FORMALIZED MATHEMATICS
Volume 6, Number 2, 1997
University of Białystok

Moore-Smith Convergence[1]

Andrzej Trybulec
Warsaw University
Białystok

Summary. The paper introduces the concept of a net (a generalized sequence). The goal is to enable the continuation of the translation of [16].

MML Identifier: `YELLOW_6`.

The notation and terminology used here are introduced in the following papers: [30], [36], [35], [13], [31], [14], [37], [38], [11], [12], [10], [26], [9], [1], [2], [33], [23], [24], [3], [4], [25], [18], [20], [39], [15], [27], [32], [21], [34], [5], [28], [6], [7], [17], [19], [29], [8], and [22].

1. Preliminaries

The scheme *SubsetEq* deals with a non empty set $\mathcal{A}$, subsets $\mathcal{B}$, $\mathcal{C}$ of $\mathcal{A}$, and a unary predicate $\mathcal{P}$, and states that:

$\mathcal{B} = \mathcal{C}$

provided the following conditions are met:

- For every element y of $\mathcal{A}$ holds $y \in \mathcal{B}$ iff $\mathcal{P}[y]$,
- For every element y of $\mathcal{A}$ holds $y \in \mathcal{C}$ iff $\mathcal{P}[y]$.

We now state the proposition

(1) For all sets X, x holds $X \longmapsto x$ is constant.

Let X, x be sets. Note that $X \longmapsto x$ is constant.

Let f be a function. Let us assume that f is non empty and constant. The value of f is defined by:

(Def. 1) There exists a set x such that $x \in \operatorname{dom} f$ and the value of $f = f(x)$.

[1]This work was partially supported by the Office of Naval Research Grant N00014-95-1-1336.

ISSN 1426–2630

Let us note that there exists a function which is non empty and constant.

Let f be a non empty constant function. Then the value of f can be characterized by the condition:

(Def. 2) There exists a set x such that $x \in \operatorname{dom} f$ and the value of $f = f(x)$.

The following propositions are true:

(2) For every non empty set X and for every set x holds the value of $X \longmapsto x = x$.

(3) For every function f holds $\overline{\overline{\operatorname{rng} f}} \subseteq \overline{\overline{\operatorname{dom} f}}$.

Let us note that every set which is universal is also transitive and a Tarski class and every set which is transitive and a Tarski class is also universal.

In the sequel x, X will be sets and T will be a universal class.

Let us consider X. The universe of X is defined as follows:

(Def. 3) The universe of $X = \mathbf{T}(X^{*\in})$.

We now state the proposition

(4) $\mathbf{T}(X)$ is a Tarski class.

Let us consider X. Note that $\mathbf{T}(X)$ is a Tarski class.

Let us consider X. Observe that the universe of X is transitive and a Tarski class.

Let us consider X. One can check that the universe of X is universal and non empty.

One can prove the following proposition

(5) For every function f such that $\operatorname{dom} f \in T$ and $\operatorname{rng} f \subseteq T$ holds $\prod f \in T$.

2. Topological spaces

Next we state the proposition

(6) Let T be a non empty topological space, A be a subset of T, and p be a point of T. Then $p \in \overline{A}$ if and only if for every neighbourhood G of p holds G meets A.

Let T be a non empty topological space. We introduce T is Hausdorff as a synonym of T is T_2.

One can verify that there exists a non empty topological space which is Hausdorff.

One can prove the following two propositions:

(7) Let X be a non empty topological space and A be a subset of the carrier of X. Then Ω_X is a neighbourhood of A.

(8) Let X be a non empty topological space, A be a subset of the carrier of X, and Y be a neighbourhood of A. Then $A \subseteq Y$.

3. 1-sorted structures

The following proposition is true

(9) Let Y be a non empty set, J be a 1-sorted yielding many sorted set indexed by Y, and i be an element of Y. Then $(\text{support}\,J)(i) =$ the carrier of $J(i)$.

Let us note that there exists a function which is non empty, constant, and 1-sorted yielding.

Let J be a 1-sorted yielding function. Let us observe that J is nonempty if and only if:

(Def. 4) For every set i such that $i \in \operatorname{rng} J$ holds i is a non empty 1-sorted structure.

We introduce J is yielding non-empty carriers as a synonym of J is nonempty.

Let X be a set and let L be a 1-sorted structure. Observe that $X \longmapsto L$ is 1-sorted yielding.

Let I be a set. Observe that there exists a 1-sorted yielding many sorted set indexed by I which is yielding non-empty carriers.

