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# Meet – Continuous Lattices [1]

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**Summary.** The aim of this work is the formalization of Chapter 0 Section 4 of [11]. In this paper the definition of meet-continuous lattices is introduced. Theorem 4.2 and Remark 4.3 are proved.

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The terminology and notation used in this paper are introduced in the following papers: [18], [21], [9], [22], [24], [23], [19], [6], [4], [14], [10], [7], [17], [5], [20], [2], [12], [1], [3], [13], [25], [8], [15], and [16].

## 1. Preliminaries

Let $X$, $Y$ be non empty sets, let $f$ be a function from $X$ into $Y$, and let $Z$ be a non empty subset of $X$. One can verify that $f^\circ Z$ is non empty.

One can check that every non empty relational structure which is reflexive and connected has g.l.b.'s and l.u.b.'s.

Let $C$ be a chain. One can verify that $\Omega_C$ is directed.

Let $X$ be a set. Note that every binary relation on $X$ which is ordering is also reflexive, antisymmetric, and transitive.

Let $X$ be a non empty set. One can verify that there exists a binary relation on $X$ which is ordering.

The following propositions are true:

(1) Let $L$ be an up-complete semilattice, and let $D$ be a non empty directed subset of $L$, and let $x$ be an element of $L$. Then $\sup \{x\} \sqcap D$ exists in $L$.

(2) Let $L$ be an up-complete sup-semilattice, and let $D$ be a non empty directed subset of $L$, and let $x$ be an element of $L$. Then $\sup \{x\} \sqcup D$ exists in $L$.

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(3) For every up-complete sup-semilattice $L$ and for all non empty directed subsets $A$, $B$ of $L$ holds $A \leq \sup(A \sqcup B)$.

(4) For every up-complete sup-semilattice $L$ and for all non empty directed subsets $A$, $B$ of $L$ holds $\sup(A \sqcup B) = \sup A \sqcup \sup B$.

(5) Let $L$ be an up-complete semilattice and let $D$ be a non empty directed subset of $[\!:L, L:\!]$. Then $\{\sup(\{x\} \sqcap \pi_2(D)) : x$ ranges over elements of $L$, $x \in \pi_1(D)\} = \{\sup X : X$ ranges over non empty directed subsets of $L$, $\bigvee_{x\,:\,\text{element of } L}\ X = \{x\} \sqcap \pi_2(D)\ \wedge\ x \in \pi_1(D)\}$.

(6) Let $L$ be a semilattice and let $D$ be a non empty directed subset of $[\!:L, L:\!]$. Then $\bigcup\{X : X$ ranges over non empty directed subsets of $L$, $\bigvee_{x\,:\,\text{element of } L}\ X = \{x\} \sqcap \pi_2(D)\ \wedge\ x \in \pi_1(D)\} = \pi_1(D) \sqcap \pi_2(D)$.

(7) Let $L$ be an up-complete semilattice and let $D$ be a non empty directed subset of $[\!:L, L:\!]$. Then $\sup \bigcup\{X : X$ ranges over non empty directed subsets of $L$, $\bigvee_{x\,:\,\text{element of } L}\ X = \{x\} \sqcap \pi_2(D)\ \wedge\ x \in \pi_1(D)\}$ exists in $L$.

(8) Let $L$ be an up-complete semilattice and let $D$ be a non empty directed subset of $[\!:L, L:\!]$. Then $\sup \{\sup X : X$ ranges over non empty directed subsets of $L$, $\bigvee_{x\,:\,\text{element of } L}\ X = \{x\} \sqcap \pi_2(D)\ \wedge\ x \in \pi_1(D)\}$ exists in $L$.

