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Some Topological Properties of Cells in R^2

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Summary. We examine the topological property of cells (rectangles) in a plane. First, some Fraenkel expressions of cells are shown. Second, it is proved that cells are closed. The last theorem asserts that the closure of the interior of a cell is the same as itself.

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The articles [7], [11], [19], [20], [24], [23], [8], [1], [21], [15], [25], [17], [18], [5], [4], [2], [22], [9], [10], [26], [16], [3], [6], [12], [14], and [13] provide the notation and terminology for this paper.

We adopt the following convention: i, j, j_1, j_2 will be natural numbers, r, s, r_2, s_1, s_2 will be real numbers, and G_1 will be a non empty topological space.

Next we state two propositions:

(1) For every subset A of the carrier of G_1 and for every point p of G_1 such that $p \in A$ and A is connected holds $A \subseteq \mathrm{Component}(p)$.

(2) Let A, B, C be subsets of the carrier of G_1. Suppose C is a component of G_1 and $A \subseteq C$ and B is connected and $\overline{A} \cap \overline{B} \neq \emptyset$. Then $B \subseteq C$.

In the sequel G_2 denotes a non empty topological space.

Next we state three propositions:

(3) Let A, B be subsets of the carrier of G_2. Suppose A is a component of G_2 and B is a component of G_2. Then $A \cup B$ is a union of components of G_2.

(4) For all subsets B_1, B_2, V of the carrier of G_1 such that $V \neq \emptyset$ holds $\mathrm{Down}(B_1 \cup B_2, V) = \mathrm{Down}(B_1, V) \cup \mathrm{Down}(B_2, V)$.

(5) For all subsets B_1, B_2, V of the carrier of G_1 such that $V \neq \emptyset$ holds $\mathrm{Down}(B_1 \cap B_2, V) = \mathrm{Down}(B_1, V) \cap \mathrm{Down}(B_2, V)$.

In the sequel f will denote a non constant standard special circular sequence and G will denote a Go-board.

We now state a number of propositions:

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(6) $(\widetilde{\mathcal{L}}(f))^{\rm c} \neq \emptyset$.

(7) Given j_1, j_2. Suppose $j_1 = \mathrm{len}$ the Go-board of f and $j_2 = \mathrm{width}$ the Go-board of f. Then the carrier of $\mathcal{E}^2_{\rm T} = \bigcup\{\mathrm{cell}($the Go-board of $f, i, j) : i \leq j_1 \ \wedge \ j \leq j_2\}$.

(8) For all subsets P_1, P_2 of the carrier of $\mathcal{E}^2_{\rm T}$ such that $P_1 = \{[r,s] : s \leq s_1\}$ and $P_2 = \{[r_2,s_2] : s_2 > s_1\}$ holds $P_1 = -P_2$.

(9) For all subsets P_1, P_2 of the carrier of $\mathcal{E}^2_{\rm T}$ such that $P_1 = \{[r,s] : s \geq s_1\}$ and $P_2 = \{[r_2,s_2] : s_2 < s_1\}$ holds $P_1 = -P_2$.

(10) For all subsets P_1, P_2 of the carrier of $\mathcal{E}^2_{\rm T}$ such that $P_1 = \{[s,r] : s \geq s_1\}$ and $P_2 = \{[s_2,r_2] : s_2 < s_1\}$ holds $P_1 = -P_2$.

(11) For all subsets P_1, P_2 of the carrier of $\mathcal{E}^2_{\rm T}$ such that $P_1 = \{[s,r] : s \leq s_1\}$ and $P_2 = \{[s_2,r_2] : s_2 > s_1\}$ holds $P_1 = -P_2$.

(12) For every subset P of the carrier of $\mathcal{E}^2_{\rm T}$ and for every s_1 such that $P = \{[r,s] : s \leq s_1\}$ holds P is closed.

(13) For every subset P of the carrier of $\mathcal{E}^2_{\rm T}$ and for every s_1 such that $P = \{[r,s] : s_1 \leq s\}$ holds P is closed.

