty \not\leq -\infty$.

(20) For every *Real number* x holds $x \leq +\infty$.

(21) For every *Real number* x holds $-\infty \leq x$.

(22) For all *Real numbers* x, y such that $x \leq y$ and $y \leq x$ holds $x = y$.

(23) For every *Real number* x such that $x \leq -\infty$ holds $x = -\infty$.

(24) For every *Real number* x such that $+\infty \leq x$ holds $x = +\infty$.

The scheme *SepR_eal* concerns a unary predicate $\mathcal{P}$, and states that:

there exists a subset X of $\mathbb{R} \cup \{-\infty, +\infty\}$ such that for every *Real number* x holds $x \in X$ if and only if $\mathcal{P}[x]$
for all values of the parameter.

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[2]The proposition (7) was either repeated or obvious.
[3]The proposition (9) was either repeated or obvious.
[4]The propositions (12)–(13) were either repeated or obvious.
[5]The proposition (15) was either repeated or obvious.

The set $\overline{\mathbb{R}}$ is defined as follows:

(Def.7) $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$.

We now state several propositions:

(25) $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$.

(26) $\overline{\mathbb{R}}$ is a non-empty set.

(27) For an arbitrary x holds x is a *Real number* if and only if $x \in \overline{\mathbb{R}}$.

(28) For every *Real number* x holds $x \leq x$.

(29) For all *Real numbers* x, y, z such that $x \leq y$ and $y \leq z$ holds $x \leq z$.

Let us note that it makes sense to consider the following constant. Then $\overline{\mathbb{R}}$ is a non-empty set.

Let x, y be *Real numbers*. The predicate $x < y$ is defined by:

(Def.8) $x \leq y$ and $x \neq y$.

The following proposition is true

(31)[6] For every *Real number* x such that $x \in \mathbb{R}$ holds $-\infty < x$ and $x < +\infty$.

Let X be a non-empty subset of $\overline{\mathbb{R}}$. A *Real number* is said to be a majorant of X if:

(Def.9) for every *Real number* x such that $x \in X$ holds $x \leq$ it.

We now state two propositions:

(33)[7] For every non-empty subset X of $\overline{\mathbb{R}}$ holds $+\infty$ is a majorant of X.

(34) For all non-empty subsets X, Y of $\overline{\mathbb{R}}$ such that $X \subseteq Y$ for every *Real number* U_1 such that U_1 is a majorant of Y holds U_1 is a majorant of X.

Let X be a non-empty subset of $\overline{\mathbb{R}}$. A *Real number* is said to be a minorant of X if:

(Def.10) for every *Real number* x such that $x \in X$ holds it $\leq x$.

We now state four propositions:

(36)[8] For every non-empty subset X of $\overline{\mathbb{R}}$ holds $-\infty$ is a minorant of X.

(37) For every non-empty subset X of $\overline{\mathbb{R}}$ such that $X = \overline{\mathbb{R}}$ holds $+\infty$ is a majorant of X.

(38) For every non-empty subset X of $\overline{\mathbb{R}}$ such that $X = \overline{\mathbb{R}}$ holds $-\infty$ is a minorant of X.

(39) For all non-empty subsets X, Y of $\overline{\mathbb{R}}$ such that $X \subseteq Y$ for every *Real number* L_1 such that L_1 is a minorant of Y holds L_1 is a minorant of X.

Let us note that it makes sense to consider the following constant. Then $\mathbb{R}$ is a non-empty subset of $\overline{\mathbb{R}}$.

One can prove the following propositions:

(41)[9] $+\infty$ is a majorant of $\mathbb{R}$.

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[6] The proposition (30) was either repeated or obvious.

[7] The proposition (32) was either repeated or obvious.

[8] The proposition (35) was either repeated or obvious.

[9] The proposition (40) was either repeated or obvious.

(42) $-\infty$ is a minorant of $\mathbb{R}$.

Let X be a non-empty subset of $\overline{\mathbb{R}}$. We say that X is upper bounded if and only if:

(Def.11) there exists a majorant U_1 of X such that $U_1 \in \mathbb{R}$.

The following two propositions are true:

(44)[10] For all non-empty subsets X, Y of $\overline{\mathbb{R}}$ such that $X \subseteq Y$ holds if Y is upper bounded, then X is upper bounded.

(45) $\mathbb{R}$ is not upper bounded.

Let X be a non-empty subset of $\overline{\mathbb{R}}$. We say that X is lower bounded if and only if:

(Def.12) there exists a minorant L_1 of X such that $L_1 \in \mathbb{R}$.