(9) Let $L$ be an up-complete semilattice and let $D$ be a non empty directed subset of $[\!:L, L:\!]$. Then $\bigsqcup_L \{\sup X : X$ ranges over non empty directed subsets of $L$, $\bigvee_{x\,:\,\text{element of } L}\ X = \{x\} \sqcap \pi_2(D)\ \wedge\ x \in \pi_1(D)\} \leq \bigsqcup_L \bigcup\{X : X$ ranges over non empty directed subsets of $L$, $\bigvee_{x\,:\,\text{element of } L}\ X = \{x\} \sqcap \pi_2(D)\ \wedge\ x \in \pi_1(D)\}$.

(10) Let $L$ be an up-complete semilattice and let $D$ be a non empty directed subset of $[\!:L, L:\!]$. Then $\bigsqcup_L \{\sup X : X$ ranges over non empty directed subsets of $L$, $\bigvee_{x\,:\,\text{element of } L}\ X = \{x\} \sqcap \pi_2(D)\ \wedge\ x \in \pi_1(D)\} = \bigsqcup_L \bigcup\{X : X$ ranges over non empty directed subsets of $L$, $\bigvee_{x\,:\,\text{element of } L}\ X = \{x\} \sqcap \pi_2(D)\ \wedge\ x \in \pi_1(D)\}$.

Let $S$, $T$ be up-complete non empty reflexive relational structures. One can verify that $[\!:S, T:\!]$ is up-complete.

The following four propositions are true:

(11) Let $S$, $T$ be non empty reflexive antisymmetric relational structures. If $[\!:S, T:\!]$ is up-complete, then $S$ is up-complete and $T$ is up-complete.

(12) Let $L$ be an up-complete antisymmetric non empty reflexive relational structure and let $D$ be a non empty directed subset of $[\!:L, L:\!]$. Then $\sup D = \langle \sup \pi_1(D), \sup \pi_2(D) \rangle$.

(13) Let $S_1$, $S_2$ be non empty relational structures, and let $D$ be a subset of $S_1$, and let $f$ be a map from $S_1$ into $S_2$. If $f$ is monotone, then $f^\circ{\downarrow}D \subseteq {\downarrow}(f^\circ D)$.

(14) Let $S_1$, $S_2$ be non empty relational structures, and let $D$ be a subset of $S_1$, and let $f$ be a map from $S_1$ into $S_2$. If $f$ is monotone, then $f^\circ{\uparrow}D \subseteq {\uparrow}(f^\circ D)$.

Let us observe that every non empty reflexive relational structure which is trivial is also distributive and complemented.

Let us note that there exists a lattice which is strict, non empty, and trivial.

One can prove the following three propositions:

(15) Let $H$ be a distributive complete lattice, and let $a$ be an element of $H$, and let $X$ be a finite subset of $H$. Then $\sup(\{a\} \sqcap X) = a \sqcap \sup X$.

(16) Let $H$ be a distributive complete lattice, and let $a$ be an element of $H$, and let $X$ be a finite subset of $H$. Then $\inf(\{a\} \sqcup X) = a \sqcup \inf X$.

(17) Let $H$ be a complete distributive lattice, and let $a$ be an element of $H$, and let $X$ be a finite subset of $H$. Then $a \sqcap \square$ preserves sup of $X$.

## 2. The properties of nets

The scheme *ExNet* concerns a non empty relational structure $\mathcal{A}$, a prenet $\mathcal{B}$ over $\mathcal{A}$, and a unary functor $\mathcal{F}$ yielding an element of the carrier of $\mathcal{A}$, and states that:

There exists a prenet $M$ over $\mathcal{A}$ such that

(i) the relational structure of $M$ = the relational structure of $\mathcal{B}$,

and

(ii) for every element $i$ of the carrier of $M$ holds (the mapping of $M)(i) = \mathcal{F}($(the mapping of $\mathcal{B})(i))$

for all values of the parameters.

The following three propositions are true:

(18) Let $L$ be a non empty relational structure and let $N$ be a prenet over $L$. If $N$ is eventually-directed, then $\operatorname{rng} \operatorname{netmap}(N, L)$ is directed.