Let I be a non empty set and let J be a relational structure yielding many sorted set indexed by I. One can verify that the carrier of $\prod J$ is functional.

Let I be a set and let J be a yielding non-empty carriers 1-sorted yielding many sorted set indexed by I. Observe that support J is non-empty.

Next we state the proposition

(10) Let T be a non empty 1-sorted structure, S be a subset of the carrier of T, and p be an element of the carrier of T. Then $p \notin S$ if and only if $p \in -S$.

4. Relational structures

Let T be a non empty relational structure and let A be a lower subset of T. Observe that $-A$ is upper.

Let T be a non empty relational structure and let A be an upper subset of T. Observe that $-A$ is lower.

Let N be a non empty relational structure. Let us observe that N is directed if and only if:

(Def. 5) For all elements x, y of N there exists an element z of N such that $x \leqslant z$ and $y \leqslant z$.

Let X be a set. Note that $2^X_\subseteq$ is directed.

Let us mention that there exists a relational structure which is non empty, directed, transitive, and strict.

Let M be a non empty set, let N be a non empty relational structure, let f be a function from M into the carrier of N, and let m be an element of M. Then $f(m)$ is an element of N.

Let I be a set. Note that there exists a relational structure yielding many sorted set indexed by I which is yielding non-empty carriers.

Let I be a non empty set and let J be a yielding non-empty carriers relational structure yielding many sorted set indexed by I. Observe that $\prod J$ is non empty.

Next we state the proposition

(11) For all relational structures R_1, R_2 holds $\Omega_{[: R_1, R_2 :]} = [: \Omega_{(R_1)}, \Omega_{(R_2)} :]$.

Let Y_1, Y_2 be directed relational structures. Observe that $[: Y_1, Y_2 :]$ is directed.

Next we state the proposition

(12) For every relational structure R holds the carrier of R = the carrier of $R^{\smile}$.

Let S be a 1-sorted structure and let N be a net structure over S. We say that N is constant if and only if:

(Def. 6) The mapping of N is constant.

Let R be a relational structure, let T be a non empty 1-sorted structure, and let p be an element of the carrier of T. The functor $R \longmapsto p$ yielding a strict net structure over T is defined by the conditions (Def. 7).

(Def. 7)(i) The relational structure of $(R \longmapsto p)$ = the relational structure of R, and

(ii) the mapping of $(R \longmapsto p)$ = (the carrier of $(R \longmapsto p)) \longmapsto p$.

Let R be a relational structure, let T be a non empty 1-sorted structure, and let p be an element of the carrier of T. Note that $R \longmapsto p$ is constant.

Let R be a non empty relational structure, let T be a non empty 1-sorted structure, and let p be an element of the carrier of T. One can verify that $R \longmapsto p$ is non empty.

Let R be a non empty directed relational structure, let T be a non empty 1-sorted structure, and let p be an element of the carrier of T. Note that $R \longmapsto p$ is directed.

Let R be a non empty transitive relational structure, let T be a non empty 1-sorted structure, and let p be an element of the carrier of T. One can check that $R \longmapsto p$ is transitive.

We now state two propositions:

(13) Let R be a relational structure, T be a non empty 1-sorted structure, and p be an element of the carrier of T. Then the carrier of $(R \longmapsto p)$ = the carrier of R.

(14) Let R be a non empty relational structure, T be a non empty 1-sorted structure, p be an element of the carrier of T, and q be an element of the carrier of $(R \longmapsto p)$.Then $(R \longmapsto p)(q) = p$.

Let T be a non empty 1-sorted structure and let N be a non empty net structure over T. Observe that the mapping of N is non empty.

5. Substructures of nets

One can prove the following propositions:

(15) Every relational structure R is a full relational substructure of R.

(16) Let R be a relational structure and S be a relational substructure of R. Then every relational substructure of S is a relational substructure of R.

Let S be a 1-sorted structure and let N be a net structure over S. A net structure over S is called a structure of a subnet of N if:

(Def. 8) It is a relational substructure of N and the mapping of it = (the mapping of N)$\restriction$(the carrier of it).

Next we state two propositions:

(17) For every 1-sorted structure S holds every net structure N over S is a structure of a subnet of N.

(18) Let Q be a 1-sorted structure, R be a net structure over Q, and S be a structure of a subnet of R. Then every structure of a subnet of S is a structure of a subnet of R.