The following two propositions are true:

(47)[11] For all non-empty subsets X, Y of $\overline{\mathbb{R}}$ such that $X \subseteq Y$ holds if Y is lower bounded, then X is lower bounded.

(48) $\mathbb{R}$ is not lower bounded.

Let X be a non-empty subset of $\overline{\mathbb{R}}$. We say that X is bounded if and only if:

(Def.13) X is upper bounded and X is lower bounded.

The following two propositions are true:

(50)[12] For all non-empty subsets X, Y of $\overline{\mathbb{R}}$ such that $X \subseteq Y$ holds if Y is bounded, then X is bounded.

(51) For every non-empty subset X of $\overline{\mathbb{R}}$ there exists a non-empty subset Y of $\overline{\mathbb{R}}$ such that for every *Real number* x holds $x \in Y$ if and only if x is a majorant of X.

Let X be a non-empty subset of $\overline{\mathbb{R}}$. The functor $\overline{X}$ yields a non-empty subset of $\overline{\mathbb{R}}$ and is defined as follows:

(Def.14) for every *Real number* x holds $x \in \overline{X}$ if and only if x is a majorant of X.

One can prove the following four propositions:

(52) For every non-empty subset X of $\overline{\mathbb{R}}$ and for every non-empty subset Y of $\overline{\mathbb{R}}$ holds $Y = \overline{X}$ if and only if for every *Real number* x holds $x \in Y$ if and only if x is a majorant of X.

(53) For every non-empty subset X of $\overline{\mathbb{R}}$ and for every *Real number* x holds $x \in \overline{X}$ if and only if x is a majorant of X.

(54) For all non-empty subsets X, Y of $\overline{\mathbb{R}}$ such that $X \subseteq Y$ for every *Real number* x such that $x \in \overline{Y}$ holds $x \in \overline{X}$.

(55) For every non-empty subset X of $\overline{\mathbb{R}}$ there exists a non-empty subset Y of $\overline{\mathbb{R}}$ such that for every *Real number* x holds $x \in Y$ if and only if x is a minorant of X.

[10]The proposition (43) was either repeated or obvious.
[11]The proposition (46) was either repeated or obvious.
[12]The proposition (49) was either repeated or obvious.

Let X be a non-empty subset of $\overline{\mathbb{R}}$. The functor $\underline{X}$ yields a non-empty subset of $\overline{\mathbb{R}}$ and is defined by:

(Def.15) for every *Real number* x holds $x \in \underline{X}$ if and only if x is a minorant of X.

We now state a number of propositions:

(56) For every non-empty subset X of $\overline{\mathbb{R}}$ and for every non-empty subset Y of $\overline{\mathbb{R}}$ holds $Y = \underline{X}$ if and only if for every *Real number* x holds $x \in Y$ if and only if x is a minorant of X.

(57) For every non-empty subset X of $\overline{\mathbb{R}}$ and for every *Real number* x holds $x \in \underline{X}$ if and only if x is a minorant of X.

(58) For all non-empty subsets X, Y of $\overline{\mathbb{R}}$ such that $X \subseteq Y$ for every *Real number* x such that $x \in \underline{Y}$ holds $x \in \underline{X}$.

(59) For every non-empty subset X of $\overline{\mathbb{R}}$ such that X is upper bounded and $X \neq \{-\infty\}$ there exists a real number x such that $x \in X$ and $x \neq -\infty$.

(60) For every non-empty subset X of $\overline{\mathbb{R}}$ such that X is lower bounded and $X \neq \{+\infty\}$ there exists a real number x such that $x \in X$ and $x \neq +\infty$.

(62)[13] For every non-empty subset X of $\overline{\mathbb{R}}$ such that X is upper bounded and $X \neq \{-\infty\}$ there exists a *Real number* U_1 such that U_1 is a majorant of X and for every *Real number* y such that y is a majorant of X holds $U_1 \leq y$.

(63) For every non-empty subset X of $\overline{\mathbb{R}}$ such that X is lower bounded and $X \neq \{+\infty\}$ there exists a *Real number* L_1 such that L_1 is a minorant of X and for every *Real number* y such that y is a minorant of X holds $y \leq L_1$.

(64) For every non-empty subset X of $\overline{\mathbb{R}}$ such that $X = \{-\infty\}$ holds X is upper bounded.

