FORMALIZED MATHEMATICS
Volume 5, Number 3, 1996
Warsaw University - Białystok

More on Segments on a Go-Board

Andrzej Trybulec
Warsaw University
Białystok

Summary. We continue the preparatory work for the Jordan Curve Theorem.

MML Identifier: GOBOARD8.

The terminology and notation used here are introduced in the following articles: [20], [23], [22], [8], [2], [18], [16], [1], [4], [3], [6], [21], [9], [10], [17], [24], [5], [7], [11], [12], [14], [19], [15], and [13].

We adopt the following rules: i, j, k will be natural numbers, p will be a point of $\mathcal{E}^2_{\rm T}$, and f will be a non constant standard special circular sequence.

One can prove the following propositions:

(1) Given k. Suppose $1 \leq k$ and $k+2 \leq \mathrm{len}\, f$. Given i, j. Suppose that
  (i) $1 \leq i$,
  (ii) $i+1 \leq \mathrm{len}$ the Go-board of f,
  (iii) $1 \leq j$,
  (iv) $j+2 \leq \mathrm{width}$ the Go-board of f,
  (v) $\pi_{k+1} f =$ (the Go-board of $f)_{i+1,j+1}$, and
  (vi) $\pi_k f =$ (the Go-board of $f)_{i+1,j}$ and $\pi_{k+2} f =$ (the Go-board of $f)_{i+1,j+2}$ or $\pi_{k+2} f =$ (the Go-board of $f)_{i+1,j}$ and $\pi_k f =$ (the Go-board of $f)_{i+1,j+2}$.

  Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f)_{i,j} +$ (the Go-board of $f)_{i+1,j+1}), \frac{1}{2} \cdot (($the Go-board of $f)_{i,j+1} +$ (the Go-board of $f)_{i+1,j+2}))$ misses $\widetilde{\mathcal{L}}(f)$.

(2) Given k. Suppose $1 \leq k$ and $k+2 \leq \mathrm{len}\, f$. Given i, j. Suppose that
  (i) $1 \leq i$,
  (ii) $i+2 \leq \mathrm{len}$ the Go-board of f,
  (iii) $1 \leq j$,
  (iv) $j+2 \leq \mathrm{width}$ the Go-board of f,
  (v) $\pi_{k+1} f =$ (the Go-board of $f)_{i+1,j+1}$, and

ISSN 1426–2630

(vi) $\pi_k f$ = (the Go-board of $f)_{i+2,j+1}$ and $\pi_{k+2} f$ = (the Go-board of $f)_{i+1,j+2}$ or $\pi_{k+2} f$ = (the Go-board of $f)_{i+2,j+1}$ and $\pi_k f$ = (the Go-board of $f)_{i+1,j+2}$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f)_{i,j} + ($the Go-board of $f)_{i+1,j+1}), \frac{1}{2} \cdot (($the Go-board of $f)_{i,j+1} + ($the Go-board of $f)_{i+1,j+2}))$ misses $\widetilde{\mathcal{L}}(f)$.

(3) Given k. Suppose $1 \leq k$ and $k+2 \leq \operatorname{len} f$. Given i, j. Suppose that

(i) $1 \leq i$,
(ii) $i+2 \leq \operatorname{len}$ the Go-board of f,
(iii) $1 \leq j$,
(iv) $j+2 \leq \operatorname{width}$ the Go-board of f,
(v) $\pi_{k+1} f$ = (the Go-board of $f)_{i+1,j+1}$, and
(vi) $\pi_k f$ = (the Go-board of $f)_{i+2,j+1}$ and $\pi_{k+2} f$ = (the Go-board of $f)_{i+1,j}$ or $\pi_{k+2} f$ = (the Go-board of $f)_{i+2,j+1}$ and $\pi_k f$ = (the Go-board of $f)_{i+1,j}$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f)_{i,j} + ($the Go-board of $f)_{i+1,j+1}), \frac{1}{2} \cdot (($the Go-board of $f)_{i,j+1} + ($the Go-board of $f)_{i+1,j+2}))$ misses $\widetilde{\mathcal{L}}(f)$.

