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Three-Argument Operations and Four-Argument Operations [1]

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Summary. The article contains the definition of three- and four-argument operations. The article is also introduces a few operation related schemes: *FuncEx3D*, *TriOpEx*, *Lambda3D*, *TriOpLambda*, *FuncEx4D*, *QuaOpEx*, *Lambda4D*, *QuaOpLambda*.

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The terminology and notation used in this paper have been introduced in the following articles: [4], [1], [2], [5], and [3]. Let f be a function, and let a, b, c be arbitrary. The functor $f(a, b, c)$ is defined by:

(Def.1) $f(a, b, c) = f(\langle a, b, c \rangle)$.

We now state the proposition

(1) For every function f and for arbitrary a, b, c holds $f(a, b, c) = f(\langle a, b, c \rangle)$.

For simplicity we adopt the following rules: A, B, C, D are non-empty sets, a is an element of A, b is an element of B, and c is an element of C. Let us consider A, B, C, D, and let f be a function from $[: A, B, C :]$ into D, and let us consider a, b, c. Then $f(a, b, c)$ is an element of D.

We adopt the following rules: X, Y, Z denote sets, T denotes a non-empty set, and x, y, z are arbitrary. One can prove the following propositions:

(2) For all functions f_1, f_2 from $[: X, Y, Z :]$ into T such that $T \neq \emptyset$ and for all x, y, z such that $x \in X$ and $y \in Y$ and $z \in Z$ holds $f_1(\langle x, y, z \rangle) = f_2(\langle x, y, z \rangle)$ holds $f_1 = f_2$.

(3) For all functions f_1, f_2 from $[: A, B, C :]$ into D such that for all a, b, c holds $f_1(\langle a, b, c \rangle) = f_2(\langle a, b, c \rangle)$ holds $f_1 = f_2$.

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(4) For all functions f_1, f_2 from $[: A, B, C :]$ into D such that for every element a of A and for every element b of B and for every element c of C holds $f_1(a, b, c) = f_2(a, b, c)$ holds $f_1 = f_2$.

Let us consider A. A ternary operation on A is a function from $[: A, A, A :]$ into A.

In this article we present several logical schemes. The scheme *FuncEx3D* concerns a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a non-empty set $\mathcal{C}$, a non-empty set $\mathcal{D}$, and a 4-ary predicate $\mathcal{P}$, and states that:

there exists a function f from $[: \mathcal{A}, \mathcal{B}, \mathcal{C} :]$ into $\mathcal{D}$ such that for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ and for every element z of $\mathcal{C}$ holds $\mathcal{P}[x, y, z, f(\langle x, y, z \rangle)]$

provided the following requirements are met:

- for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ and for every element z of $\mathcal{C}$ there exists an element t of $\mathcal{D}$ such that $\mathcal{P}[x, y, z, t]$,
- for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ and for every element z of $\mathcal{C}$ and for all elements t_1, t_2 of $\mathcal{D}$ such that $\mathcal{P}[x, y, z, t_1]$ and $\mathcal{P}[x, y, z, t_2]$ holds $t_1 = t_2$.

The scheme *TriOpEx* concerns a non-empty set $\mathcal{A}$, and a 4-ary predicate $\mathcal{P}$, and states that:

there exists a ternary operation o on $\mathcal{A}$ such that for all elements a, b, c of $\mathcal{A}$ holds $\mathcal{P}[a, b, c, o(a, b, c)]$

provided the parameters meet the following requirements:

- for every elements x, y, z of $\mathcal{A}$ there exists an element t of $\mathcal{A}$ such that $\mathcal{P}[x, y, z, t]$,
- for all elements x, y, z of $\mathcal{A}$ and for all elements t_1, t_2 of $\mathcal{A}$ such that $\mathcal{P}[x, y, z, t_1]$ and $\mathcal{P}[x, y, z, t_2]$ holds $t_1 = t_2$.

The scheme *Lambda3D* concerns a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a non-empty set $\mathcal{C}$, a non-empty set $\mathcal{D}$, and a ternary functor $\mathcal{F}$ yielding an element of $\mathcal{D}$ and states that:

there exists a function f from $[: \mathcal{A}, \mathcal{B}, \mathcal{C} :]$ into $\mathcal{D}$ such that for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ and for every element z of $\mathcal{C}$ holds $f(\langle x, y, z \rangle) = \mathcal{F}(x, y, z)$

for all values of the parameters.

The scheme *TriOpLambda* concerns a non-empty set $\mathcal{A}$ and a ternary functor $\mathcal{F}$ yielding an element of $\mathcal{A}$ and states that:

there exists a ternary operation o on $\mathcal{A}$ such that for all elements a, b, c of $\mathcal{A}$ holds $o(a, b, c) = \mathcal{F}(a, b, c)$

for all values of the parameters.

Let f be a function, and let a, b, c, d be arbitrary. The functor $f(a, b, c, d)$ is defined as follows:

(Def.2) $f(a, b, c, d) = f(\langle a, b, c, d \rangle)$.