(65) For every non-empty subset X of $\overline{\mathbb{R}}$ such that $X = \{+\infty\}$ holds X is lower bounded.

(66) For every non-empty subset X of $\overline{\mathbb{R}}$ such that $X = \{-\infty\}$ there exists a *Real number* U_1 such that U_1 is a majorant of X and for every *Real number* y such that y is a majorant of X holds $U_1 \leq y$.

(67) For every non-empty subset X of $\overline{\mathbb{R}}$ such that $X = \{+\infty\}$ there exists a *Real number* L_1 such that L_1 is a minorant of X and for every *Real number* y such that y is a minorant of X holds $y \leq L_1$.

(68) For every non-empty subset X of $\overline{\mathbb{R}}$ such that X is upper bounded there exists a *Real number* U_1 such that U_1 is a majorant of X and for every *Real number* y such that y is a majorant of X holds $U_1 \leq y$.

(69) For every non-empty subset X of $\overline{\mathbb{R}}$ such that X is lower bounded there exists a *Real number* L_1 such that L_1 is a minorant of X and for every *Real number* y such that y is a minorant of X holds $y \leq L_1$.

[13] The proposition (61) was either repeated or obvious.

(70) For every non-empty subset X of $\overline{\mathbb{R}}$ such that X is not upper bounded for every *Real number* y such that y is a majorant of X holds $y = +\infty$.

(71) For every non-empty subset X of $\overline{\mathbb{R}}$ such that X is not lower bounded for every *Real number* y such that y is a minorant of X holds $y = -\infty$.

(72) For every non-empty subset X of $\overline{\mathbb{R}}$ there exists a *Real number* U_1 such that U_1 is a majorant of X and for every *Real number* y such that y is a majorant of X holds $U_1 \leq y$.

(73) For every non-empty subset X of $\overline{\mathbb{R}}$ there exists a *Real number* L_1 such that L_1 is a minorant of X and for every *Real number* y such that y is a minorant of X holds $y \leq L_1$.

Let X be a non-empty subset of $\overline{\mathbb{R}}$. The functor $\sup X$ yields a *Real number* and is defined as follows:

(Def.16) $\sup X$ is a majorant of X and for every *Real number* y such that y is a majorant of X holds $\sup X \leq y$.

The following propositions are true:

(74) For every non-empty subset X of $\overline{\mathbb{R}}$ and for every *Real number* S holds $S = \sup X$ if and only if S is a majorant of X and for every *Real number* y such that y is a majorant of X holds $S \leq y$.

(75) For every non-empty subset X of $\overline{\mathbb{R}}$ holds $\sup X$ is a majorant of X and for every *Real number* y such that y is a majorant of X holds $\sup X \leq y$.

(76) For every non-empty subset X of $\overline{\mathbb{R}}$ and for every *Real number* x such that $x \in X$ holds $x \leq \sup X$.

Let X be a non-empty subset of $\overline{\mathbb{R}}$. The functor $\inf X$ yields a *Real number* and is defined by:

(Def.17) $\inf X$ is a minorant of X and for every *Real number* y such that y is a minorant of X holds $y \leq \inf X$.

The following propositions are true:

(77) For every non-empty subset X of $\overline{\mathbb{R}}$ and for every *Real number* S holds $S = \inf X$ if and only if S is a minorant of X and for every *Real number* y such that y is a minorant of X holds $y \leq S$.

(78) For every non-empty subset X of $\overline{\mathbb{R}}$ holds $\inf X$ is a minorant of X and for every *Real number* y such that y is a minorant of X holds $y \leq \inf X$.

(79) For every non-empty subset X of $\overline{\mathbb{R}}$ and for every *Real number* x such that $x \in X$ holds $\inf X \leq x$.

(80) For every non-empty subset X of $\overline{\mathbb{R}}$ and for every majorant x of X such that $x \in X$ holds $x = \sup X$.

(81) For every non-empty subset X of $\overline{\mathbb{R}}$ and for every minorant x of X such that $x \in X$ holds $x = \inf X$.

(82) For every non-empty subset X of $\overline{\mathbb{R}}$ holds $\sup X = \inf \overline{X}$ and $\inf X = \sup \underline{X}$.

(83) For every non-empty subset X of $\overline{\mathbb{R}}$ such that X is upper bounded and $X \neq \{-\infty\}$ holds $\sup X \in \mathbb{R}$.

(84) For every non-empty subset X of $\overline{\mathbb{R}}$ such that X is lower bounded and $X \neq \{+\infty\}$ holds $\inf X \in \mathbb{R}$.