(4) Given k. Suppose $1 \leq k$ and $k+2 \leq \operatorname{len} f$. Given i, j. Suppose that

(i) $1 \leq i$,
(ii) $i+1 \leq \operatorname{len}$ the Go-board of f,
(iii) $1 \leq j$,
(iv) $j+2 \leq \operatorname{width}$ the Go-board of f,
(v) $\pi_{k+1} f$ = (the Go-board of $f)_{i,j+1}$, and
(vi) $\pi_k f$ = (the Go-board of $f)_{i,j}$ and $\pi_{k+2} f$ = (the Go-board of $f)_{i,j+2}$ or $\pi_{k+2} f$ = (the Go-board of $f)_{i,j}$ and $\pi_k f$ = (the Go-board of $f)_{i,j+2}$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f)_{i,j} + ($the Go-board of $f)_{i+1,j+1}), \frac{1}{2} \cdot (($the Go-board of $f)_{i,j+1} + ($the Go-board of $f)_{i+1,j+2}))$ misses $\widetilde{\mathcal{L}}(f)$.

(5) Given k. Suppose $1 \leq k$ and $k+2 \leq \operatorname{len} f$. Given i, j. Suppose that

(i) $1 \leq i$,
(ii) $i+2 \leq \operatorname{len}$ the Go-board of f,
(iii) $1 \leq j$,
(iv) $j+2 \leq \operatorname{width}$ the Go-board of f,
(v) $\pi_{k+1} f$ = (the Go-board of $f)_{i+1,j+1}$, and
(vi) $\pi_k f$ = (the Go-board of $f)_{i,j+1}$ and $\pi_{k+2} f$ = (the Go-board of $f)_{i+1,j+2}$ or $\pi_{k+2} f$ = (the Go-board of $f)_{i,j+1}$ and $\pi_k f$ = (the Go-board of $f)_{i+1,j+2}$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f)_{i+1,j} + ($the Go-board of $f)_{i+2,j+1}), \frac{1}{2} \cdot (($the Go-board of $f)_{i+1,j+1} + ($the Go-board of $f)_{i+2,j+2}))$ misses $\widetilde{\mathcal{L}}(f)$.

(6) Given k. Suppose $1 \leq k$ and $k+2 \leq \operatorname{len} f$. Given i, j. Suppose that

(i) $1 \leq i$,
(ii) $i+2 \leq \operatorname{len}$ the Go-board of f,
(iii) $1 \leq j$,
(iv) $j+2 \leq \operatorname{width}$ the Go-board of f,
(v) $\pi_{k+1} f$ = (the Go-board of $f)_{i+1,j+1}$, and

(vi) $\pi_k f = (\text{the Go-board of } f)_{i,j+1}$ and $\pi_{k+2} f = (\text{the Go-board of } f)_{i+1,j}$ or $\pi_{k+2} f = (\text{the Go-board of } f)_{i,j+1}$ and $\pi_k f = (\text{the Go-board of } f)_{i+1,j}$.

Then $\mathcal{L}(\frac{1}{2} \cdot ((\text{the Go-board of } f)_{i+1,j} + (\text{the Go-board of } f)_{i+2,j+1}), \frac{1}{2} \cdot ((\text{the Go-board of } f)_{i+1,j+1} + (\text{the Go-board of } f)_{i+2,j+2}))$ misses $\widetilde{\mathcal{L}}(f)$.

(7) Given k. Suppose $1 \leq k$ and $k+2 \leq \operatorname{len} f$. Given i. Suppose that

(i) $1 \leq i$,

(ii) $i+2 \leq \operatorname{len}$ the Go-board of f,

(iii) $\pi_{k+1} f = (\text{the Go-board of } f)_{i+1,1}$, and

(iv) $\pi_k f = (\text{the Go-board of } f)_{i+2,1}$ and $\pi_{k+2} f = (\text{the Go-board of } f)_{i+1,2}$ or $\pi_{k+2} f = (\text{the Go-board of } f)_{i+2,1}$ and $\pi_k f = (\text{the Go-board of } f)_{i+1,2}$.

