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Sequences in $\mathcal{E}^N_{\rm T}$

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The papers [12], [3], [4], [11], [8], [10], [1], [2], [5], [6], [9], and [7] provide the notation and terminology for this paper.

For simplicity we adopt the following rules: f denotes a function, N, n, m denote natural numbers, q, r, r_1, r_2 denote real numbers, x is arbitrary, and w, w_1, w_2, g denote points of $\mathcal{E}^N_{\rm T}$.

Let us consider N. A sequence in $\mathcal{E}^N_{\rm T}$ is a function from $\mathbb{N}$ into the carrier of $\mathcal{E}^N_{\rm T}$.

In the sequel s_1, s_2, s_3, s_4, s'_1 are sequences in $\mathcal{E}^N_{\rm T}$.

Next we state two propositions:

(1) f is a sequence in $\mathcal{E}^N_{\rm T}$ if and only if $\mathrm{dom}\, f = \mathbb{N}$ and for every x such that $x \in \mathbb{N}$ holds $f(x)$ is a point of $\mathcal{E}^N_{\rm T}$.

(2) f is a sequence in $\mathcal{E}^N_{\rm T}$ iff $\mathrm{dom}\, f = \mathbb{N}$ and for every n holds $f(n)$ is a point of $\mathcal{E}^N_{\rm T}$.

Let us consider N, s_1, n. Then $s_1(n)$ is a point of $\mathcal{E}^N_{\rm T}$.

Let us consider N. A sequence in $\mathcal{E}^N_{\rm T}$ is non-zero if:

(Def.1) $\mathrm{rng}\,\mathrm{it} \subseteq (\text{the carrier of } \mathcal{E}^N_{\rm T}) \setminus \{0_{\mathcal{E}^N_{\rm T}}\}$.

We now state several propositions:

(3) s_1 is non-zero iff for every x such that $x \in \mathbb{N}$ holds $s_1(x) \neq 0_{\mathcal{E}^N_{\rm T}}$.

(4) s_1 is non-zero iff for every n holds $s_1(n) \neq 0_{\mathcal{E}^N_{\rm T}}$.

(5) For all N, s_1, s_2 such that for every x such that $x \in \mathbb{N}$ holds $s_1(x) = s_2(x)$ holds $s_1 = s_2$.

(6) For all N, s_1, s_2 such that for every n holds $s_1(n) = s_2(n)$ holds $s_1 = s_2$.

(7) For every point w of $\mathcal{E}^N_{\rm T}$ there exists s_1 such that $\mathrm{rng}\, s_1 = \{w\}$.

The scheme *ExTopRealNSeq* deals with a natural number $\mathcal{A}$ and a unary functor $\mathcal{F}$ yielding a point of $\mathcal{E}^{\mathcal{A}}_{\rm T}$, and states that:

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There exists a sequence s_1 in $\mathcal{E}^{\mathcal{A}}_{\mathrm{T}}$ such that for every n holds $s_1(n) = \mathcal{F}(n)$

for all values of the parameters.

Let us consider N, s_2, s_3. The functor $s_2 + s_3$ yielding a sequence in $\mathcal{E}^{N}_{\mathrm{T}}$ is defined by:

(Def.2) For every n holds $(s_2 + s_3)(n) = s_2(n) + s_3(n)$.

Let us consider r, N, s_1. The functor $r \cdot s_1$ yields a sequence in $\mathcal{E}^{N}_{\mathrm{T}}$ and is defined by:

(Def.3) For every n holds $(r \cdot s_1)(n) = r \cdot s_1(n)$.

Let us consider N, s_1. The functor $-s_1$ yields a sequence in $\mathcal{E}^{N}_{\mathrm{T}}$ and is defined as follows:

(Def.4) For every n holds $(-s_1)(n) = -s_1(n)$.

Let us consider N, s_2, s_3. The functor $s_2 - s_3$ yields a sequence in $\mathcal{E}^{N}_{\mathrm{T}}$ and is defined by:

(Def.5) $s_2 - s_3 = s_2 + -s_3$.

Let us consider N and let x be a point of $\mathcal{E}^{N}_{\mathrm{T}}$. The functor $|x|$ yields a real number and is defined by:

(Def.6) There exists a finite sequence y of elements of $\mathbb{R}$ such that $x = y$ and $|x| = |y|$.

Let us consider N, s_1. The functor $|s_1|$ yielding a sequence of real numbers is defined by:

(Def.7) For every n holds $|s_1|(n) = |s_1(n)|$.

We now state a number of propositions:

(8) $|r| \cdot |w| = |r \cdot w|$.

(9) $|r \cdot s_1| = |r|\,|s_1|$.

(10) $s_2 + s_3 = s_3 + s_2$.