Let S be a 1-sorted structure, let N be a net structure over S, and let M be a structure of a subnet of N. We say that M is full if and only if:

(Def. 9) M is a full relational substructure of N.

Let S be a 1-sorted structure and let N be a net structure over S. Note that there exists a structure of a subnet of N which is full and strict.

Let S be a 1-sorted structure and let N be a non empty net structure over S. Note that there exists a structure of a subnet of N which is full, non empty, and strict.

One can prove the following three propositions:

(19) Let S be a 1-sorted structure, N be a net structure over S, and M be a structure of a subnet of N. Then the carrier of $M \subseteq$ the carrier of N.

(20) Let S be a 1-sorted structure, N be a net structure over S, M be a structure of a subnet of N, x, y be elements of N, and i, j be elements of the carrier of M. If $x = i$ and $y = j$ and $i \leqslant j$, then $x \leqslant y$.

(21) Let S be a 1-sorted structure, N be a non empty net structure over S, M be a non empty full structure of a subnet of N, x, y be elements of N, and i, j be elements of the carrier of M. If $x = i$ and $y = j$ and $x \leqslant y$, then $i \leqslant j$.

6. More about nets

Let T be a non empty 1-sorted structure. One can verify that there exists a net in T which is constant and strict.

Let T be a non empty 1-sorted structure and let N be a constant net structure over T. One can verify that the mapping of N is constant.

Let T be a non empty 1-sorted structure and let N be a net structure over T. Let us assume that N is constant and non empty. The value of N yields an element of T and is defined as follows:

(Def. 10) The value of $N =$ the value of the mapping of N.

Let T be a non empty 1-sorted structure and let N be a constant non empty net structure over T. Then the value of N can be characterized by the condition:

(Def. 11) The value of $N =$ the value of the mapping of N.

Next we state the proposition

(22) Let R be a non empty relational structure, T be a non empty 1-sorted structure, and p be an element of the carrier of T. Then the value of $R \longmapsto p = p$.

Let T be a non empty 1-sorted structure and let N be a net in T. A net in T is said to be a subnet of N if it satisfies the condition (Def. 12).

(Def. 12) There exists a map f from it into N such that
(i) the mapping of it $=$ (the mapping of N) $\cdot f$, and
(ii) for every element m of N there exists an element n of it such that for every element p of it such that $n \leqslant p$ holds $m \leqslant f(p)$.

We now state several propositions:

(23) For every non empty 1-sorted structure T holds every net N in T is a subnet of N.

(24) Let T be a non empty 1-sorted structure and N_1, N_2, N_3 be nets in T. Suppose N_1 is a subnet of N_2 and N_2 is a subnet of N_3. Then N_1 is a subnet of N_3.

(25) Let T be a non empty 1-sorted structure, N be a constant net in T, and i be an element of the carrier of N. Then $N(i) =$ the value of N.

(26) Let L be a non empty 1-sorted structure, N be a net in L, and X, Y be sets. If N is eventually in X and eventually in Y, then X meets Y.

(27) Let S be a non empty 1-sorted structure, N be a net in S, M be a subnet of N, and given X. If M is often in X, then N is often in X.

(28) Let S be a non empty 1-sorted structure, N be a net in S, and given X. If N is eventually in X, then N is often in X.

(29) For every non empty 1-sorted structure S holds every net in S is eventually in the carrier of S.

7. The restriction of a net

Let S be a 1-sorted structure, let N be a net structure over S, and let us consider X. The functor $N^{-1}(X)$ yields a strict structure of a subnet of N and is defined by:

(Def. 13) $N^{-1}(X)$ is a full relational substructure of N and the carrier of $N^{-1}(X) =$ (the mapping of $N)^{-1}(X)$.

Let S be a 1-sorted structure, let N be a transitive net structure over S, and let us consider X. One can verify that $N^{-1}(X)$ is transitive and full.

We now state three propositions:

(30) Let S be a non empty 1-sorted structure, N be a net in S, and given X. If N is often in X, then $N^{-1}(X)$ is non empty and directed.

(31) Let S be a non empty 1-sorted structure, N be a net in S, and given X. If N is often in X, then $N^{-1}(X)$ is a subnet of N.

(32) Let S be a non empty 1-sorted structure, N be a net in S, given X, and M be a subnet of N. If $M = N^{-1}(X)$, then M is eventually in X.

8. The universe of nets

Let X be a non empty 1-sorted structure. The functor NetUniv(X) is defined by the condition (Def. 14).