Then $\mathcal{L}(\frac{1}{2} \cdot ((\text{the Go-board of } f)_{i,1} + (\text{the Go-board of } f)_{i+1,1}) - [0, 1], \frac{1}{2} \cdot ((\text{the Go-board of } f)_{i,1} + (\text{the Go-board of } f)_{i+1,2}))$ misses $\widetilde{\mathcal{L}}(f)$.

(8) Given k. Suppose $1 \leq k$ and $k+2 \leq \operatorname{len} f$. Given i. Suppose that

(i) $1 \leq i$,

(ii) $i+2 \leq \operatorname{len}$ the Go-board of f,

(iii) $\pi_{k+1} f = (\text{the Go-board of } f)_{i+1,1}$, and

(iv) $\pi_k f = (\text{the Go-board of } f)_{i,1}$ and $\pi_{k+2} f = (\text{the Go-board of } f)_{i+1,2}$ or $\pi_{k+2} f = (\text{the Go-board of } f)_{i,1}$ and $\pi_k f = (\text{the Go-board of } f)_{i+1,2}$.

Then $\mathcal{L}(\frac{1}{2} \cdot ((\text{the Go-board of } f)_{i+1,1} + (\text{the Go-board of } f)_{i+2,1}) - [0, 1], \frac{1}{2} \cdot ((\text{the Go-board of } f)_{i+1,1} + (\text{the Go-board of } f)_{i+2,2}))$ misses $\widetilde{\mathcal{L}}(f)$.

(9) Given k. Suppose $1 \leq k$ and $k+2 \leq \operatorname{len} f$. Given i. Suppose that

(i) $1 \leq i$,

(ii) $i+2 \leq \operatorname{len}$ the Go-board of f,

(iii) $\pi_{k+1} f = (\text{the Go-board of } f)_{i+1,\text{width the Go-board of } f}$, and

(iv) $\pi_k f = (\text{the Go-board of } f)_{i+2,\text{width the Go-board of } f}$ and $\pi_{k+2} f = (\text{the Go-board of } f)_{i+1,\text{width the Go-board of } f -' 1}$ or $\pi_{k+2} f = (\text{the Go-board of } f)_{i+2,\text{width the Go-board of } f}$ and $\pi_k f = (\text{the Go-board of } f)_{i+1,\text{width the Go-board of } f -' 1}$.

Then $\mathcal{L}(\frac{1}{2} \cdot ((\text{the Go-board of } f)_{i,\text{width the Go-board of } f -' 1} + (\text{the Go-board of } f)_{i+1,\text{width the Go-board of } f}), \frac{1}{2} \cdot ((\text{the Go-board of } f)_{i,\text{width the Go-board of } f} + (\text{the Go-board of } f)_{i+1,\text{width the Go-board of } f}) + [0, 1])$ misses $\widetilde{\mathcal{L}}(f)$.

(10) Given k. Suppose $1 \leq k$ and $k+2 \leq \operatorname{len} f$. Given i. Suppose that

(i) $1 \leq i$,

(ii) $i+2 \leq \operatorname{len}$ the Go-board of f,

(iii) $\pi_{k+1} f = (\text{the Go-board of } f)_{i+1,\text{width the Go-board of } f}$, and

(iv) $\pi_k f = (\text{the Go-board of } f)_{i,\text{width the Go-board of } f}$ and $\pi_{k+2} f = (\text{the Go-board of } f)_{i+1,\text{width the Go-board of } f -' 1}$ or $\pi_{k+2} f = (\text{the Go-board of } f)_{i,\text{width the Go-board of } f}$ and $\pi_k f = (\text{the Go-board of } f)_{i+1,\text{width the Go-board of } f -' 1}$.

