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The First Part of Jordan's Theorem for Special Polygons

Yatsuka Nakamura
Shinshu University
Nagano

Andrzej Trybulec
Warsaw University
Białystok

Summary. We prove here the first part of Jordan's theorem for special polygons, i.e., the complement of a special polygon is the union of two components (a left component and a right component). At this stage, we do not know if the two components are different from each other.

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The articles [7], [11], [5], [21], [24], [23], [8], [1], [16], [25], [18], [19], [4], [3], [2], [22], [9], [10], [26], [17], [6], [12], [15], [20], [14], and [13] provide the notation and terminology for this paper.

We adopt the following convention: i, j, k_1, k_2, i_1, i_2, j_1, j_2 will be natural numbers and f will be a non constant standard special circular sequence.

The following propositions are true:

(1) $(\widetilde{\mathcal{L}}(f))^{\rm c} \neq \emptyset$.

(2) For all i, j such that $i \leq$ len the Go-board of f and $j \leq$ width the Go-board of f holds Int cell(the Go-board of f, $i, j) \subseteq (\widetilde{\mathcal{L}}(f))^{\rm c}$.

(3) Given i, j. Suppose $i \leq$ len the Go-board of f and $j \leq$ width the Go-board of f. Then $\overline{\mathrm{Down}(\mathrm{Int\,cell}(\text{the Go-board of } f, i, j), (\widetilde{\mathcal{L}}(f))^{\rm c})} =$ cell(the Go-board of f, $i, j) \cap (\widetilde{\mathcal{L}}(f))^{\rm c}$.

(4) Given i, j. Suppose $i \leq$ len the Go-board of f and $j \leq$ width the Go-board of f. Then $\mathrm{Down}(\mathrm{Int\,cell}(\text{the Go-board of } f, i, j), (\widetilde{\mathcal{L}}(f))^{\rm c})$ is connected and $\mathrm{Down}(\mathrm{Int\,cell}(\text{the Go-board of } f, i, j), (\widetilde{\mathcal{L}}(f))^{\rm c}) =$ Int cell(the Go-board of f, $i, j)$.

(5) $(\widetilde{\mathcal{L}}(f))^{\rm c} = \bigcup\{\overline{\mathrm{Down}(\mathrm{Int\,cell}(\text{the Go-board of } f, i, j), (\widetilde{\mathcal{L}}(f))^{\rm c})} : i \leq$ len the Go-board of $f \wedge j \leq$ width the Go-board of $f\}$.

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(6) Down(LeftComp(f), $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$) $\cup$ Down(RightComp(f), $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$) is a union of components of $(\mathcal{E}^2_{\mathrm{T}}) \restriction (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ and Down(LeftComp(f), $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$) = LeftComp($f$) and Down(RightComp($f$), $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$) = RightComp($f$).

(7) Given i_1, j_1, i_2, j_2. Suppose that
(i) $i_1 \leq$ len the Go-board of f,
(ii) $j_1 \leq$ width the Go-board of f,
(iii) $i_2 \leq$ len the Go-board of f,
(iv) $j_2 \leq$ width the Go-board of f, and
(v) i_1, j_1, i_2, and j_2 are adjacent.
Then Int cell(the Go-board of f, i_1, j_1) $\subseteq$ LeftComp(f) $\cup$ RightComp(f) if and only if Int cell(the Go-board of f, i_2, j_2) $\subseteq$ LeftComp(f) $\cup$ RightComp(f).

(8) Let F_1, F_2 be finite sequences of elements of $\mathbb{N}$. Suppose that
(i) len F_1 = len F_2,
(ii) there exists i such that $i \in \mathrm{dom}\, F_1$ and Int cell(the Go-board of f, $\pi_i F_1, \pi_i F_2$) $\subseteq$ LeftComp(f) $\cup$ RightComp(f),
(iii) for every i such that $1 \leq i$ and $i <$ len F_1 holds $\pi_i F_1$, $\pi_i F_2$, $\pi_{i+1} F_1$, and $\pi_{i+1} F_2$ are adjacent, and
(iv) for all i, k_1, k_2 such that $i \in \mathrm{dom}\, F_1$ and $k_1 = F_1(i)$ and $k_2 = F_2(i)$ holds $k_1 \leq$ len the Go-board of f and $k_2 \leq$ width the Go-board of f.
Given i. If $i \in \mathrm{dom}\, F_1$, then Int cell(the Go-board of f, $\pi_i F_1, \pi_i F_2$) $\subseteq$ LeftComp(f) $\cup$ RightComp(f).

(9) There exist i, j such that $i \leq$ len the Go-board of f and $j \leq$ width the Go-board of f and Int cell(the Go-board of f, i, j) $\subseteq$ LeftComp(f) $\cup$ RightComp(f).

(10) For all i, j such that $i \leq$ len the Go-board of f and $j \leq$ width the Go-board of f holds Int cell(the Go-board of f, i, j) $\subseteq$ LeftComp(f) $\cup$ RightComp(f).

(11) $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ = LeftComp(f) $\cup$ RightComp(f).

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