One can prove the following proposition

(5) For every function f and for arbitrary a, b, c, d holds $f(a, b, c, d) = f(\langle a, b, c, d \rangle)$.

For simplicity we adopt the following rules: A, B, C, D, E will be non-empty sets, a will be an element of A, b will be an element of B, c will be an element of C, and d will be an element of D. Let us consider A, B, C, D, E, and let f be a function from $[: A, B, C, D :]$ into E, and let us consider a, b, c, d. Then $f(a, b, c, d)$ is an element of E.

We adopt the following rules: X, Y, Z, S will be sets, T will be a non-empty set, and x, y, z, s will be arbitrary. The following three propositions are true:

(6) Let f_1, f_2 be functions from $[: X, Y, Z, S :]$ into T. Then if $T \neq \emptyset$ and for all x, y, z, s such that $x \in X$ and $y \in Y$ and $z \in Z$ and $s \in S$ holds $f_1(\langle x, y, z, s\rangle) = f_2(\langle x, y, z, s\rangle)$, then $f_1 = f_2$.

(7) For all functions f_1, f_2 from $[: A, B, C, D :]$ into E such that for all a, b, c, d holds $f_1(\langle a, b, c, d\rangle) = f_2(\langle a, b, c, d\rangle)$ holds $f_1 = f_2$.

(8) For all functions f_1, f_2 from $[: A, B, C, D :]$ into E such that for every element a of A and for every element b of B and for every element c of C and for every element d of D holds $f_1(a, b, c, d) = f_2(a, b, c, d)$ holds $f_1 = f_2$.

Let us consider A. A quadrary operation on A is a function from $[: A, A, A, A :]$ into A.

Now we present four schemes. The scheme *FuncEx4D* concerns a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a non-empty set $\mathcal{C}$, a non-empty set $\mathcal{D}$, a non-empty set $\mathcal{E}$, and a 5-ary predicate $\mathcal{P}$, and states that:

there exists a function f from $[: \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D} :]$ into $\mathcal{E}$ such that for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ and for every element z of $\mathcal{C}$ and for every element s of $\mathcal{D}$ holds $\mathcal{P}[x, y, z, s, f(\langle x, y, z, s\rangle)]$

provided the parameters have the following properties:

- for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ and for every element z of $\mathcal{C}$ and for every element s of $\mathcal{D}$ there exists an element t of $\mathcal{E}$ such that $\mathcal{P}[x, y, z, s, t]$,
- for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ and for every element z of $\mathcal{C}$ and for every element s of $\mathcal{D}$ and for all elements t_1, t_2 of $\mathcal{E}$ such that $\mathcal{P}[x, y, z, s, t_1]$ and $\mathcal{P}[x, y, z, s, t_2]$ holds $t_1 = t_2$.

The scheme *QuaOpEx* deals with a non-empty set $\mathcal{A}$, and a 5-ary predicate $\mathcal{P}$, and states that:

there exists a quadrary operation o on $\mathcal{A}$ such that for all elements a, b, c, d of $\mathcal{A}$ holds $\mathcal{P}[a, b, c, d, o(a, b, c, d)]$

provided the parameters meet the following requirements:

- for every elements x, y, z, s of $\mathcal{A}$ there exists an element t of $\mathcal{A}$ such that $\mathcal{P}[x, y, z, s, t]$,
- for all elements x, y, z, s of $\mathcal{A}$ and for all elements t_1, t_2 of $\mathcal{A}$ such that $\mathcal{P}[x, y, z, s, t_1]$ and $\mathcal{P}[x, y, z, s, t_2]$ holds $t_1 = t_2$.

The scheme *Lambda4D* concerns a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a non-empty set $\mathcal{C}$, a non-empty set $\mathcal{D}$, a non-empty set $\mathcal{E}$, and a 4-ary functor $\mathcal{F}$ yielding an element of $\mathcal{E}$ and states that:

there exists a function f from $[: \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D} :]$ into $\mathcal{E}$ such that for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ and for every element z of $\mathcal{C}$ and for every element s of $\mathcal{D}$ holds $f(\langle x, y, z, s \rangle) = \mathcal{F}(x, y, z, s)$
for all values of the parameters.

The scheme *QuaOpLambda* deals with a non-empty set $\mathcal{A}$ and a 4-ary functor $\mathcal{F}$ yielding an element of $\mathcal{A}$ and states that:

there exists a quadrary operation o on $\mathcal{A}$ such that for all elements a, b, c, d of $\mathcal{A}$ holds $o(a, b, c, d) = \mathcal{F}(a, b, c, d)$
for all values of the parameters.

References

[1] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.
[2] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.
[3] Andrzej Trybulec. Domains and their Cartesian products. *Formalized Mathematics*, 1(**1**):115–122, 1990.
[4] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[5] Andrzej Trybulec. Tuples, projections and Cartesian products. *Formalized Mathematics*, 1(**1**):97–105, 1990.

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