(14) For every subset P of the carrier of $\mathcal{E}^2_{\rm T}$ and for every s_1 such that $P = \{[s,r] : s \leq s_1\}$ holds P is closed.

(15) For every subset P of the carrier of $\mathcal{E}^2_{\rm T}$ and for every s_1 such that $P = \{[s,r] : s_1 \leq s\}$ holds P is closed.

(16) For every j holds $\mathrm{hstrip}(G,j)$ is closed.

(17) For every i holds $\mathrm{vstrip}(G,i)$ is closed.

(18) $\mathrm{vstrip}(G,0) = \{[r,s] : r \leq (G_{1,1})_{\mathbf{1}}\}$.

(19) $\mathrm{vstrip}(G,\mathrm{len}\,G) = \{[r,s] : (G_{\mathrm{len}\,G,1})_{\mathbf{1}} \leq r\}$.

(20) If $1 \leq i$ and $i < \mathrm{len}\,G$, then $\mathrm{vstrip}(G,i) = \{[r,s] : (G_{i,1})_{\mathbf{1}} \leq r \ \wedge \ r \leq (G_{i+1,1})_{\mathbf{1}}\}$.

(21) $\mathrm{hstrip}(G,0) = \{[r,s] : s \leq (G_{1,1})_{\mathbf{2}}\}$.

(22) $\mathrm{hstrip}(G,\mathrm{width}\,G) = \{[r,s] : (G_{1,\mathrm{width}\,G})_{\mathbf{2}} \leq s\}$.

(23) If $1 \leq j$ and $j < \mathrm{width}\,G$, then $\mathrm{hstrip}(G,j) = \{[r,s] : (G_{1,j})_{\mathbf{2}} \leq s \ \wedge \ s \leq (G_{1,j+1})_{\mathbf{2}}\}$.

(24) $\mathrm{cell}(G,0,0) = \{[r,s] : r \leq (G_{1,1})_{\mathbf{1}} \ \wedge \ s \leq (G_{1,1})_{\mathbf{2}}\}$.

(25) $\mathrm{cell}(G,0,\mathrm{width}\,G) = \{[r,s] : r \leq (G_{1,1})_{\mathbf{1}} \ \wedge \ (G_{1,\mathrm{width}\,G})_{\mathbf{2}} \leq s\}$.

(26) If $1 \leq j$ and $j < \mathrm{width}\,G$, then $\mathrm{cell}(G,0,j) = \{[r,s] : r \leq (G_{1,1})_{\mathbf{1}} \ \wedge \ (G_{1,j})_{\mathbf{2}} \leq s \ \wedge \ s \leq (G_{1,j+1})_{\mathbf{2}}\}$.

(27) $\mathrm{cell}(G,\mathrm{len}\,G,0) = \{[r,s] : (G_{\mathrm{len}\,G,1})_{\mathbf{1}} \leq r \ \wedge \ s \leq (G_{1,1})_{\mathbf{2}}\}$.

(28) $\mathrm{cell}(G,\mathrm{len}\,G,\mathrm{width}\,G) = \{[r,s] : (G_{\mathrm{len}\,G,1})_{\mathbf{1}} \leq r \ \wedge \ (G_{1,\mathrm{width}\,G})_{\mathbf{2}} \leq s\}$.

(29) If $1 \leq j$ and $j < \mathrm{width}\,G$, then $\mathrm{cell}(G,\mathrm{len}\,G,j) = \{[r,s] : (G_{\mathrm{len}\,G,1})_{\mathbf{1}} \leq r \ \wedge \ (G_{1,j})_{\mathbf{2}} \leq s \ \wedge \ s \leq (G_{1,j+1})_{\mathbf{2}}\}$.

(30) If $1 \leq i$ and $i < \mathrm{len}\,G$, then $\mathrm{cell}(G,i,0) = \{[r,s] : (G_{i,1})_{\mathbf{1}} \leq r \ \wedge \ r \leq (G_{i+1,1})_{\mathbf{1}} \ \wedge \ s \leq (G_{1,1})_{\mathbf{2}}\}$.