Then $\mathcal{L}(\frac{1}{2} \cdot ((\text{the Go-board of } f)_{i+1,\text{width the Go-board of } f -' 1} + (\text{the Go-board of } f)_{i+2,\text{width the Go-board of } f}), \frac{1}{2} \cdot ((\text{the Go-board of } f)_{i+1,\text{width the Go-board of } f} + (\text{the Go-board of } f)_{i+2,\text{width the Go-board of } f}) + [0, 1])$ misses $\widetilde{\mathcal{L}}(f)$.

(11) Given k. Suppose $1 \leq k$ and $k+2 \leq \operatorname{len} f$. Given i, j. Suppose that

(i) $1 \leq j$,
(ii) $j + 1 \leq \operatorname{width}$ the Go-board of f,
(iii) $1 \leq i$,
(iv) $i + 2 \leq \operatorname{len}$ the Go-board of f,
(v) $\pi_{k+1}f =$ (the Go-board of $f)_{i+1,j+1}$, and
(vi) $\pi_k f =$ (the Go-board of $f)_{i,j+1}$ and $\pi_{k+2}f =$ (the Go-board of $f)_{i+2,j+1}$ or $\pi_{k+2}f =$ (the Go-board of $f)_{i,j+1}$ and $\pi_k f =$ (the Go-board of $f)_{i+2,j+1}$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f)_{i,j} + ($the Go-board of $f)_{i+1,j+1}), \frac{1}{2} \cdot (($the Go-board of $f)_{i+1,j} + ($the Go-board of $f)_{i+2,j+1}))$ misses $\widetilde{\mathcal{L}}(f)$.

(12) Given k. Suppose $1 \leq k$ and $k + 2 \leq \operatorname{len} f$. Given j, i. Suppose that
(i) $1 \leq j$,
(ii) $j + 2 \leq \operatorname{width}$ the Go-board of f,
(iii) $1 \leq i$,
(iv) $i + 2 \leq \operatorname{len}$ the Go-board of f,
(v) $\pi_{k+1}f =$ (the Go-board of $f)_{i+1,j+1}$, and
(vi) $\pi_k f =$ (the Go-board of $f)_{i+1,j+2}$ and $\pi_{k+2}f =$ (the Go-board of $f)_{i+2,j+1}$ or $\pi_{k+2}f =$ (the Go-board of $f)_{i+1,j+2}$ and $\pi_k f =$ (the Go-board of $f)_{i+2,j+1}$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f)_{i,j} + ($the Go-board of $f)_{i+1,j+1}), \frac{1}{2} \cdot (($the Go-board of $f)_{i+1,j} + ($the Go-board of $f)_{i+2,j+1}))$ misses $\widetilde{\mathcal{L}}(f)$.

(13) Given k. Suppose $1 \leq k$ and $k + 2 \leq \operatorname{len} f$. Given j, i. Suppose that
(i) $1 \leq j$,
(ii) $j + 2 \leq \operatorname{width}$ the Go-board of f,
(iii) $1 \leq i$,
(iv) $i + 2 \leq \operatorname{len}$ the Go-board of f,
(v) $\pi_{k+1}f =$ (the Go-board of $f)_{i+1,j+1}$, and
(vi) $\pi_k f =$ (the Go-board of $f)_{i+1,j+2}$ and $\pi_{k+2}f =$ (the Go-board of $f)_{i,j+1}$ or $\pi_{k+2}f =$ (the Go-board of $f)_{i+1,j+2}$ and $\pi_k f =$ (the Go-board of $f)_{i,j+1}$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f)_{i,j} + ($the Go-board of $f)_{i+1,j+1}), \frac{1}{2} \cdot (($the Go-board of $f)_{i+1,j} + ($the Go-board of $f)_{i+2,j+1}))$ misses $\widetilde{\mathcal{L}}(f)$.