(19) Let $L$ be a non empty reflexive relational structure, and let $D$ be a non empty directed subset of $L$, and let $n$ be a function from $D$ into the carrier of $L$. Then $\langle D,$ (the internal relation of $L) |^2(D), n\rangle$ is a prenet over $L$.

(20) Let $L$ be a non empty reflexive relational structure, and let $D$ be a non empty directed subset of $L$, and let $n$ be a function from $D$ into the carrier of $L$, and let $N$ be a prenet over $L$. Suppose $n = \mathrm{id}_D$ and $N = \langle D,$ (the internal relation of $L) |^2(D), n\rangle$. Then $N$ is eventually-directed.

Let $L$ be a non empty relational structure and let $N$ be a net structure over $L$. The functor $\sup N$ yielding an element of $L$ is defined by:

(Def. 1) $\sup N = \mathrm{Sup}($the mapping of $N)$.

Let $L$ be a non empty relational structure, let $J$ be a set, and let $f$ be a function from $J$ into the carrier of $L$. The functor $\mathrm{FinSups}(f)$ yields a prenet over $L$ and is defined by the condition (Def. 2).

(Def. 2) There exists a function $g$ from $\mathrm{Fin}\, J$ into the carrier of $L$ such that for every element $x$ of $\mathrm{Fin}\, J$ holds $g(x) = \sup(f^\circ x)$ and $\mathrm{FinSups}(f) = \langle \mathrm{Fin}\, J, \subseteq_{\mathrm{Fin}\, J}, g\rangle$.

The following proposition is true

(21) Let $L$ be a non empty relational structure, and let $J$, $x$ be sets, and let $f$ be a function from $J$ into the carrier of $L$. Then $x$ is an element of $\mathrm{FinSups}(f)$ if and only if $x$ is an element of $\mathrm{Fin}\,J$.

Let $L$ be a complete antisymmetric non empty reflexive relational structure, let $J$ be a set, and let $f$ be a function from $J$ into the carrier of $L$. Note that $\mathrm{FinSups}(f)$ is monotone.

Let $L$ be a non empty relational structure, let $x$ be an element of $L$, and let $N$ be a non empty net structure over $L$. The functor $x \sqcap N$ yielding a strict net structure over $L$ is defined by the conditions (Def. 3).

(Def. 3) (i) The relational structure of $x \sqcap N =$ the relational structure of $N$, and

(ii) for every element $i$ of the carrier of $x \sqcap N$ there exists an element $y$ of $L$ such that $y =$ (the mapping of $N$)($i$) and (the mapping of $x \sqcap N$)($i$) $= x \sqcap y$.

We now state the proposition

(22) Let $L$ be a non empty relational structure, and let $N$ be a non empty net structure over $L$, and let $x$ be an element of $L$, and let $y$ be a set. Then $y$ is an element of $N$ if and only if $y$ is an element of $x \sqcap N$.

Let $L$ be a non empty relational structure, let $x$ be an element of $L$, and let $N$ be a non empty net structure over $L$. Observe that $x \sqcap N$ is non empty.

Let $L$ be a non empty relational structure, let $x$ be an element of $L$, and let $N$ be a prenet over $L$. Note that $x \sqcap N$ is directed.

Next we state several propositions:

(23) Let $L$ be a non empty relational structure, and let $x$ be an element of $L$, and let $F$ be a non empty net structure over $L$. Then $\mathrm{rng}$ (the mapping of $x \sqcap F$) $= \{x\} \sqcap \mathrm{rng}$ (the mapping of $F$).

(24) Let $L$ be a non empty relational structure, and let $J$ be a set, and let $f$ be a function from $J$ into the carrier of $L$. If for every set $x$ holds sup $f^{\circ}x$ exists in $L$, then $\mathrm{rng}\,\mathrm{netmap}(\mathrm{FinSups}(f), L) \subseteq \mathrm{finsups}(\mathrm{rng}\,f)$.