Let x be a *Real number*. Then $\{x\}$ is a non-empty subset of $\overline{\mathbb{R}}$.

Let x, y be *Real numbers*. Then $\{x, y\}$ is a non-empty subset of $\overline{\mathbb{R}}$.

We now state a number of propositions:

(85) For every *Real number* x holds $\sup\{x\} = x$.

(86) For every *Real number* x holds $\inf\{x\} = x$.

(87) $\sup\{-\infty\} = -\infty$.

(88) $\sup\{+\infty\} = +\infty$.

(89) $\inf\{-\infty\} = -\infty$.

(90) $\inf\{+\infty\} = +\infty$.

(91) For all non-empty subsets X, Y of $\overline{\mathbb{R}}$ such that $X \subseteq Y$ holds $\sup X \leq \sup Y$.

(92) For all *Real numbers* x, y and for every *Real number* a such that $x \leq a$ and $y \leq a$ holds $\sup\{x, y\} \leq a$.

(93) For all *Real numbers* x, y holds if $x \leq y$, then $\sup\{x, y\} = y$ but if $y \leq x$, then $\sup\{x, y\} = x$.

(94) For all non-empty subsets X, Y of $\overline{\mathbb{R}}$ such that $X \subseteq Y$ holds $\inf Y \leq \inf X$.

(95) For all *Real numbers* x, y and for every *Real number* a such that $a \leq x$ and $a \leq y$ holds $a \leq \inf\{x, y\}$.

(96) For all *Real numbers* x, y holds if $x \leq y$, then $\inf\{x, y\} = x$ but if $y \leq x$, then $\inf\{x, y\} = y$.

(97) For every non-empty subset X of $\overline{\mathbb{R}}$ and for every *Real number* x such that there exists a *Real number* y such that $y \in X$ and $x \leq y$ holds $x \leq \sup X$.

(98) For every non-empty subset X of $\overline{\mathbb{R}}$ and for every *Real number* x such that there exists a *Real number* y such that $y \in X$ and $y \leq x$ holds $\inf X \leq x$.

(99) For all non-empty subsets X, Y of $\overline{\mathbb{R}}$ such that for every *Real number* x such that $x \in X$ there exists a *Real number* y such that $y \in Y$ and $x \leq y$ holds $\sup X \leq \sup Y$.

(100) For all non-empty subsets X, Y of $\overline{\mathbb{R}}$ such that for every *Real number* y such that $y \in Y$ there exists a *Real number* x such that $x \in X$ and $x \leq y$ holds $\inf X \leq \inf Y$.

Let X, Y be non-empty subsets of $\overline{\mathbb{R}}$. Then $X \cup Y$ is a non-empty subset of $\overline{\mathbb{R}}$.

One can prove the following propositions:

(101) For all non-empty subsets X, Y of $\overline{\mathbb{R}}$ and for every majorant U_2 of X and for every majorant U_3 of Y holds $\sup\{U_2, U_3\}$ is a majorant of $X \cup Y$.

(102) For all non-empty subsets X, Y of $\overline{\mathbb{R}}$ and for every minorant L_2 of X and for every minorant L_3 of Y holds $\inf\{L_2, L_3\}$ is a minorant of $X \cup Y$.

(103) For all non-empty subsets X, Y, S of $\overline{\mathbb{R}}$ and for every majorant U_2 of X and for every majorant U_3 of Y such that $S = X \cap Y$ holds $\inf\{U_2, U_3\}$ is a majorant of S.

(104) For all non-empty subsets X, Y, S of $\overline{\mathbb{R}}$ and for every minorant L_2 of X and for every minorant L_3 of Y such that $S = X \cap Y$ holds $\sup\{L_2, L_3\}$ is a minorant of S.

(105) For all non-empty subsets X, Y of $\overline{\mathbb{R}}$ holds $\sup(X \cup Y) = \sup\{\sup X, \sup Y\}$.

(106) For all non-empty subsets X, Y of $\overline{\mathbb{R}}$ holds $\inf(X \cup Y) = \inf\{\inf X, \inf Y\}$.

(107) For all non-empty subsets X, Y, S of $\overline{\mathbb{R}}$ such that $S = X \cap Y$ holds $\sup S \leq \inf\{\sup X, \sup Y\}$.

(108) For all non-empty subsets X, Y, S of $\overline{\mathbb{R}}$ such that $S = X \cap Y$ holds $\sup\{\inf X, \inf Y\} \leq \inf S$.