(14) Given k. Suppose $1 \leq k$ and $k + 2 \leq \operatorname{len} f$. Given j, i. Suppose that
(i) $1 \leq j$,
(ii) $j + 1 \leq \operatorname{width}$ the Go-board of f,
(iii) $1 \leq i$,
(iv) $i + 2 \leq \operatorname{len}$ the Go-board of f,
(v) $\pi_{k+1}f =$ (the Go-board of $f)_{i+1,j}$, and
(vi) $\pi_k f =$ (the Go-board of $f)_{i,j}$ and $\pi_{k+2}f =$ (the Go-board of $f)_{i+2,j}$ or $\pi_{k+2}f =$ (the Go-board of $f)_{i,j}$ and $\pi_k f =$ (the Go-board of $f)_{i+2,j}$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f)_{i,j} + ($the Go-board of $f)_{i+1,j+1}), \frac{1}{2} \cdot (($the Go-board of $f)_{i+1,j} + ($the Go-board of $f)_{i+2,j+1}))$ misses $\widetilde{\mathcal{L}}(f)$.

(15) Given k. Suppose $1 \leq k$ and $k + 2 \leq \operatorname{len} f$. Given j, i. Suppose that

(i) $1 \leq j$,
(ii) $j + 2 \leq$ width the Go-board of f,
(iii) $1 \leq i$,
(iv) $i + 2 \leq$ len the Go-board of f,
(v) $\pi_{k+1}f =$ (the Go-board of $f)_{i+1,j+1}$, and
(vi) $\pi_k f =$ (the Go-board of $f)_{i+1,j}$ and $\pi_{k+2}f =$ (the Go-board of $f)_{i+2,j+1}$ or $\pi_{k+2}f =$ (the Go-board of $f)_{i+1,j}$ and $\pi_k f =$ (the Go-board of $f)_{i+2,j+1}$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f)_{i,j+1} + ($the Go-board of $f)_{i+1,j+2}), \frac{1}{2} \cdot (($the Go-board of $f)_{i+1,j+1} + ($the Go-board of $f)_{i+2,j+2}))$ misses $\widetilde{\mathcal{L}}(f)$.

(16) Given k. Suppose $1 \leq k$ and $k + 2 \leq \operatorname{len} f$. Given j, i. Suppose that
(i) $1 \leq j$,
(ii) $j + 2 \leq$ width the Go-board of f,
(iii) $1 \leq i$,
(iv) $i + 2 \leq$ len the Go-board of f,
(v) $\pi_{k+1}f =$ (the Go-board of $f)_{i+1,j+1}$, and
(vi) $\pi_k f =$ (the Go-board of $f)_{i+1,j}$ and $\pi_{k+2}f =$ (the Go-board of $f)_{i,j+1}$ or $\pi_{k+2}f =$ (the Go-board of $f)_{i+1,j}$ and $\pi_k f =$ (the Go-board of $f)_{i,j+1}$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f)_{i,j+1} + ($the Go-board of $f)_{i+1,j+2}), \frac{1}{2} \cdot (($the Go-board of $f)_{i+1,j+1} + ($the Go-board of $f)_{i+2,j+2}))$ misses $\widetilde{\mathcal{L}}(f)$.

(17) Given k. Suppose $1 \leq k$ and $k + 2 \leq \operatorname{len} f$. Given j. Suppose that
(i) $1 \leq j$,
(ii) $j + 2 \leq$ width the Go-board of f,
(iii) $\pi_{k+1}f =$ (the Go-board of $f)_{1,j+1}$, and
(iv) $\pi_k f =$ (the Go-board of $f)_{1,j+2}$ and $\pi_{k+2}f =$ (the Go-board of $f)_{2,j+1}$ or $\pi_{k+2}f =$ (the Go-board of $f)_{1,j+2}$ and $\pi_k f =$ (the Go-board of $f)_{2,j+1}$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f)_{1,j} + ($the Go-board of $f)_{1,j+1}) - [1, 0], \frac{1}{2} \cdot (($the Go-board of $f)_{1,j} + ($the Go-board of $f)_{2,j+1}))$ misses $\widetilde{\mathcal{L}}(f)$.

