FORMALIZED MATHEMATICS
Volume 5, Number 3, 1996
Warsaw University - Białystok

Replacement of Subtrees in a Tree

Oleg Okhotnikov
Ural University
Ekaterinburg

Summary. This paper is based on previous works [1], [3] in which the operation replacement of subtree in a tree has been defined. We extend this notion for arbitrary non empty antichain.

MML Identifier: `TREES_A`.

The notation and terminology used in this paper are introduced in the following papers: [8], [9], [6], [10], [5], [7], [4], [1], [3], and [2].

We follow the rules: T, T_1 will denote trees, P will denote an antichain of prefixes of T, and p, q, r will denote finite sequences of elements of $\mathbb{N}$.

We now state the proposition

(1) For all finite sequences p, q, r, s such that $p \frown q = s \frown r$ holds p and s are comparable.

Let us consider T, T_1 and let us consider P. Let us assume that $P \neq \emptyset$. The functor $T(P/T_1)$ yields a tree and is defined as follows:

(Def.1) $q \in T(P/T_1)$ iff $q \in T$ and for every p such that $p \in P$ holds $p \not\preceq q$ or there exist p, r such that $p \in P$ and $r \in T_1$ and $q = p \frown r$.

One can prove the following propositions:

(2) Suppose $P \neq \emptyset$. Then $T(P/T_1) = \{t_1 : t_1$ ranges over elements of T, $\bigwedge_p \ p \in P \ \Rightarrow \ p \not\preceq t_1\} \cup \{p \frown s : p$ ranges over elements of T, s ranges over elements of T_1, $p \in P\}$.

(3) $\{t_1 : t_1$ ranges over elements of T, $\bigwedge_p \ p \in P \ \Rightarrow \ p \npreceq t_1\} \subseteq \{t_1 : t_1$ ranges over elements of T, $\bigwedge_p \ p \in P \ \Rightarrow \ p \not\preceq t_1\}$.

(4) $P \subseteq \{t_1 : t_1$ ranges over elements of T, $\bigwedge_p \ p \in P \ \Rightarrow \ p \not\preceq t_1\}$.

(5) $\{t_1 : t_1$ ranges over elements of T, $\bigwedge_p \ p \in P \ \Rightarrow \ p \not\preceq t_1\} \setminus \{t_1 : t_1$ ranges over elements of T, $\bigwedge_p \ p \in P \ \Rightarrow \ p \npreceq t_1\} = P$.

(6) For all T, T_1, P holds $P \subseteq \{p \frown s : p$ ranges over elements of T, s ranges over elements of T_1, $p \in P\}$.

ISSN 1426–2630

(7) Suppose $P \neq \emptyset$. Then $T(P/T_1) = \{t_1 : t_1$ ranges over elements of T, $\bigwedge_p\ p \in P\ \Rightarrow\ p \not\preceq t_1\} \cup \{p \frown s : p$ ranges over elements of T, s ranges over elements of T_1, $p \in P\}$.

(8) If $p \in P$ and $q \in T_1$, then $p \frown q \in T(P/T_1)$.

(9) If $p \in P$, then $T_1 = T(P/T_1) \restriction p$.

Let us consider T. Observe that there exists an antichain of prefixes of T which is non empty.

Let us consider T and let t be an element of T. Then $\{t\}$ is a non empty antichain of prefixes of T.

In the sequel t will be an element of T.

We now state the proposition

(10) $T(\{t\}/T_1) = T(t/T_1)$.

In the sequel T, T_1 denote decorated trees, P denotes an antichain of prefixes of $\operatorname{dom} T$, and t denotes an element of $\operatorname{dom} T$.

Let us consider T, P, T_1. Let us assume that $P \neq \emptyset$. The functor $T(P/T_1)$ yields a decorated tree and is defined by the conditions (Def.2).

(Def.2) (i) $\operatorname{dom}(T(P/T_1)) = (\operatorname{dom} T)(P/\operatorname{dom} T_1)$, and

(ii) for every q such that $q \in (\operatorname{dom} T)(P/\operatorname{dom} T_1)$ holds for every p such that $p \in P$ holds $p \not\preceq q$ and $T(P/T_1)(q) = T(q)$ or there exist p, r such that $p \in P$ and $r \in \operatorname{dom} T_1$ and $q = p \frown r$ and $T(P/T_1)(q) = T_1(r)$.