(Def. 14) Let given x. Then $x \in$ NetUniv(X) if and only if there exists a strict net N in X such that $N = x$ and the carrier of $N \in$ the universe of the carrier of X.

Let X be a non empty 1-sorted structure. One can check that NetUniv(X) is non empty.

9. Parametrized families of nets, iteration

Let X be a set and let T be a 1-sorted structure. A many sorted set indexed by X is said to be a net set of X, T if:

(Def. 15) For every set i such that $i \in \operatorname{rng}$ it holds i is a net in T.

The following proposition is true

(33) Let X be a set, T be a 1-sorted structure, and F be a many sorted set indexed by X. Then F is a net set of X, T if and only if for every set i such that $i \in X$ holds $F(i)$ is a net in T.

Let X be a non empty set, let T be a 1-sorted structure, let J be a net set of X, T, and let i be an element of X. Then $J(i)$ is a net in T.

Let X be a set and let T be a 1-sorted structure. One can check that every net set of X, T is relational structure yielding.

Let T be a 1-sorted structure and let Y be a net in T. Observe that every net set of the carrier of Y, T is yielding non-empty carriers.

Let T be a non empty 1-sorted structure, let Y be a net in T, and let J be a net set of the carrier of Y, T. One can check that $\prod J$ is directed and transitive.

Let X be a set and let T be a 1-sorted structure. Observe that every net set of X, T is yielding non-empty carriers.

Let X be a set and let T be a 1-sorted structure. One can check that there exists a net set of X, T which is yielding non-empty carriers.

Let T be a non empty 1-sorted structure, let Y be a net in T, and let J be a net set of the carrier of Y, T. The functor Iterated(J) yielding a strict net in T is defined by the conditions (Def. 16).

(Def. 16)(i) The relational structure of Iterated(J) = $[\![Y, \prod J]\!]$, and

(ii) for every element i of the carrier of Y and for every function f such that $i \in$ the carrier of Y and $f \in$ the carrier of $\prod J$ holds (the mapping of Iterated(J))(i, f) = (the mapping of $J(i)$)($f(i)$).

We now state four propositions:

(34) Let T be a non empty 1-sorted structure, Y be a net in T, and J be a net set of the carrier of Y, T. Suppose $Y \in$ NetUniv(T) and for every element i of the carrier of Y holds $J(i) \in$ NetUniv(T). Then Iterated(J) $\in$ NetUniv(T).

(35) Let T be a non empty 1-sorted structure, N be a net in T, and J be a net set of the carrier of N, T. Then the carrier of Iterated(J) = $[\![$ the carrier of N, $\prod$ support J $]\!]$.

(36) Let T be a non empty 1-sorted structure, N be a net in T, J be a net set of the carrier of N, T, i be an element of the carrier of N, f be an element of the carrier of $\prod J$, and x be an element of the carrier of Iterated(J). If $x = \langle i, f \rangle$, then (Iterated(J))(x) = (the mapping of $J(i)$)($f(i)$).

(37) Let T be a non empty 1-sorted structure, Y be a net in T, and J be a net set of the carrier of Y, T. Then rng (the mapping of Iterated(J)) $\subseteq$ $\bigcup\{$rng (the mapping of $J(i)$): i ranges over elements of $Y\}$.

10. Poset of open neighbourhoods

Let T be a non empty topological space and let p be a point of T. The open neighbourhoods of p constitute a relational structure and is defined as follows:

(Def. 17) The open neighbourhoods of p = $(\langle\{V, V$ ranges over subsets of T: $p \in V \wedge V$ is open$\}, \subseteq\rangle)^{\smile}$.

Let T be a non empty topological space and let p be a point of T. One can check that the open neighbourhoods of p is non empty.

One can prove the following propositions:

(38) Let T be a non empty topological space, p be a point of T, and x be an element of the carrier of the open neighbourhoods of p. Then there exists a subset W of T such that $W = x$ and $p \in W$ and W is open.

(39) Let T be a non empty topological space, p be a point of T, and x be a subset of the carrier of T. Then $x \in$ the carrier of the open neighbourhoods of p if and only if $p \in x$ and x is open.

(40) Let T be a non empty topological space, p be a point of T, and x, y be elements of the carrier of the open neighbourhoods of p. Then $x \leqslant y$ if and only if $y \subseteq x$.

Let T be a non empty topological space and let p be a point of T. Note that the open neighbourhoods of p is transitive and directed.