(18) Given k. Suppose $1 \leq k$ and $k + 2 \leq \operatorname{len} f$. Given j. Suppose that
(i) $1 \leq j$,
(ii) $j + 2 \leq$ width the Go-board of f,
(iii) $\pi_{k+1}f =$ (the Go-board of $f)_{1,j+1}$, and
(iv) $\pi_k f =$ (the Go-board of $f)_{1,j}$ and $\pi_{k+2}f =$ (the Go-board of $f)_{2,j+1}$ or $\pi_{k+2}f =$ (the Go-board of $f)_{1,j}$ and $\pi_k f =$ (the Go-board of $f)_{2,j+1}$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f)_{1,j+1} + ($the Go-board of $f)_{1,j+2}) - [1, 0], \frac{1}{2} \cdot (($the Go-board of $f)_{1,j+1} + ($the Go-board of $f)_{2,j+2}))$ misses $\widetilde{\mathcal{L}}(f)$.

(19) Given k. Suppose $1 \leq k$ and $k + 2 \leq \operatorname{len} f$. Given j. Suppose that
(i) $1 \leq j$,
(ii) $j + 2 \leq$ width the Go-board of f,
(iii) $\pi_{k+1}f =$ (the Go-board of $f)_{\text{len the Go-board of } f,\, j+1}$, and
(iv) $\pi_k f =$ (the Go-board of $f)_{\text{len the Go-board of } f,\, j+2}$ and $\pi_{k+2}f =$ (the Go-board of $f)_{\text{len the Go-board of } f -' 1, j+1}$ or $\pi_{k+2}f =$ (the Go-

board of $f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f,\,j+2}$ and $\pi_k f$ = (the Go-board of $f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f-'1,j+1}$.

Then $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of }f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f-'1,j}+(\text{the Go-board of }f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f,\,j+1}),\frac{1}{2}\cdot((\text{the Go-board of }f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f,\,j}+(\text{the Go-board of }f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f,\,j+1})+[1,0])$ misses $\widetilde{\mathcal{L}}(f)$.

(20) Given k. Suppose $1\leq k$ and $k+2\leq \mathrm{len}\,f$. Given j. Suppose that

(i) $1\leq j$,

(ii) $j+2\leq$ width the Go-board of f,

(iii) $\pi_{k+1}f=$ (the Go-board of $f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f,\,j+1}$, and

(iv) $\pi_k f$ = (the Go-board of $f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f,\,j}$ and $\pi_{k+2}f$ = (the Go-board of $f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f-'1,j+1}$ or $\pi_{k+2}f$ = (the Go-board of $f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f,\,j}$ and $\pi_k f$ = (the Go-board of $f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f-'1,j+1}$.

Then $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of }f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f-'1,j+1}+(\text{the Go-board of }f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f,\,j+2}),\frac{1}{2}\cdot((\text{the Go-board of }f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f,\,j+1}+(\text{the Go-board of }f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f,\,j+2})+[1,0])$ misses $\widetilde{\mathcal{L}}(f)$.

In the sequel P will be a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$.

We now state a number of propositions:

(21) If for every p such that $p\in P$ holds $p_{\mathbf{1}}<((\text{the Go-board of }f)_{1,1})_{\mathbf{1}}$, then P misses $\widetilde{\mathcal{L}}(f)$.

(22) If for every p such that $p\in P$ holds $p_{\mathbf{1}}>((\text{the Go-board of }f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f,\,1})_{\mathbf{1}}$, then P misses $\widetilde{\mathcal{L}}(f)$.

(23) If for every p such that $p\in P$ holds $p_{\mathbf{2}}<((\text{the Go-board of }f)_{1,1})_{\mathbf{2}}$, then P misses $\widetilde{\mathcal{L}}(f)$.

(24) If for every p such that $p\in P$ holds $p_{\mathbf{2}}>((\text{the Go-board of }f)_{1,\mathrm{width\,the\,Go\text{-}board\,of}\,f})_{\mathbf{2}}$, then P misses $\widetilde{\mathcal{L}}(f)$.