(25) Let $L$ be a non empty reflexive antisymmetric relational structure, and let $J$ be a set, and let $f$ be a function from $J$ into the carrier of $L$. Then $\mathrm{rng}\,f \subseteq \mathrm{rng}\,\mathrm{netmap}(\mathrm{FinSups}(f), L)$.

(26) Let $L$ be a non empty reflexive antisymmetric relational structure, and let $J$ be a set, and let $f$ be a function from $J$ into the carrier of $L$. Suppose sup $\mathrm{rng}\,f$ exists in $L$ and sup $\mathrm{rng}\,\mathrm{netmap}(\mathrm{FinSups}(f), L)$ exists in $L$ and for every element $x$ of $\mathrm{Fin}\,J$ holds sup $f^{\circ}x$ exists in $L$. Then $\mathrm{Sup}(f) = \sup \mathrm{FinSups}(f)$.

(27) Let $L$ be an antisymmetric transitive relational structure with g.l.b.'s, and let $N$ be a prenet over $L$, and let $x$ be an element of $L$. If $N$ is eventually-directed, then $x \sqcap N$ is eventually-directed.

(28) Let $L$ be an up-complete semilattice. Suppose that for every element $x$ of $L$ and for every non empty directed subset $E$ of $L$ such that $x \leq \sup E$ holds $x \leq \sup(\{x\} \sqcap E)$. Let $D$ be a non empty directed subset of $L$ and let $x$ be an element of $L$. Then $x \sqcap \sup D = \sup(\{x\} \sqcap D)$.

(29) Let $L$ be a poset with l.u.b.'s. Suppose that for every directed subset $X$ of $L$ and for every element $x$ of $L$ holds $x \sqcap \sup X = \sup(\{x\} \sqcap X)$. Let $X$ be a subset of $L$ and let $x$ be an element of $L$. If $\sup X$ exists in $L$, then $x \sqcap \sup X = \sup(\{x\} \sqcap \mathrm{finsups}(X))$.

(30) Let $L$ be an up-complete lattice. Suppose that for every subset $X$ of $L$ and for every element $x$ of $L$ holds $x \sqcap \sup X = \sup(\{x\} \sqcap \mathrm{finsups}(X))$. Let $X$ be a non empty directed subset of $L$ and let $x$ be an element of $L$. Then $x \sqcap \sup X = \sup(\{x\} \sqcap X)$.

## 3. On the inf and sup operation

Let $L$ be a non empty relational structure. The functor $\mathrm{inf\_op}(L)$ yields a map from $[\!: L, L :\!]$ into $L$ and is defined as follows:

(Def. 4) For all elements $x$, $y$ of $L$ holds $(\mathrm{inf\_op}(L))(\langle x, y\rangle) = x \sqcap y$.

One can prove the following proposition

(31) For every non empty relational structure $L$ and for every element $x$ of $[\!: L, L :\!]$ holds $(\mathrm{inf\_op}(L))(x) = x_{\mathbf{1}} \sqcap x_{\mathbf{2}}$.

Let $L$ be a transitive antisymmetric relational structure with g.l.b.'s. Note that $\mathrm{inf\_op}(L)$ is monotone.

The following two propositions are true:

(32) For every non empty relational structure $S$ and for all subsets $D_1$, $D_2$ of $S$ holds $(\mathrm{inf\_op}(S))^\circ[\!: D_1, D_2 :\!] = D_1 \sqcap D_2$.

(33) For every up-complete semilattice $L$ and for every non empty directed subset $D$ of $[\!: L, L :\!]$ holds $\sup((\mathrm{inf\_op}(L))^\circ D) = \sup(\pi_1(D) \sqcap \pi_2(D))$.

Let $L$ be a non empty relational structure. The functor $\mathrm{sup\_op}(L)$ yielding a map from $[\!: L, L :\!]$ into $L$ is defined by:

(Def. 5) For all elements $x$, $y$ of $L$ holds $(\mathrm{sup\_op}(L))(\langle x, y\rangle) = x \sqcup y$.