11. Nets in topological spaces

Let T be a non empty topological space and let N be a net in T. The functor $\mathrm{Lim}\, N$ yields a subset of T and is defined as follows:

(Def. 18) For every point p of T holds $p \in \mathrm{Lim}\, N$ iff for every neighbourhood V of p holds N is eventually in V.

The following four propositions are true:

(41) For every non empty topological space T and for every net N in T and for every subnet Y of N holds $\mathrm{Lim}\, N \subseteq \mathrm{Lim}\, Y$.

(42) For every non empty topological space T and for every constant net N in T holds the value of $N \in \mathrm{Lim}\, N$.

(43) Let T be a non empty topological space, N be a net in T, and p be a point of T. Suppose $p \in \mathrm{Lim}\, N$. Let d be an element of N. Then there exists a subset S of T such that $S = \{N(c), c$ ranges over elements of N: $d \leqslant c\}$ and $p \in \overline{S}$.

(44) Let T be a non empty topological space. Then T is Hausdorff if and only if for every net N in T and for all points p, q of T such that $p \in \mathrm{Lim}\, N$ and $q \in \mathrm{Lim}\, N$ holds $p = q$.

Let T be a Hausdorff non empty topological space and let N be a net in T. Observe that $\mathrm{Lim}\, N$ is trivial.

Let T be a non empty topological space and let N be a net in T. We say that N is convergent if and only if:

(Def. 19) $\mathrm{Lim}\, N \neq \emptyset$.

Let T be a non empty topological space. Observe that every net in T which is constant is also convergent.

Let T be a non empty topological space. Note that there exists a net in T which is convergent and strict.

Let T be a Hausdorff non empty topological space and let N be a convergent net in T. The functor $\lim N$ yielding an element of T is defined as follows:

(Def. 20) $\lim N \in \mathrm{Lim}\, N$.

One can prove the following propositions:

(45) For every Hausdorff non empty topological space T and for every constant net N in T holds $\lim N =$ the value of N.

(46) Let T be a non empty topological space, N be a net in T, and p be a point of T. Suppose $p \notin \mathrm{Lim}\, N$. Then it is not true that there exists a subnet Y of N and there exists a subnet Z of Y such that $p \in \mathrm{Lim}\, Z$.

(47) Let T be a non empty topological space and N be a net in T. Suppose $N \in \mathrm{NetUniv}(T)$. Let p be a point of T. Suppose $p \notin \mathrm{Lim}\, N$. Then there exists a subnet Y of N such that $Y \in \mathrm{NetUniv}(T)$ and it is not true that there exists a subnet Z of Y such that $p \in \mathrm{Lim}\, Z$.

(48) Let T be a non empty topological space, N be a net in T, and p be a point of T. Suppose $p \in \mathrm{Lim}\, N$. Let J be a net set of the carrier of N, T. Suppose that for every element i of the carrier of N holds $N(i) \in \mathrm{Lim}\, J(i)$. Then $p \in \mathrm{Lim}\, \mathrm{Iterated}(J)$.

12. Convergence classes

Let S be a non empty 1-sorted structure. Convergence class of S is defined as follows:

(Def. 21) It $\subseteq [\!:\mathrm{NetUniv}(S), \text{the carrier of } S:\!]$.

Let S be a non empty 1-sorted structure. Note that every convergence class of S is relation-like.

Let T be a non empty topological space. The functor Convergence(T) yielding a convergence class of T is defined as follows:

(Def. 22) For every net N in T and for every point p of T holds $\langle N, p\rangle \in \mathrm{Convergence}(T)$ iff $N \in \mathrm{NetUniv}(T)$ and $p \in \mathrm{Lim}\, N$.

Let T be a non empty 1-sorted structure and let C be a convergence class of T. We say that C has (CONSTANTS) property if and only if:

(Def. 23) For every constant net N in T such that $N \in \mathrm{NetUniv}(T)$ holds $\langle N, \text{the value of } N\rangle \in C$.

We say that C has (SUBNETS) property if and only if the condition (Def. 24) is satisfied.

(Def. 24) Let N be a net in T and Y be a subnet of N. Suppose $Y \in \mathrm{NetUniv}(T)$. Let p be an element of the carrier of T. If $\langle N, p\rangle \in C$, then $\langle Y, p\rangle \in C$.

We say that C has (DIVERGENCE) property if and only if the condition (Def. 25) is satisfied.

