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Decomposing a Go-Board into Cells

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The articles [20], [23], [6], [22], [9], [2], [14], [17], [18], [24], [1], [5], [3], [4], [21], [10], [11], [16], [15], [7], [8], [12], [13], and [19] provide the terminology and notation for this paper.

For simplicity we follow a convention: q will be a point of $\mathcal{E}^2_{\rm T}$, i, i_1, i_2, j, j_1, j_2, k will be natural numbers, r, s will be real numbers, and G will be a Go-board.

We now state the proposition

(1) Let M be a tabular finite sequence and given i, j. If $\langle i, j\rangle \in$ the indices of M, then $1 \leq i$ and $i \leq \operatorname{len} M$ and $1 \leq j$ and $j \leq \operatorname{width} M$.

Let us consider G, i. The functor $\operatorname{vstrip}(G,i)$ yielding a subset of the carrier of $\mathcal{E}^2_{\rm T}$ is defined as follows:

(Def.1) (i) $\operatorname{vstrip}(G,i) = \{[r,s] : (G_{i,1})_{\mathbf{1}} \leq r \ \wedge\ r \leq (G_{i+1,1})_{\mathbf{1}}\}$ if $1 \leq i$ and $i < \operatorname{len} G$,

(ii) $\operatorname{vstrip}(G,i) = \{[r,s] : (G_{i,1})_{\mathbf{1}} \leq r\}$ if $i \geq \operatorname{len} G$,

(iii) $\operatorname{vstrip}(G,i) = \{[r,s] : r \leq (G_{i+1,1})_{\mathbf{1}}\}$, otherwise.

The functor $\operatorname{hstrip}(G,i)$ yields a subset of the carrier of $\mathcal{E}^2_{\rm T}$ and is defined by:

(Def.2) (i) $\operatorname{hstrip}(G,i) = \{[r,s] : (G_{1,i})_{\mathbf{2}} \leq s \ \wedge\ s \leq (G_{1,i+1})_{\mathbf{2}}\}$ if $1 \leq i$ and $i < \operatorname{width} G$,

(ii) $\operatorname{hstrip}(G,i) = \{[r,s] : (G_{1,i})_{\mathbf{2}} \leq s\}$ if $i \geq \operatorname{width} G$,

(iii) $\operatorname{hstrip}(G,i) = \{[r,s] : s \leq (G_{1,i+1})_{\mathbf{2}}\}$, otherwise.

We now state a number of propositions:

(2) If $1 \leq j$ and $j \leq \operatorname{width} G$ and $1 \leq i$ and $i \leq \operatorname{len} G$, then $(G_{i,j})_{\mathbf{2}} = (G_{1,j})_{\mathbf{2}}$.

(3) If $1 \leq j$ and $j \leq \operatorname{width} G$ and $1 \leq i$ and $i \leq \operatorname{len} G$, then $(G_{i,j})_{\mathbf{1}} = (G_{i,1})_{\mathbf{1}}$.

(4) If $1 \leq j$ and $j \leq \operatorname{width} G$ and $1 \leq i_1$ and $i_1 < i_2$ and $i_2 \leq \operatorname{len} G$, then $(G_{i_1,j})_{\mathbf{1}} < (G_{i_2,j})_{\mathbf{1}}$.

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(5) If $1 \leq j_1$ and $j_1 < j_2$ and $j_2 \leq \mathrm{width}\, G$ and $1 \leq i$ and $i \leq \mathrm{len}\, G$, then $(G_{i,j_1})_{\mathbf{2}} < (G_{i,j_2})_{\mathbf{2}}$.

(6) If $1 \leq j$ and $j < \mathrm{width}\, G$ and $1 \leq i$ and $i \leq \mathrm{len}\, G$, then $\mathrm{hstrip}(G, j) = \{[r, s] : (G_{i,j})_{\mathbf{2}} \leq s \ \wedge \ s \leq (G_{i,j+1})_{\mathbf{2}}\}$.