We now state the proposition

(34) For every non empty relational structure $L$ and for every element $x$ of $[\!: L, L :\!]$ holds $(\mathrm{sup\_op}(L))(x) = x_{\mathbf{1}} \sqcup x_{\mathbf{2}}$.

Let $L$ be a transitive antisymmetric relational structure with l.u.b.'s. Observe that $\mathrm{sup\_op}(L)$ is monotone.

The following two propositions are true:

(35) For every non empty relational structure $S$ and for all subsets $D_1$, $D_2$ of $S$ holds $(\mathrm{sup\_op}(S))^\circ[\!: D_1, D_2 :\!] = D_1 \sqcup D_2$.

(36) For every complete non empty poset $L$ and for every non empty filtered subset $D$ of $[\!: L, L :\!]$ holds $\inf((\mathrm{sup\_op}(L))^\circ D) = \inf(\pi_1(D) \sqcup \pi_2(D))$.

## 4. Meet-continuous lattices

Let $R$ be a non empty reflexive relational structure. We say that $R$ satisfies MC if and only if:

(Def. 6) For every element $x$ of $R$ and for every non empty directed subset $D$ of $R$ holds $x \sqcap \sup D = \sup(\{x\} \sqcap D)$.

Let $R$ be a non empty reflexive relational structure. We say that $R$ is meet-continuous if and only if:

(Def. 7) $R$ is up-complete and satisfies MC.

One can check that every non empty reflexive relational structure which is trivial satisfies MC.

Let us observe that every non empty reflexive relational structure which is meet-continuous is also up-complete and satisfies MC and every non empty reflexive relational structure which is up-complete and satisfies MC is also meet-continuous.

Let us observe that there exists a lattice which is strict, non empty, and trivial.

Next we state two propositions:

(37) Let $S$ be a non empty reflexive relational structure. Suppose that for every subset $X$ of $S$ and for every element $x$ of $S$ holds $x \sqcap \sup X = \bigsqcup_S \{x \sqcap y : y$ ranges over elements of $S$, $y \in X\}$. Then $S$ satisfies MC.

(38) Let $L$ be an up-complete semilattice. If $\mathrm{SupMap}(L)$ is meet-preserving, then for all ideals $I_1$, $I_2$ of $L$ holds $\sup I_1 \sqcap \sup I_2 = \sup(I_1 \sqcap I_2)$.

Let $L$ be an up-complete sup-semilattice. Note that $\mathrm{SupMap}(L)$ is join-preserving.

One can prove the following propositions:

(39) Let $L$ be an up-complete semilattice. If for all ideals $I_1$, $I_2$ of $L$ holds $\sup I_1 \sqcap \sup I_2 = \sup(I_1 \sqcap I_2)$, then $\mathrm{SupMap}(L)$ is meet-preserving.

(40) Let $L$ be an up-complete semilattice. Suppose that for all ideals $I_1$, $I_2$ of $L$ holds $\sup I_1 \sqcap \sup I_2 = \sup(I_1 \sqcap I_2)$. Let $D_1$, $D_2$ be directed non empty subsets of $L$. Then $\sup D_1 \sqcap \sup D_2 = \sup(D_1 \sqcap D_2)$.

(41) Let $L$ be a non empty reflexive relational structure. Suppose $L$ satisfies MC. Let $x$ be an element of $L$ and let $N$ be a non empty prenet over $L$. If $N$ is eventually-directed, then $x \sqcap \sup N = \sup(\{x\} \sqcap \mathrm{rng}\,\mathrm{netmap}(N, L))$.

(42) Let $L$ be a non empty reflexive relational structure. Suppose that for every element $x$ of $L$ and for every prenet $N$ over $L$ such that $N$ is eventually-directed holds $x \sqcap \sup N = \sup(\{x\} \sqcap \mathrm{rng}\,\mathrm{netmap}(N, L))$. Then $L$ satisfies MC.

