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Heine–Borel's Covering Theorem

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Summary. Heine–Borel's covering theorem, also known as Borel–Lebesgue theorem [3], is proved. Some useful theorems on real inequalities, intervals, sequences and notion of power sequence which are necessary for the theorem are also proved.

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The terminology and notation used in this paper have been introduced in the following articles: [23], [11], [1], [5], [6], [12], [9], [4], [24], [18], [19], [8], [7], [2], [20], [16], [13], [15], [14], [21], [22], [17], and [10]. We follow a convention: a, b, x, y, z denote real numbers and k, n denote natural numbers. We now state several propositions:

(1) For every subspace A of the metric space of real numbers and for all points p, q of A and for all x, y such that $x = p$ and $y = q$ holds $\rho(p,q) = |x - y|$.

(2) If $x \leq y$ and $y \leq z$, then $[x,y] \cup [y,z] = [x,z]$.

(3) If $x \geq 0$ and $a + x \leq b$, then $a \leq b$.

(4) If $x \geq 0$ and $a - x \geq b$, then $a \geq b$.

(5) If $x > 0$, then $x^k > 0$.

In the sequel s_1 will be a sequence of real numbers. Next we state the proposition

(6) If s_1 is increasing and $\operatorname{rng} s_1 \subseteq \mathbb{N}$, then $n \leq s_1(n)$.

Let us consider s_1, k. The functor k^{s_1} yielding a sequence of real numbers is defined by:

(Def.1) for every n holds $k^{s_1}(n) = k^{s_1(n)}$.

We now state several propositions:

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(7) $2^n \geq n + 1$.

(8) $2^n > n$.

(9) If s_1 is divergent to $+\infty$, then 2^{s_1} is divergent to $+\infty$.

(10) For every topological space T such that the carrier of T is finite holds T is compact.

(11) If $a \leq b$, then $[a, b]_{\rm T}$ is compact.

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