(7) If $1 \leq i$ and $i \leq \mathrm{len}\, G$, then $\mathrm{hstrip}(G, \mathrm{width}\, G) = \{[r, s] : (G_{i,\mathrm{width}\, G})_{\mathbf{2}} \leq s\}$.

(8) If $1 \leq i$ and $i \leq \mathrm{len}\, G$, then $\mathrm{hstrip}(G, 0) = \{[r, s] : s \leq (G_{i,1})_{\mathbf{2}}\}$.

(9) If $1 \leq i$ and $i < \mathrm{len}\, G$ and $1 \leq j$ and $j \leq \mathrm{width}\, G$, then $\mathrm{vstrip}(G, i) = \{[r, s] : (G_{i,j})_{\mathbf{1}} \leq r \ \wedge \ r \leq (G_{i+1,j})_{\mathbf{1}}\}$.

(10) If $1 \leq j$ and $j \leq \mathrm{width}\, G$, then $\mathrm{vstrip}(G, \mathrm{len}\, G) = \{[r, s] : (G_{\mathrm{len}\, G,j})_{\mathbf{1}} \leq r\}$.

(11) If $1 \leq j$ and $j \leq \mathrm{width}\, G$, then $\mathrm{vstrip}(G, 0) = \{[r, s] : r \leq (G_{1,j})_{\mathbf{1}}\}$.

Let G be a Go-board and let us consider i, j. The functor $\mathrm{cell}(G, i, j)$ yields a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ and is defined as follows:

(Def.3) $\mathrm{cell}(G, i, j) = \mathrm{vstrip}(G, i) \cap \mathrm{hstrip}(G, j)$.

A finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ is s.c.c. if:

(Def.4) For all i, j such that $i + 1 < j$ but $i > 1$ and $j < \mathrm{len\,it}$ or $j + 1 < \mathrm{len\,it}$ holds $\mathcal{L}(\mathrm{it}, i) \cap \mathcal{L}(\mathrm{it}, j) = \emptyset$.

A non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ is standard if:

(Def.5) It is a sequence which elements belong to the Go-board of it.

One can verify that there exists a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ which is non constant special unfolded circular s.c.c. and standard.

We now state two propositions:

(12) Let f be a standard non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $k \in \mathrm{dom}\, f$. Then there exist i, j such that $\langle i, j \rangle \in$ the indices of the Go-board of f and $\pi_k f = (\text{the Go-board of } f)_{i,j}$.

(13) Let f be a standard non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ and let n be a natural number. Suppose $n \in \mathrm{dom}\, f$ and $n + 1 \in \mathrm{dom}\, f$. Let m, k, i, j be natural numbers. Suppose that

(i) $\langle m, k \rangle \in$ the indices of the Go-board of f,

(ii) $\langle i, j \rangle \in$ the indices of the Go-board of f,

(iii) $\pi_n f = (\text{the Go-board of } f)_{m,k}$, and

(iv) $\pi_{n+1} f = (\text{the Go-board of } f)_{i,j}$.

Then $|m - i| + |k - j| = 1$.

A special circular sequence is a special unfolded circular s.c.c. non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$.

In the sequel f is a standard special circular sequence.

Let us consider f, k. Let us assume that $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$. The functor $\mathrm{rightcell}(f, k)$ yielding a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ is defined by the condition (Def.6).

(Def.6) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose that

(i) $\langle i_1, j_1 \rangle \in$ the indices of the Go-board of f,

(ii) $\langle i_2, j_2\rangle \in$ the indices of the Go-board of f,
(iii) $\pi_k f =$ (the Go-board of $f)_{i_1,j_1}$, and
(iv) $\pi_{k+1} f =$ (the Go-board of $f)_{i_2,j_2}$.
Then
(v) $i_1 = i_2$ and $j_1 + 1 = j_2$ and rightcell$(f, k) =$ cell(the Go-board of f, i_1, j_1), or
(vi) $i_1 + 1 = i_2$ and $j_1 = j_2$ and rightcell$(f, k) =$ cell(the Go-board of f, $i_1, j_1 -' 1$), or
(vii) $i_1 = i_2 + 1$ and $j_1 = j_2$ and rightcell$(f, k) =$ cell(the Go-board of f, i_2, j_2), or
(viii) $i_1 = i_2$ and $j_1 = j_2 + 1$ and rightcell$(f, k) =$ cell(the Go-board of f, $i_1 -' 1, j_2$).

