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The Topological Space $\mathcal{E}^2_{\rm T}$. Simple Closed Curves

Agata Darmochwał
Warsaw University

Yatsuka Nakamura
Shinshu University

Summary. Continuation of [13]. The fact that the unit square is compact is shown in the beginning of the article. Next the notion of simple closed curve is introduced. It is proved that any simple closed curve can be divided into two independent parts which are homeomorphic to unit interval $\mathbb{I}$.

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The notation and terminology used here have been introduced in the following articles: [22], [21], [14], [1], [24], [20], [6], [7], [18], [4], [8], [23], [17], [25], [11], [16], [9], [19], [2], [5], [15], [3], [10], [12], and [13]. We follow the rules: p_1, p_2, q_1, q_2 will denote points of $\mathcal{E}^2_{\rm T}$ and P, Q, P_1, P_2 will denote subsets of $\mathcal{E}^2_{\rm T}$. The following propositions are true:

(1) If $p_1 \neq p_2$ and $p_1 \in \square_{\mathcal{E}^2}$ and $p_2 \in \square_{\mathcal{E}^2}$, then there exist P_1, P_2 such that P_1 is an arc from p_1 to p_2 and P_2 is an arc from p_1 to p_2 and $\square_{\mathcal{E}^2} = P_1 \cup P_2$ and $P_1 \cap P_2 = \{p_1, p_2\}$.

(2) $\square_{\mathcal{E}^2}$ is compact.

(3) For every map f from $(\mathcal{E}^2_{\rm T}) \restriction Q$ into $(\mathcal{E}^2_{\rm T}) \restriction P$ such that f is a homeomorphism and Q is an arc from q_1 to q_2 and $P \neq \emptyset$ and for all p_1, p_2 such that $p_1 = f(q_1)$ and $p_2 = f(q_2)$ holds P is an arc from p_1 to p_2.

Let us consider P. We say that P is a simple closed curve if and only if:

(Def.1) $P \neq \emptyset$ and there exists a map f from $(\mathcal{E}^2_{\rm T}) \restriction \square_{\mathcal{E}^2}$ into $(\mathcal{E}^2_{\rm T}) \restriction P$ such that f is a homeomorphism.

Next we state two propositions:

(4) If P is a simple closed curve, then there exist p_1, p_2 such that $p_1 \neq p_2$ and $p_1 \in P$ and $p_2 \in P$.

(5) P is a simple closed curve if and only if the following conditions are satisfied:

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(i) there exist p_1, p_2 such that $p_1 \neq p_2$ and $p_1 \in P$ and $p_2 \in P$,
(ii) for all p_1, p_2 such that $p_1 \neq p_2$ and $p_1 \in P$ and $p_2 \in P$ there exist P_1, P_2 such that P_1 is an arc from p_1 to p_2 and P_2 is an arc from p_1 to p_2 and $P = P_1 \cup P_2$ and $P_1 \cap P_2 = \{p_1, p_2\}$.

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