(43) Let $L$ be an up-complete antisymmetric non empty reflexive relational structure. Suppose $\mathrm{inf\_op}(L)$ is directed-sups-preserving. Let $D_1$, $D_2$ be non empty directed subsets of $L$. Then $\sup D_1 \sqcap \sup D_2 = \sup(D_1 \sqcap D_2)$.

(44) Let $L$ be a non empty reflexive antisymmetric relational structure. If for all non empty directed subsets $D_1$, $D_2$ of $L$ holds $\sup D_1 \sqcap \sup D_2 = \sup(D_1 \sqcap D_2)$, then $L$ satisfies MC.

(45) Let $L$ be an antisymmetric non empty reflexive relational structure with g.l.b.'s, satisfying MC, and let $x$ be an element of $L$, and let $D$ be a non empty directed subset of $L$. If $x \leq \sup D$, then $x = \sup(\{x\} \sqcap D)$.

(46) Let $L$ be an up-complete semilattice. Suppose that for every element $x$ of $L$ and for every non empty directed subset $E$ of $L$ such that $x \leq \sup E$ holds $x \leq \sup(\{x\} \sqcap E)$. Then $\inf\_\mathrm{op}(L)$ is directed-sups-preserving.

(47) Let $L$ be a complete antisymmetric non empty reflexive relational structure. Suppose that for every element $x$ of $L$ and for every prenet $N$ over $L$ such that $N$ is eventually-directed holds $x \sqcap \sup N = \sup(\{x\} \sqcap \mathrm{rng}\,\mathrm{netmap}(N, L))$. Let $x$ be an element of $L$, and let $J$ be a set, and let $f$ be a function from $J$ into the carrier of $L$. Then $x \sqcap \mathrm{Sup}(f) = \sup(x \sqcap \mathrm{FinSups}(f))$.

(48) Let $L$ be a complete semilattice. Suppose that for every element $x$ of $L$ and for every set $J$ and for every function $f$ from $J$ into the carrier of $L$ holds $x \sqcap \mathrm{Sup}(f) = \sup(x \sqcap \mathrm{FinSups}(f))$. Let $x$ be an element of $L$ and let $N$ be a prenet over $L$. If $N$ is eventually-directed, then $x \sqcap \sup N = \sup(\{x\} \sqcap \mathrm{rng}\,\mathrm{netmap}(N, L))$.

(49) For every up-complete lattice $L$ holds $L$ is meet-continuous iff $\mathrm{SupMap}(L)$ is meet-preserving and join-preserving.

Let $L$ be a meet-continuous lattice. One can verify that $\mathrm{SupMap}(L)$ is meet-preserving and join-preserving.

We now state four propositions:

(50) Let $L$ be an up-complete lattice. Then $L$ is meet-continuous if and only if for all ideals $I_1$, $I_2$ of $L$ holds $\sup I_1 \sqcap \sup I_2 = \sup(I_1 \sqcap I_2)$.

(51) Let $L$ be an up-complete lattice. Then $L$ is meet-continuous if and only if for all non empty directed subsets $D_1$, $D_2$ of $L$ holds $\sup D_1 \sqcap \sup D_2 = \sup(D_1 \sqcap D_2)$.

(52) Let $L$ be an up-complete lattice. Then $L$ is meet-continuous if and only if for every element $x$ of $L$ and for every non empty directed subset $D$ of $L$ such that $x \leq \sup D$ holds $x = \sup(\{x\} \sqcap D)$.

(53) For every up-complete semilattice $L$ holds $L$ is meet-continuous iff $\inf\_\mathrm{op}(L)$ is directed-sups-preserving.

Let $L$ be a meet-continuous semilattice. Observe that $\inf\_\mathrm{op}(L)$ is directed-sups-preserving.