(11) $(s_2 + s_3) + s_4 = s_2 + (s_3 + s_4)$.

(12) $-s_1 = (-1) \cdot s_1$.

(13) $r \cdot (s_2 + s_3) = r \cdot s_2 + r \cdot s_3$.

(14) $(r \cdot q) \cdot s_1 = r \cdot (q \cdot s_1)$.

(15) $r \cdot (s_2 - s_3) = r \cdot s_2 - r \cdot s_3$.

(16) $s_2 - (s_3 + s_4) = s_2 - s_3 - s_4$.

(17) $1 \cdot s_1 = s_1$.

(18) $--s_1 = s_1$.

(19) $s_2 - -s_3 = s_2 + s_3$.

(20) $s_2 - (s_3 - s_4) = (s_2 - s_3) + s_4$.

(21) $s_2 + (s_3 - s_4) = (s_2 + s_3) - s_4$.

(22) If $r \neq 0$ and s_1 is non-zero, then $r \cdot s_1$ is non-zero.

(23) If s_1 is non-zero, then $-s_1$ is non-zero.

(24) $|0_{\mathcal{E}^{N}_{\mathrm{T}}}| = 0$.

(25) If $|w| = 0$, then $w = 0_{\mathcal{E}^N_{\rm T}}$.

(26) $|w| \geq 0$.

(27) $|-w| = |w|$.

(28) $|w_1 - w_2| = |w_2 - w_1|$.

(29) $|w_1 - w_2| = 0$ iff $w_1 = w_2$.

(30) $|w_1 + w_2| \leq |w_1| + |w_2|$.

(31) $|w_1 - w_2| \leq |w_1| + |w_2|$.

(32) $|w_1| - |w_2| \leq |w_1 + w_2|$.

(33) $|w_1| - |w_2| \leq |w_1 - w_2|$.

(34) If $w_1 \neq w_2$, then $|w_1 - w_2| > 0$.

(35) $|w_1 - w_2| \leq |w_1 - w| + |w - w_2|$.

(36) If $0 \leq |w_1|$ and $0 \leq r_1$ and $|w_1| < |w_2|$ and $r_1 < r_2$, then $|w_1| \cdot r_1 < |w_2| \cdot r_2$.

(38)[1] $-|w| < r$ and $r < |w|$ iff $|r| < |w|$.

Let us consider N. A sequence in $\mathcal{E}^N_{\rm T}$ is bounded if:

(Def.8) There exists r such that for every n holds $|{\rm it}(n)| < r$.

The following proposition is true

(39) For every n there exists r such that $0 < r$ and for every m such that $m \leq n$ holds $|s_1(m)| < r$.

Let us consider N. A sequence in $\mathcal{E}^N_{\rm T}$ is convergent if:

(Def.9) There exists g such that for every r such that $0 < r$ there exists n such that for every m such that $n \leq m$ holds $|{\rm it}(m) - g| < r$.

Let us consider N, s_1. Let us assume that s_1 is convergent. The functor $\lim s_1$ yields a point of $\mathcal{E}^N_{\rm T}$ and is defined by:

(Def.10) For every r such that $0 < r$ there exists n such that for every m such that $n \leq m$ holds $|s_1(m) - \lim s_1| < r$.

The following propositions are true:

(40) Suppose s_1 is convergent. Then $\lim s_1 = g$ if and only if for every r such that $0 < r$ there exists n such that for every m such that $n \leq m$ holds $|s_1(m) - g| < r$.

(41) If s_1 is convergent and s'_1 is convergent, then $s_1 + s'_1$ is convergent.

(42) If s_1 is convergent and s'_1 is convergent, then $\lim(s_1 + s'_1) = \lim s_1 + \lim s'_1$.

(43) If s_1 is convergent, then $r \cdot s_1$ is convergent.

(44) If s_1 is convergent, then $\lim(r \cdot s_1) = r \cdot \lim s_1$.

(45) If s_1 is convergent, then $-s_1$ is convergent.

(46) If s_1 is convergent, then $\lim(-s_1) = -\lim s_1$.

(47) If s_1 is convergent and s'_1 is convergent, then $s_1 - s'_1$ is convergent.

[1]The proposition (37) has been removed.

(48) If s_1 is convergent and s'_1 is convergent, then $\lim(s_1 - s'_1) = \lim s_1 - \lim s'_1$.

(50)[2] If s_1 is convergent, then s_1 is bounded.

(51) If s_1 is convergent, then if $\lim s_1 \neq 0_{\mathcal{E}^N_{\rm T}}$, then there exists n such that for every m such that $n \leq m$ holds $\frac{|\lim s_1|}{2} < |s_1(m)|$.

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[2]The proposition (49) has been removed.