The functor leftcell(f, k) yielding a subset of the carrier of $\mathcal{E}^2_{\rm T}$ is defined by the condition (Def.7).

(Def.7) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose that
(i) $\langle i_1, j_1\rangle \in$ the indices of the Go-board of f,
(ii) $\langle i_2, j_2\rangle \in$ the indices of the Go-board of f,
(iii) $\pi_k f =$ (the Go-board of $f)_{i_1,j_1}$, and
(iv) $\pi_{k+1} f =$ (the Go-board of $f)_{i_2,j_2}$.
Then
(v) $i_1 = i_2$ and $j_1 + 1 = j_2$ and leftcell$(f, k) =$ cell(the Go-board of f, $i_1 -' 1, j_1$), or
(vi) $i_1 + 1 = i_2$ and $j_1 = j_2$ and leftcell$(f, k) =$ cell(the Go-board of f, i_1, j_1), or
(vii) $i_1 = i_2 + 1$ and $j_1 = j_2$ and leftcell$(f, k) =$ cell(the Go-board of f, $i_2, j_2 -' 1$), or
(viii) $i_1 = i_2$ and $j_1 = j_2 + 1$ and leftcell$(f, k) =$ cell(the Go-board of f, i_1, j_2).

Next we state a number of propositions:

(14) If $i < \operatorname{len} G$ and $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(G_{i+1,j}, G_{i+1,j+1}) \subseteq$ vstrip(G, i).

(15) If $1 \leq i$ and $i \leq \operatorname{len} G$ and $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(G_{i,j}, G_{i,j+1}) \subseteq$ vstrip(G, i).

(16) If $j < \operatorname{width} G$ and $1 \leq i$ and $i < \operatorname{len} G$, then $\mathcal{L}(G_{i,j+1}, G_{i+1,j+1}) \subseteq$ hstrip(G, j).

(17) If $1 \leq j$ and $j \leq \operatorname{width} G$ and $1 \leq i$ and $i < \operatorname{len} G$, then $\mathcal{L}(G_{i,j}, G_{i+1,j}) \subseteq$ hstrip(G, j).

(18) If $1 \leq i$ and $i \leq \operatorname{len} G$ and $1 \leq j$ and $j + 1 \leq \operatorname{width} G$, then $\mathcal{L}(G_{i,j}, G_{i,j+1}) \subseteq$ hstrip(G, j).

(19) If $i < \operatorname{len} G$ and $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(G_{i+1,j}, G_{i+1,j+1}) \subseteq$ cell(G, i, j).

(20) If $1 \leq i$ and $i \leq \operatorname{len} G$ and $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(G_{i,j}, G_{i,j+1}) \subseteq$ cell(G, i, j).

(21) If $1 \leq j$ and $j \leq \operatorname{width} G$ and $1 \leq i$ and $i+1 \leq \operatorname{len} G$, then $\mathcal{L}(G_{i,j}, G_{i+1,j}) \subseteq \operatorname{vstrip}(G, i)$.

(22) If $j < \operatorname{width} G$ and $1 \leq i$ and $i < \operatorname{len} G$, then $\mathcal{L}(G_{i,j+1}, G_{i+1,j+1}) \subseteq \operatorname{cell}(G, i, j)$.

(23) If $1 \leq i$ and $i < \operatorname{len} G$ and $1 \leq j$ and $j \leq \operatorname{width} G$, then $\mathcal{L}(G_{i,j}, G_{i+1,j}) \subseteq \operatorname{cell}(G, i, j)$.

(24) If $i+1 \leq \operatorname{len} G$, then $\operatorname{vstrip}(G, i) \cap \operatorname{vstrip}(G, i+1) = \{q : q_{\mathbf{1}} = (G_{i+1,1})_{\mathbf{1}}\}$.

