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Ternary Fields [1]

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Summary. The article contains part 3 of the set of papers concerning the theory of algebraic structures, based on the book [11] pp. 13-15 (pages 6-8 for English edition).

First the basic structure (F, 0, 1, T) is defined, where T is a ternary operation on F (three-argument operations have been introduced in the article [9]). Following it, the basic axioms of a Ternary Field are displayed, the mode is defined and its existence proved. The basic properties of a Ternary Field are also contemplated there.

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The articles [13], [12], [3], [4], [1], [2], [6], [5], [7], [8], [10], and [9] provide the notation and terminology for this paper. We consider ternary field structures which are systems

⟨a carrier, a zero, a unity, a operation⟩,

where the carrier is a non-empty set, the zero is an element of the carrier, the unity is an element of the carrier, and the operation is a ternary operation on the carrier.

In the sequel F denotes a ternary field structure. Let us consider F. A scalar of F is an element of the carrier of F.

In the sequel a, b, c are scalars of F. Let us consider F, a, b, c. The functor $\mathrm{T}(a,b,c)$ yields a scalar of F and is defined by:

(Def.1) $\mathrm{T}(a,b,c) =$ (the operation of F)(a, b, c).

Let us consider F. The functor 0_F yielding a scalar of F is defined as follows:

(Def.2) $0_F =$ the zero of F.

Let us consider F. The functor 1_F yields a scalar of F and is defined by:

(Def.3) $1_F =$ the unity of F.

The ternary operation $\mathrm{T}_{\mathbb{R}}$ on $\mathbb{R}$ is defined as follows:

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(Def.4) for all real numbers a, b, c holds $\mathrm{T}_{\mathbb{R}}(a, b, c) = a \cdot b + c$.

The ternary field structure $\mathbb{R}_{\mathrm{t}}$ is defined by:

(Def.5) $\mathbb{R}_{\mathrm{t}} = \langle \mathbb{R}, 0, 1, \mathrm{T}_{\mathbb{R}} \rangle$.

Let a, b, c be scalars of $\mathbb{R}_{\mathrm{t}}$. The functor $\mathrm{T}^e(a,b,c)$ yields a scalar of $\mathbb{R}_{\mathrm{t}}$ and is defined by:

(Def.6) $\mathrm{T}^e(a,b,c) =$ (the operation of $\mathbb{R}_{\mathrm{t}}$)(a, b, c).

We now state several propositions:

(1) For every scalar a of $\mathbb{R}_{\mathrm{t}}$ holds a is a real number.

(2) For every real number a holds a is a scalar of $\mathbb{R}_{\mathrm{t}}$.

(3) For all real numbers u, u', v, v' such that $u \neq u'$ there exists a real number x such that $u \cdot x + v = u' \cdot x + v'$.

(5)[2] For all scalars u, a, v of $\mathbb{R}_{\mathrm{t}}$ and for all real numbers z, x, y such that $u = z$ and $a = x$ and $v = y$ holds $\mathrm{T}(u,a,v) = z \cdot x + y$.

(6) $0 = 0_{\mathbb{R}_{\mathrm{t}}}$.

(7) $1 = 1_{\mathbb{R}_{\mathrm{t}}}$.

A ternary field structure is called a ternary field if:

(Def.7) (i) $0_{\mathrm{it}} \neq 1_{\mathrm{it}}$,

(ii) for every scalar a of it holds $\mathrm{T}(a, 1_{\mathrm{it}}, 0_{\mathrm{it}}) = a$,

(iii) for every scalar a of it holds $\mathrm{T}(1_{\mathrm{it}}, a, 0_{\mathrm{it}}) = a$,

(iv) for all scalars a, b of it holds $\mathrm{T}(a, 0_{\mathrm{it}}, b) = b$,

(v) for all scalars a, b of it holds $\mathrm{T}(0_{\mathrm{it}}, a, b) = b$,

(vi) for every scalars u, a, b of it there exists a scalar v of it such that $\mathrm{T}(u,a,v) = b$,

(vii) for all scalars u, a, v, v' of it such that $\mathrm{T}(u,a,v) = \mathrm{T}(u,a,v')$ holds $v = v'$,

(viii) for all scalars a, a' of it such that $a \neq a'$ for every scalars b, b' of it there exist scalars u, v of it such that $\mathrm{T}(u,a,v) = b$ and $\mathrm{T}(u,a',v) = b'$,

(ix) for all scalars u, u' of it such that $u \neq u'$ for every scalars v, v' of it there exists a scalar a of it such that $\mathrm{T}(u,a,v) = \mathrm{T}(u',a,v')$,

(x) for all scalars a, a', u, u', v, v' of it such that $\mathrm{T}(u,a,v) = \mathrm{T}(u',a,v')$ and $\mathrm{T}(u,a',v) = \mathrm{T}(u',a',v')$ holds $a = a'$ or $u = u'$.

We adopt the following convention: F is a ternary field and a, a', b, c, x, x', u, u', v, v' are scalars of F. We now state several propositions:

(8) If $a \neq a'$ and $\mathrm{T}(u,a,v) = \mathrm{T}(u',a,v')$ and $\mathrm{T}(u,a',v) = \mathrm{T}(u',a',v')$, then $u = u'$ and $v = v'$.

(9) For every a, b, c there exists x such that $\mathrm{T}(a,b,x) = c$.

(10) If $\mathrm{T}(a,b,x) = \mathrm{T}(a,b,x')$, then $x = x'$.

(11) If $a \neq 0_F$, then for every b, c there exists x such that $\mathrm{T}(a,x,b) = c$.

(12) If $a \neq 0_F$ and $\mathrm{T}(a,x,b) = \mathrm{T}(a,x',b)$, then $x = x'$.

(13) If $a \neq 0_F$, then for every b, c there exists x such that $\mathrm{T}(x,a,b) = c$.

[2]The proposition (4) was either repeated or obvious.

(14) If $a \neq 0_F$ and $\mathrm{T}(x, a, b) = \mathrm{T}(x', a, b)$, then $x = x'$.

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