Let X be a non-empty set. A set is called a non-empty set of non-empty subsets of X if:

(Def.18) it is a non-empty subset of 2^X and for every set A such that $A \in$ it holds A is a non-empty set.

Let F be a non-empty set of non-empty subsets of $\overline{\mathbb{R}}$. The functor $\sup_{\overline{\mathbb{R}}} F$ yielding a non-empty subset of $\overline{\mathbb{R}}$ is defined as follows:

(Def.19) for every *Real number* a holds $a \in \sup_{\overline{\mathbb{R}}} F$ if and only if there exists a non-empty subset A of $\overline{\mathbb{R}}$ such that $A \in F$ and $a = \sup A$.

We now state several propositions:

(110)[14] For every non-empty set F of non-empty subsets of $\overline{\mathbb{R}}$ and for every non-empty subset S of $\overline{\mathbb{R}}$ holds $S = \sup_{\overline{\mathbb{R}}} F$ if and only if for every *Real number* a holds $a \in S$ if and only if there exists a non-empty subset A of $\overline{\mathbb{R}}$ such that $A \in F$ and $a = \sup A$.

(111) For every non-empty set F of non-empty subsets of $\overline{\mathbb{R}}$ and for every *Real number* a holds $a \in \sup_{\overline{\mathbb{R}}} F$ if and only if there exists a non-empty subset A of $\overline{\mathbb{R}}$ such that $A \in F$ and $a = \sup A$.

(112) For every non-empty set F of non-empty subsets of $\overline{\mathbb{R}}$ and for every non-empty subset S of $\overline{\mathbb{R}}$ such that $S = \bigcup F$ holds $\sup S$ is a majorant of $\sup_{\overline{\mathbb{R}}} F$.

(113) For every non-empty set F of non-empty subsets of $\overline{\mathbb{R}}$ and for every non-empty subset S of $\overline{\mathbb{R}}$ such that $S = \bigcup F$ holds $\sup(\sup_{\overline{\mathbb{R}}} F)$ is a majorant of S.

[14]The proposition (109) was either repeated or obvious.

(114) For every non-empty set F of non-empty subsets of $\overline{\mathbb{R}}$ and for every non-empty subset S of $\overline{\mathbb{R}}$ such that $S = \bigcup F$ holds $\sup S = \sup(\sup_{\overline{\mathbb{R}}} F)$.

Let F be a non-empty set of non-empty subsets of $\overline{\mathbb{R}}$. The functor $\inf_{\overline{\mathbb{R}}} F$ yields a non-empty subset of $\overline{\mathbb{R}}$ and is defined as follows:

(Def.20) for every *Real number* a holds $a \in \inf_{\overline{\mathbb{R}}} F$ if and only if there exists a non-empty subset A of $\overline{\mathbb{R}}$ such that $A \in F$ and $a = \inf A$.

We now state several propositions:

(115) For every non-empty set F of non-empty subsets of $\overline{\mathbb{R}}$ and for every non-empty subset S of $\overline{\mathbb{R}}$ holds $S = \inf_{\overline{\mathbb{R}}} F$ if and only if for every *Real number* a holds $a \in S$ if and only if there exists a non-empty subset A of $\overline{\mathbb{R}}$ such that $A \in F$ and $a = \inf A$.

(116) For every non-empty set F of non-empty subsets of $\overline{\mathbb{R}}$ and for every *Real number* a holds $a \in \inf_{\overline{\mathbb{R}}} F$ if and only if there exists a non-empty subset A of $\overline{\mathbb{R}}$ such that $A \in F$ and $a = \inf A$.

(117) For every non-empty set F of non-empty subsets of $\overline{\mathbb{R}}$ and for every non-empty subset S of $\overline{\mathbb{R}}$ such that $S = \bigcup F$ holds $\inf S$ is a minorant of $\inf_{\overline{\mathbb{R}}} F$.

(118) For every non-empty set F of non-empty subsets of $\overline{\mathbb{R}}$ and for every non-empty subset S of $\overline{\mathbb{R}}$ such that $S = \bigcup F$ holds $\inf(\inf_{\overline{\mathbb{R}}} F)$ is a minorant of S.

(119) For every non-empty set F of non-empty subsets of $\overline{\mathbb{R}}$ and for every non-empty subset S of $\overline{\mathbb{R}}$ such that $S = \bigcup F$ holds $\inf S = \inf(\inf_{\overline{\mathbb{R}}} F)$.

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