(25) Given i. Suppose $1\leq i$ and $i+2\leq$ len the Go-board of f. Then $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of }f)_{i,1}+(\text{the Go-board of }f)_{i+1,1})-[0,1],\frac{1}{2}\cdot((\text{the Go-board of }f)_{i+1,1}+(\text{the Go-board of }f)_{i+2,1})-[0,1])$ misses $\widetilde{\mathcal{L}}(f)$.

(26) $\mathcal{L}((\text{the Go-board of }f)_{1,1}-[1,1],\frac{1}{2}\cdot((\text{the Go-board of }f)_{1,1}+(\text{the Go-board of }f)_{2,1})-[0,1])$ misses $\widetilde{\mathcal{L}}(f)$.

(27) $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of }f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f-'1,1}+(\text{the Go-board of }f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f,\,1})-[0,1],(\text{the Go-board of }f)_{\mathrm{len\,the\,Go\text{-}board\,of}\,f,\,1}+[1,-1])$ misses $\widetilde{\mathcal{L}}(f)$.

(28) Given i. Suppose $1\leq i$ and $i+2\leq$ len the Go-board of f. Then $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of }f)_{i,\mathrm{width\,the\,Go\text{-}board\,of}\,f}+(\text{the Go-board of }f)_{i+1,\mathrm{width\,the\,Go\text{-}board\,of}\,f})+[0,1],\frac{1}{2}\cdot((\text{the Go-board of }f)_{i+1,\mathrm{width\,the\,Go\text{-}board\,of}\,f}+(\text{the Go-board of }f)_{i+2,\mathrm{width\,the\,Go\text{-}board\,of}\,f})+[0,1])$ misses $\widetilde{\mathcal{L}}(f)$.

(29) $\mathcal{L}((\text{the Go-board of }f)_{1,\mathrm{width\,the\,Go\text{-}board\,of}\,f}+[-1,1],\frac{1}{2}\cdot((\text{the Go-board of }f)_{1,\mathrm{width\,the\,Go\text{-}board\,of}\,f}+(\text{the Go-board of }f)_{2,\mathrm{width\,the\,Go\text{-}board\,of}\,f})+[0,$

1]) misses $\widetilde{\mathcal{L}}(f)$.

(30) $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of } f)_{\text{len the Go-board of } f -' 1, \text{width the Go-board of } f} + (\text{the Go-board of } f)_{\text{len the Go-board of } f,\ \text{width the Go-board of } f}) + [0,1], (\text{the Go-board of } f)_{\text{len the Go-board of } f,\ \text{width the Go-board of } f} + [1,1])$ misses $\widetilde{\mathcal{L}}(f)$.

(31) Given j. Suppose $1 \leq j$ and $j+2 \leq$ width the Go-board of f. Then $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of } f)_{1,j} + (\text{the Go-board of } f)_{1,j+1}) - [1,0], \frac{1}{2}\cdot((\text{the Go-board of } f)_{1,j+1} + (\text{the Go-board of } f)_{1,j+2}) - [1,0])$ misses $\widetilde{\mathcal{L}}(f)$.

(32) $\mathcal{L}((\text{the Go-board of } f)_{1,1} - [1,1], \frac{1}{2}\cdot((\text{the Go-board of } f)_{1,1} + (\text{the Go-board of } f)_{1,2}) - [1,0])$ misses $\widetilde{\mathcal{L}}(f)$.

(33) $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of } f)_{1, \text{width the Go-board of } f -' 1} + (\text{the Go-board of } f)_{1, \text{width the Go-board of } f}) - [1,0], (\text{the Go-board of } f)_{1, \text{width the Go-board of } f} + [-1,1])$ misses $\widetilde{\mathcal{L}}(f)$.

(34) Given j. Suppose $1 \leq j$ and $j+2 \leq$ width the Go-board of f. Then $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of } f)_{\text{len the Go-board of } f,\ j} + (\text{the Go-board of } f)_{\text{len the Go-board of } f,\ j+1}) + [1,0], \frac{1}{2}\cdot((\text{the Go-board of } f)_{\text{len the Go-board of } f,\ j+1} + (\text{the Go-board of } f)_{\text{len the Go-board of } f,\ j+2}) + [1,0])$ misses $\widetilde{\mathcal{L}}(f)$.