We now state several propositions:

(11) If $P \neq \emptyset$, then $\operatorname{dom}(T(P/T_1)) = (\operatorname{dom} T)(P/\operatorname{dom} T_1)$.

(12) If $p \in \operatorname{dom} T$, then $\operatorname{dom}(T(p/T_1)) = (\operatorname{dom} T)(p/\operatorname{dom} T_1)$.

(13) Suppose $P \neq \emptyset$. Given q. Suppose $q \in \operatorname{dom}(T(P/T_1))$. Then for every p such that $p \in P$ holds $p \not\preceq q$ and $T(P/T_1)(q) = T(q)$ or there exist p, r such that $p \in P$ and $r \in \operatorname{dom} T_1$ and $q = p \frown r$ and $T(P/T_1)(q) = T_1(r)$.

(14) Suppose $p \in \operatorname{dom} T$. Given q. Suppose $q \in \operatorname{dom}(T(p/T_1))$. Then $p \not\preceq q$ and $T(p/T_1)(q) = T(q)$ or there exists r such that $r \in \operatorname{dom} T_1$ and $q = p \frown r$ and $T(p/T_1)(q) = T_1(r)$.

(15) Suppose $P \neq \emptyset$. Given q. Suppose $q \in \operatorname{dom}(T(P/T_1))$ and $q \in \{t_1 : t_1$ ranges over elements of $\operatorname{dom} T$, $\bigwedge_p\ p \in P\ \Rightarrow\ p \not\preceq t_1\}$. Then $T(P/T_1)(q) = T(q)$.

(16) If $p \in \operatorname{dom} T$, then for every q such that $q \in \operatorname{dom}(T(p/T_1))$ and $q \in \{t_1 : t_1$ ranges over elements of $\operatorname{dom} T, p \not\preceq t_1\}$ holds $T(p/T_1)(q) = T(q)$.

(17) Suppose $P \neq \emptyset$. Given q. Suppose $q \in \operatorname{dom}(T(P/T_1))$ and $q \in \{p \frown s : p$ ranges over elements of $\operatorname{dom} T, s$ ranges over elements of $\operatorname{dom} T_1, p \in P\}$. Then there exists an element p' of $\operatorname{dom} T$ and there exists an element r of $\operatorname{dom} T_1$ such that $q = p' \frown r$ and $p' \in P$ and $T(P/T_1)(q) = T_1(r)$.

(18) Suppose $p \in \operatorname{dom} T$. Given q. Suppose $q \in \operatorname{dom}(T(p/T_1))$ and $q \in \{p \frown s : s$ ranges over elements of $\operatorname{dom} T_1, s = s\}$. Then there exists an element r of $\operatorname{dom} T_1$ such that $q = p \frown r$ and $T(p/T_1)(q) = T_1(r)$.

(19) $T(\{t\}/T_1) = T(t/T_1)$.

In the sequel D will denote a non empty set, T, T_1 will denote trees decorated with elements of D, and P will denote an antichain of prefixes of $\operatorname{dom} T$.

Let us consider D, T, P, T_1. Let us assume that $P \neq \emptyset$. The functor $T(P/T_1)$ yields a tree decorated with elements of D and is defined by:

(Def.3) $T(P/T_1) = T(P/T_1)$.

Acknowledgments

The author wishes to thank to G. Bancerek for his assistance during the preparation of this paper.

References

[1] Grzegorz Bancerek. Introduction to trees. *Formalized Mathematics*, 1(**2**):421–427, 1990.
[2] Grzegorz Bancerek. Joining of decorated trees. *Formalized Mathematics*, 4(**1**):77–82, 1993.
[3] Grzegorz Bancerek. König's lemma. *Formalized Mathematics*, 2(**3**):397–402, 1991.
[4] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.
[5] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.
[6] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(**1**):47–53, 1990.
[7] Agata Darmochwał. Finite sets. *Formalized Mathematics*, 1(**1**):165–167, 1990.
[8] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[9] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.
[10] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.

Received October 1, 1995