(31) If $1 \leq i$ and $i < \mathrm{len}\, G$, then $\mathrm{cell}(G, i, \mathrm{width}\, G) = \{[r, s] : (G_{i,1})_{\mathbf{1}} \leq r \ \wedge \ r \leq (G_{i+1,1})_{\mathbf{1}} \ \wedge \ (G_{1,\mathrm{width}\, G})_{\mathbf{2}} \leq s\}$.

(32) If $1 \leq i$ and $i < \mathrm{len}\, G$ and $1 \leq j$ and $j < \mathrm{width}\, G$, then $\mathrm{cell}(G, i, j) = \{[r, s] : (G_{i,1})_{\mathbf{1}} \leq r \ \wedge \ r \leq (G_{i+1,1})_{\mathbf{1}} \ \wedge \ (G_{1,j})_{\mathbf{2}} \leq s \ \wedge \ s \leq (G_{1,j+1})_{\mathbf{2}}\}$.

(33) For all i, j holds $\mathrm{cell}(G, i, j)$ is closed.

(34) $1 \leq \mathrm{len}\, G$ and $1 \leq \mathrm{width}\, G$.

(35) For all i, j such that $i \leq \mathrm{len}\, G$ and $j \leq \mathrm{width}\, G$ holds $\mathrm{cell}(G, i, j) = \overline{\mathrm{Int}\, \mathrm{cell}(G, i, j)}$.

References

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.

[2] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.

[3] Leszek Borys. Paracompact and metrizable spaces. *Formalized Mathematics*, 2(**4**):481–485, 1991.

[4] Czesław Byliński. Binary operations. *Formalized Mathematics*, 1(**1**):175–180, 1990.

[5] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[6] Agata Darmochwał. The Euclidean space. *Formalized Mathematics*, 2(**4**):599–603, 1991.

[7] Agata Darmochwał and Yatsuka Nakamura. The topological space $\mathcal{E}^2_{\mathrm{T}}$. Arcs, line segments and special polygonal arcs. *Formalized Mathematics*, 2(**5**):617–621, 1991.

[8] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.

[9] Katarzyna Jankowska. Matrices. Abelian group of matrices. *Formalized Mathematics*, 2(**4**):475–480, 1991.

[10] Stanisława Kanas, Adam Lecko, and Mariusz Startek. Metric spaces. *Formalized Mathematics*, 1(**3**):607–610, 1990.

[11] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-board - Part I. *Formalized Mathematics*, 3(**1**):107–115, 1992.

[12] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-board - Part II. *Formalized Mathematics*, 3(**1**):117–121, 1992.

[13] Yatsuka Nakamura and Andrzej Trybulec. Components and unions of components. *Formalized Mathematics*, 5(**4**):513–517, 1996.

[14] Yatsuka Nakamura and Andrzej Trybulec. Decomposing a Go-Board into cells. *Formalized Mathematics*, 5(**3**):323–328, 1996.

[15] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Formalized Mathematics*, 4(**1**):83–86, 1993.

[16] Beata Padlewska. Connected spaces. *Formalized Mathematics*, 1(**1**):239–244, 1990.

[17] Beata Padlewska. Families of sets. *Formalized Mathematics*, 1(**1**):147–152, 1990.

[18] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Formalized Mathematics*, 1(**1**):223–230, 1990.

[19] Andrzej Trybulec. On the decomposition of finite sequences. *Formalized Mathematics*, 5(**3**):317–322, 1996.

[20] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.

[21] Andrzej Trybulec and Czesław Byliński. Some properties of real numbers. *Formalized Mathematics*, 1(**3**):445–449, 1990.

[22] Wojciech A. Trybulec. Pigeon hole principle. *Formalized Mathematics*, 1(**3**):575–579, 1990.

[23] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.

[24] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.

[25] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.
[26] Mirosław Wysocki and Agata Darmochwał. Subsets of topological spaces. *Formalized Mathematics*, 1(**1**):231–237, 1990.

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