(35) $\mathcal{L}((\text{the Go-board of } f)_{\text{len the Go-board of } f,\ 1} + [1,-1], \frac{1}{2}\cdot((\text{the Go-board of } f)_{\text{len the Go-board of } f,\ 1} + (\text{the Go-board of } f)_{\text{len the Go-board of } f,\ 2}) + [1,0])$ misses $\widetilde{\mathcal{L}}(f)$.

(36) $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of } f)_{\text{len the Go-board of } f,\ \text{width the Go-board of } f -' 1} + (\text{the Go-board of } f)_{\text{len the Go-board of } f,\ \text{width the Go-board of } f}) + [1,0], (\text{the Go-board of } f)_{\text{len the Go-board of } f,\ \text{width the Go-board of } f} + [1,1])$ misses $\widetilde{\mathcal{L}}(f)$.

(37) If $1 \leq k$ and $k+1 \leq \operatorname{len} f$, then Int leftcell$(f,k)$ misses $\widetilde{\mathcal{L}}(f)$.

(38) If $1 \leq k$ and $k+1 \leq \operatorname{len} f$, then Int rightcell$(f,k)$ misses $\widetilde{\mathcal{L}}(f)$.

References

[1] Grzegorz Bancerek. Cardinal numbers. *Formalized Mathematics*, 1(**2**):377–382, 1990.

[2] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.

[3] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.

[4] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[5] Agata Darmochwał. The Euclidean space. *Formalized Mathematics*, 2(**4**):599–603, 1991.

[6] Agata Darmochwał. Finite sets. *Formalized Mathematics*, 1(**1**):165–167, 1990.

[7] Agata Darmochwał and Yatsuka Nakamura. The topological space $\mathcal{E}^2_{\mathrm{T}}$. Arcs, line segments and special polygonal arcs. *Formalized Mathematics*, 2(**5**):617–621, 1991.

[8] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.

[9] Katarzyna Jankowska. Matrices. Abelian group of matrices. *Formalized Mathematics*, 2(**4**):475–480, 1991.

[10] Stanisława Kanas, Adam Lecko, and Mariusz Startek. Metric spaces. *Formalized Mathematics*, 1(**3**):607–610, 1990.

[11] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-board - part I. *Formalized Mathematics*, 3(**1**):107–115, 1992.

[12] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-board - part II. *Formalized Mathematics*, 3(**1**):117–121, 1992.

[13] Rafał Kwiatek and Grzegorz Zwara. The divisibility of integers and integer relative primes. *Formalized Mathematics*, 1(**5**):829–832, 1990.

[14] Yatsuka Nakamura and Czesław Byliński. Extremal properties of vertices on special polygons, part I. *Formalized Mathematics*, 5(**1**):97–102, 1996.

[15] Yatsuka Nakamura and Andrzej Trybulec. Decomposing a Go-Board into cells. *Formalized Mathematics*, 5(**3**):323–328, 1996.

[16] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Formalized Mathematics*, 4(**1**):83–86, 1993.

[17] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Formalized Mathematics*, 1(**1**):223–230, 1990.

[18] Jan Popiołek. Some properties of functions modul and signum. *Formalized Mathematics*, 1(**2**):263–264, 1990.

[19] Andrzej Trybulec. On the decomposition of finite sequences. *Formalized Mathematics*, 5(**3**):317–322, 1996.

[20] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.

[21] Wojciech A. Trybulec. Pigeon hole principle. *Formalized Mathematics*, 1(**3**):575–579, 1990.

[22] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.

[23] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.

[24] Mirosław Wysocki and Agata Darmochwał. Subsets of topological spaces. *Formalized Mathematics*, 1(**1**):231–237, 1990.

Received October 17, 1995