(25) If $j+1 \leq \operatorname{width} G$, then $\operatorname{hstrip}(G, j) \cap \operatorname{hstrip}(G, j+1) = \{q : q_{\mathbf{2}} = (G_{1,j+1})_{\mathbf{2}}\}$.

(26) For every Go-board G such that $i < \operatorname{len} G$ and $1 \leq j$ and $j < \operatorname{width} G$ holds $\operatorname{cell}(G, i, j) \cap \operatorname{cell}(G, i+1, j) = \mathcal{L}(G_{i+1,j}, G_{i+1,j+1})$.

(27) For every Go-board G such that $j < \operatorname{width} G$ and $1 \leq i$ and $i < \operatorname{len} G$ holds $\operatorname{cell}(G, i, j) \cap \operatorname{cell}(G, i, j+1) = \mathcal{L}(G_{i,j+1}, G_{i+1,j+1})$.

(28) Suppose that
- (i) $1 \leq k$,
- (ii) $k+1 \leq \operatorname{len} f$,
- (iii) $\langle i+1, j \rangle \in$ the indices of the Go-board of f,
- (iv) $\langle i+1, j+1 \rangle \in$ the indices of the Go-board of f,
- (v) $\pi_k f =$ (the Go-board of $f)_{i+1,j}$, and
- (vi) $\pi_{k+1} f =$ (the Go-board of $f)_{i+1,j+1}$.

Then $\operatorname{leftcell}(f, k) = \operatorname{cell}($the Go-board of f, $i, j)$ and $\operatorname{rightcell}(f, k) = \operatorname{cell}($the Go-board of f, $i+1, j)$.

(29) Suppose that
- (i) $1 \leq k$,
- (ii) $k+1 \leq \operatorname{len} f$,
- (iii) $\langle i, j+1 \rangle \in$ the indices of the Go-board of f,
- (iv) $\langle i+1, j+1 \rangle \in$ the indices of the Go-board of f,
- (v) $\pi_k f =$ (the Go-board of $f)_{i,j+1}$, and
- (vi) $\pi_{k+1} f =$ (the Go-board of $f)_{i+1,j+1}$.

Then $\operatorname{leftcell}(f, k) = \operatorname{cell}($the Go-board of f, $i, j+1)$ and $\operatorname{rightcell}(f, k) = \operatorname{cell}($the Go-board of f, $i, j)$.

(30) Suppose that
- (i) $1 \leq k$,
- (ii) $k+1 \leq \operatorname{len} f$,
- (iii) $\langle i, j+1 \rangle \in$ the indices of the Go-board of f,
- (iv) $\langle i+1, j+1 \rangle \in$ the indices of the Go-board of f,
- (v) $\pi_k f =$ (the Go-board of $f)_{i+1,j+1}$, and
- (vi) $\pi_{k+1} f =$ (the Go-board of $f)_{i,j+1}$.

Then $\operatorname{leftcell}(f, k) = \operatorname{cell}($the Go-board of f, $i, j)$ and $\operatorname{rightcell}(f, k) = \operatorname{cell}($the Go-board of f, $i, j+1)$.

(31) Suppose that
- (i) $1 \leq k$,
- (ii) $k+1 \leq \operatorname{len} f$,

(iii) $\langle i+1, j+1\rangle \in$ the indices of the Go-board of f,
(iv) $\langle i+1, j\rangle \in$ the indices of the Go-board of f,
(v) $\pi_k f =$ (the Go-board of $f)_{i+1,j+1}$, and
(vi) $\pi_{k+1} f =$ (the Go-board of $f)_{i+1,j}$.

Then leftcell$(f, k) =$ cell(the Go-board of f, $i+1, j$) and rightcell$(f, k) =$ cell(the Go-board of f, i, j).

(32) If $1 \leq k$ and $k+1 \leq \operatorname{len} f$, then leftcell$(f, k) \cap$ rightcell$(f, k) = \mathcal{L}(f, k)$.

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