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Elementary Variants of Affine Configurational Theorems [1]

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Summary. We present elementary versions of Pappus, Major Desargues and Minor Desargues Axioms (i.e. statements formulated entirely in the language of points and parallelism of segments). Evidently they are consequences of appropriate configurational axioms introduced in the article [2]. In particular it follows that there exists an affine plane satisfying all of them.

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The terminology and notation used in this paper have been introduced in the following papers: [1], [3], [2], and [4]. In the sequel S_1 will be an affine plane. The following propositions are true:

(1) If S_1 satisfies **PAP**, then for all elements a_1, a_2, a_3, b_1, b_2, b_3 of the points of S_1 such that $a_1, a_2 \parallel a_1, a_3$ and $b_1, b_2 \parallel b_1, b_3$ and $a_1, b_2 \parallel a_2, b_1$ and $a_2, b_3 \parallel a_3, b_2$ holds $a_3, b_1 \parallel a_1, b_3$.

(2) Suppose S_1 satisfies **DES**. Let o, a, a', b, b', c, c' be elements of the points of S_1. Then if $o, a \nparallel o, b$ and $o, a \nparallel o, c$ and $o, a \parallel o, a'$ and $o, b \parallel o, b'$ and $o, c \parallel o, c'$ and $a, b \parallel a', b'$ and $a, c \parallel a', c'$, then $b, c \parallel b', c'$.

(3) Suppose S_1 satisfies **des**. Let a, a', b, b', c, c' be elements of the points of S_1. Then if $a, a' \nparallel a, b$ and $a, a' \nparallel a, c$ and $a, a' \parallel b, b'$ and $a, a' \parallel c, c'$ and $a, b \parallel a', b'$ and $a, c \parallel a', c'$, then $b, c \parallel b', c'$.

(4) If S_1 satisfies Fano Axiom, then for all elements a, b, c, d of the points of S_1 such that $a, b \nparallel a, c$ and $a, b \parallel c, d$ and $a, c \parallel b, d$ holds $a, d \nparallel b, c$.

(5) There exists S_1 such that for all elements o, a, a', b, b', c, c' of the points of S_1 such that $o, a \nparallel o, b$ and $o, a \nparallel o, c$ and $o, a \parallel o, a'$ and $o, b \parallel o, b'$ and $o, c \parallel o, c'$ and $a, b \parallel a', b'$ and $a, c \parallel a', c'$ holds $b, c \parallel b', c'$ and for

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all elements a, a', b, b', c, c' of the points of S_1 such that $a, a' \nparallel a, b$ and $a, a' \nparallel a, c$ and $a, a' \parallel b, b'$ and $a, a' \parallel c, c'$ and $a, b \parallel a', b'$ and $a, c \parallel a', c'$ holds $b, c \parallel b', c'$ and for all elements a_1, a_2, a_3, b_1, b_2, b_3 of the points of S_1 such that $a_1, a_2 \parallel a_1, a_3$ and $b_1, b_2 \parallel b_1, b_3$ and $a_1, b_2 \parallel a_2, b_1$ and $a_2, b_3 \parallel a_3, b_2$ holds $a_3, b_1 \parallel a_1, b_3$ and for all elements a, b, c, d of the points of S_1 such that $a, b \nparallel a, c$ and $a, b \parallel c, d$ and $a, c \parallel b, d$ holds $a, d \nparallel b, c$.

(6) For every elements o, a of the points of S_1 there exists an element p of the points of S_1 such that for all elements b, c of the points of S_1 holds $o, a \parallel o, p$ and there exists an element d of the points of S_1 such that if $o, p \parallel o, b$, then $o, c \parallel o, d$ and $p, c \parallel b, d$.

References

[1] Henryk Oryszczyszyn and Krzysztof Prażmowski. Analytical ordered affine spaces. *Formalized Mathematics*, 1(**3**):601–605, 1990.
[2] Henryk Oryszczyszyn and Krzysztof Prażmowski. Classical configurations in affine planes. *Formalized Mathematics*, 1(**4**):625–633, 1990.
[3] Henryk Oryszczyszyn and Krzysztof Prażmowski. Parallelity and lines in affine spaces. *Formalized Mathematics*, 1(**3**):617–621, 1990.
[4] Krzysztof Prażmowski. Fanoian, Pappian and Desarguesian affine spaces. *Formalized Mathematics*, 2(**3**):341–346, 1991.

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