The following two propositions are true:

(54) Let $L$ be an up-complete semilattice. Then $L$ is meet-continuous if and only if for every element $x$ of $L$ and for every non empty prenet $N$ over $L$ such that $N$ is eventually-directed holds $x \sqcap \sup N = \sup(\{x\} \sqcap \mathrm{rng}\,\mathrm{netmap}(N, L))$.

(55) Let $L$ be a complete semilattice. Then $L$ is meet-continuous if and only if for every element $x$ of $L$ and for every set $J$ and for every function $f$ from $J$ into the carrier of $L$ holds $x \sqcap \mathrm{Sup}(f) = \sup(x \sqcap \mathrm{FinSups}(f))$.

Let $L$ be a meet-continuous semilattice and let $x$ be an element of $L$. One can verify that $x \sqcap \Box$ is directed-sups-preserving.

The following proposition is true

(56) For every complete non empty poset $H$ holds $H$ is Heyting iff $H$ is meet-continuous and distributive.

Let us mention that every non empty poset which is complete and Heyting is also meet-continuous and distributive and every non empty poset which is complete, meet-continuous, and distributive is also Heyting.

## References


[1] Grzegorz Bancerek. Bounds in posets and relational substructures. *Formalized Mathematics*, 6(**1**):81–91, 1997.

[2] Grzegorz Bancerek. Complete lattices. *Formalized Mathematics*, 2(**5**):719–725, 1991.

[3] Grzegorz Bancerek. Directed sets, nets, ideals, filters, and maps. *Formalized Mathematics*, 6(**1**):93–107, 1997.

[4] Grzegorz Bancerek. The well ordering relations. *Formalized Mathematics*, 1(**1**):123–129, 1990.

[5] Józef Białas. Group and field definitions. *Formalized Mathematics*, 1(**3**):433–439, 1990.

[6] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[7] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.

[8] Czesław Byliński. Galois connections. *Formalized Mathematics*, 6(**1**):131–143, 1997.

[9] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(**1**):47–53, 1990.

[10] Agata Darmochwał. Finite sets. *Formalized Mathematics*, 1(**1**):165–167, 1990.

[11] G. Gierz, K.H. Hofmann, K. Keimel, J.D. Lawson, M. Mislove, and D.S. Scott. *A Compendium of Continuous Lattices*. Springer-Verlag, Berlin, Heidelberg, New York, 1980.

[12] Adam Grabowski. On the category of posets. *Formalized Mathematics*, 5(**4**):501–505, 1996.

[13] Adam Grabowski and Robert Milewski. Boolean posets, posets under inclusion and products of relational structures. *Formalized Mathematics*, 6(**1**):117–121, 1997.

[14] Krzysztof Hryniewiecki. Relations of tolerance. *Formalized Mathematics*, 2(**1**):105–109, 1991.

[15] Artur Korniłowicz. Cartesian products of relations and relational structures. *Formalized Mathematics*, 6(**1**):145–152, 1997.

[16] Artur Korniłowicz. Definitions and properties of the join and meet of subsets. *Formalized Mathematics*, 6(**1**):153–158, 1997.

[17] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Formalized Mathematics*, 1(**1**):223–230, 1990.

[18] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.

[19] Andrzej Trybulec and Agata Darmochwał. Boolean domains. *Formalized Mathematics*, 1(**1**):187–190, 1990.

[20] Wojciech A. Trybulec. Partially ordered sets. *Formalized Mathematics*, 1(**2**):313–319, 1990.

[21] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.

[22] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.
[23] Edmund Woronowicz. Relations defined on sets. *Formalized Mathematics*, 1(**1**):181–186, 1990.
[24] Edmund Woronowicz and Anna Zalewska. Properties of binary relations. *Formalized Mathematics*, 1(**1**):85–89, 1990.
[25] Mariusz Żynel and Czesław Byliński. Properties of relational structures, posets, lattices and maps. *Formalized Mathematics*, 6(**1**):123–130, 1997.

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