FORMALIZED MATHEMATICS
Volume 5, Number **1**, 1996
Warsaw University - Białystok

Solvable Groups

Katarzyna Zawadzka
Warsaw University
Białystok

Summary. The concept of solvable group is introduced. Some theorems concerning heirdom of solvability are proved.

MML Identifier: `GRSOLV_1`.

The articles [7], [13], [3], [4], [11], [6], [5], [2], [1], [9], [10], [8], and [12] provide the terminology and notation for this paper.

In this paper G denotes a group and i denotes a natural number.

A group is solvable if it satisfies the condition (Def.1).

(Def.1) There exists a finite sequence F of elements of SubGr it such that

(i) $\operatorname{len} F > 0$,

(ii) $F(1) = \Omega_{\rm it}$,

(iii) $F(\operatorname{len} F) = \{\mathbf{1}\}_{\rm it}$, and

(iv) for every i such that $i \in \operatorname{dom} F$ and $i+1 \in \operatorname{dom} F$ and for all strict subgroups G_1, G_2 of it such that $G_1 = F(i)$ and $G_2 = F(i+1)$ holds G_2 is a strict normal subgroup of G_1 and for every normal subgroup N of G_1 such that $N = G_2$ holds ${}^{G_1}/_{N}$ is commutative.

One can check that there exists a group which is solvable and strict.

One can prove the following propositions:

(1) Let G be a strict group and let H, F_1, F_2 be strict subgroups of G. Suppose F_1 is a normal subgroup of F_2. Then $F_1 \cap H$ is a normal subgroup of $F_2 \cap H$.

(2) Let G be a strict group, and let F_2 be a strict subgroup of G, and let F_1 be a strict normal subgroup of F_2, and let a, b be elements of F_2. Then $a \cdot F_1 \cdot (b \cdot F_1) = (a \cdot b) \cdot F_1$.

(3) Let G be a strict group, and let H, F_2 be strict subgroups of G, and let F_1 be a strict normal subgroup of F_2, and let G_2 be a strict subgroup of G. Suppose $G_2 = H \cap F_2$. Let G_1 be a normal subgroup of G_2. Suppose

ISSN 0777–4028

$G_1 = H \cap F_1$. Then there exists a subgroup G_3 of ${}^{F_2}/_{F_1}$ such that ${}^{G_2}/_{G_1}$ and G_3 are isomorphic.

(4) Let G be a strict group, and let H, F_2 be strict subgroups of G, and let F_1 be a strict normal subgroup of F_2, and let G_2 be a strict subgroup of G. Suppose $G_2 = F_2 \cap H$. Let G_1 be a normal subgroup of G_2. Suppose $G_1 = F_1 \cap H$. Then there exists a subgroup G_3 of ${}^{F_2}/_{F_1}$ such that ${}^{G_2}/_{G_1}$ and G_3 are isomorphic.

(5) For every solvable strict group G holds every strict subgroup of G is solvable.

(6) Let G be a strict group. Given a finite sequence F of elements of $\mathrm{SubGr}\, G$ such that

(i) $\mathrm{len}\, F > 0$,

(ii) $F(1) = \Omega_G$,

(iii) $F(\mathrm{len}\, F) = \{\mathbf{1}\}_G$, and

(iv) for every i such that $i \in \mathrm{dom}\, F$ and $i + 1 \in \mathrm{dom}\, F$ and for all strict subgroups G_1, G_2 of G such that $G_1 = F(i)$ and $G_2 = F(i + 1)$ holds G_2 is a strict normal subgroup of G_1 and for every normal subgroup N of G_1 such that $N = G_2$ holds ${}^{G_1}/_{N}$ is a cyclic group.

Then G is solvable.

(7) Every strict commutative group is strict and solvable.

Let G, H be strict groups, let g be a homomorphism from G to H, and let A be a subgroup of G. The functor $g \upharpoonright A$ yielding a homomorphism from A to H is defined as follows:

(Def.2) $g \upharpoonright A = g \upharpoonright$ (the carrier of A).

Let G, H be strict groups, let g be a homomorphism from G to H, and let A be a subgroup of G. The functor $g^\circ A$ yields a strict subgroup of H and is defined as follows:

(Def.3) $g^\circ A = \mathrm{Im}(g \upharpoonright A)$.

Next we state a number of propositions:

(8) Let G, H be strict groups, and let g be a homomorphism from G to H, and let A be a subgroup of G. Then $\mathrm{rng}(g \upharpoonright A) = g^\circ$(the carrier of A).

(9) Let G, H be strict groups, and let g be a homomorphism from G to H, and let A be a strict subgroup of G. Then the carrier of $g^\circ A = g^\circ$(the carrier of A).

(10) Let G, H be strict groups, and let h be a homomorphism from G to H, and let A be a strict subgroup of G. Then $\mathrm{Im}(h \upharpoonright A)$ is a strict subgroup of $\mathrm{Im}\, h$.

(11) Let G, H be strict groups, and let h be a homomorphism from G to H, and let A be a strict subgroup of G. Then $h^\circ A$ is a strict subgroup of $\mathrm{Im}\, h$.

(12) For all strict groups G, H and for every homomorphism h from G to H holds $h^\circ(\{\mathbf{1}\}_G) = \{\mathbf{1}\}_H$ and $h^\circ(\Omega_G) = \Omega_{\mathrm{Im}\, h}$.

(13) Let G, H be strict groups, and let h be a homomorphism from G to H, and let A, B be strict subgroups of G. If A is a subgroup of B, then $h^\circ A$ is a subgroup of $h^\circ B$.

(14) Let G, H be strict groups, and let h be a homomorphism from G to H, and let A be a strict subgroup of G, and let a be an element of G. Then $h(a) \cdot h^\circ A = h^\circ(a \cdot A)$ and $h^\circ A \cdot h(a) = h^\circ(A \cdot a)$.

(15) Let G, H be strict groups, and let h be a homomorphism from G to H, and let A, B be subsets of G. Then $h^\circ A \cdot h^\circ B = h^\circ(A \cdot B)$.

(16) Let G, H be strict groups, and let h be a homomorphism from G to H, and let A, B be strict subgroups of G. Suppose A is a strict normal subgroup of B. Then $h^\circ A$ is a strict normal subgroup of $h^\circ B$.

(17) Let G, H be strict groups and let h be a homomorphism from G to H. If G is a solvable group, then $\mathrm{Im}\, h$ is solvable.

References

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.

[2] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.

[3] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[4] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.

[5] Eugeniusz Kusak, Wojciech Leończuk, and Michał Muzalewski. Abelian groups, fields and vector spaces. *Formalized Mathematics*, 1(**2**):335–342, 1990.

[6] Dariusz Surowik. Cyclic groups and some of their properties - part I. *Formalized Mathematics*, 2(**5**):623–627, 1991.

[7] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.

[8] Wojciech A. Trybulec. Classes of conjugation. Normal subgroups. *Formalized Mathematics*, 1(**5**):955–962, 1990.

[9] Wojciech A. Trybulec. Groups. *Formalized Mathematics*, 1(**5**):821–827, 1990.

[10] Wojciech A. Trybulec. Subgroup and cosets of subgroups. *Formalized Mathematics*, 1(**5**):855–864, 1990.

[11] Wojciech A. Trybulec. Vectors in real linear space. *Formalized Mathematics*, 1(**2**):291–296, 1990.

[12] Wojciech A. Trybulec and Michał J. Trybulec. Homomorphisms and isomorphisms of groups. Quotient group. *Formalized Mathematics*, 2(**4**):573–578, 1991.

[13] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.

Received October 23, 1994