(Def. 25) Let X be a net in T and p be an element of the carrier of T. Suppose $X \in \mathrm{NetUniv}(T)$ and $\langle X, p\rangle \notin C$. Then there exists a subnet Y of X such that $Y \in \mathrm{NetUniv}(T)$ and it is not true that there exists a subnet Z of Y such that $\langle Z, p\rangle \in C$.

We say that C has (ITERATED LIMITS) property if and only if the condition (Def. 26) is satisfied.

(Def. 26) Let X be a net in T and p be an element of the carrier of T. Suppose $\langle X, p\rangle \in C$. Let J be a net set of the carrier of X, T. Suppose that for every element i of the carrier of X holds $\langle J(i), X(i)\rangle \in C$. Then $\langle \mathrm{Iterated}(J), p\rangle \in C$.

Let T be a non empty topological space. Note that Convergence(T) has (CONSTANTS) property, (SUBNETS) property, (DIVERGENCE) property, and (ITERATED LIMITS) property.

Let S be a non empty 1-sorted structure and let C be a convergence class of S. The functor ConvergenceSpace(C) yielding a strict topological structure is defined by the conditions (Def. 27).

(Def. 27)(i) The carrier of ConvergenceSpace(C) = the carrier of S, and

(ii) the topology of ConvergenceSpace(C) = $\{V, V$ ranges over subsets of the carrier of S: $\bigwedge_{p\,:\,\text{element of the carrier of } S} (p \in V \Rightarrow \bigwedge_{N\,:\,\text{net in } S} (\langle N, p\rangle \in C \Rightarrow N$ is eventually in $V))\}$.

Let S be a non empty 1-sorted structure and let C be a convergence class of S. Observe that ConvergenceSpace(C) is non empty.

Let S be a non empty 1-sorted structure and let C be a convergence class of S. Note that ConvergenceSpace(C) is topological space-like.

One can prove the following proposition

(49) For every non empty 1-sorted structure S and for every convergence class C of S holds $C \subseteq$ Convergence(ConvergenceSpace(C)).

Let T be a non empty 1-sorted structure and let C be a convergence class of T. We say that C is topological if and only if:

(Def. 28) C has (CONSTANTS) property, (SUBNETS) property, (DIVERGENCE) property, and (ITERATED LIMITS) property.

Let T be a non empty 1-sorted structure. One can check that there exists a convergence class of T which is non empty and topological.

Let T be a non empty 1-sorted structure. One can verify that every convergence class of T which is topological has (CONSTANTS) property, (SUBNETS) property, (DIVERGENCE) property, and (ITERATED LIMITS) property and every convergence class of T which has (CONSTANTS) property, (SUBNETS) property, (DIVERGENCE) property, and (ITERATED LIMITS) property is topological.

The following propositions are true:

(50) Let T be a non empty 1-sorted structure, C be a topological convergence class of T, and S be a subset of ConvergenceSpace(C) **qua** non empty topological space. Then S is open if and only if for every element p of the carrier of T such that $p \in S$ and for every net N in T such that $\langle N, p\rangle \in C$ holds N is eventually in S.

(51) Let T be a non empty 1-sorted structure, C be a topological convergence class of T, and S be a subset of ConvergenceSpace(C) **qua** non empty topological space. Then S is closed if and only if for every element p of the carrier of T and for every net N in T such that $\langle N, p\rangle \in C$ and N is often in S holds $p \in S$.

(52) Let T be a non empty 1-sorted structure, C be a topological convergence class of T, S be a subset of ConvergenceSpace(C), and p be a point of ConvergenceSpace(C). Suppose $p \in \overline{S}$. Then there exists a net N in

ConvergenceSpace(C) such that $\langle N, p\rangle \in C$ and rng (the mapping of N) $\subseteq S$ and $p \in \mathrm{Lim}\, N$.

(53) Let T be a non empty 1-sorted structure and C be a convergence class of T. Then Convergence(ConvergenceSpace(C)) $= C$ if and only if C is topological.

References

[1] Grzegorz Bancerek. Cardinal numbers. *Formalized Mathematics*, 1(**2**):377–382, 1990.
[2] Grzegorz Bancerek. König's theorem. *Formalized Mathematics*, 1(**3**):589–593, 1990.
[3] Grzegorz Bancerek. The ordinal numbers. *Formalized Mathematics*, 1(**1**):91–96, 1990.
[4] Grzegorz Bancerek. Tarski's classes and ranks. *Formalized Mathematics*, 1(**3**):563–567, 1990.
[5] Grzegorz Bancerek. Complete lattices. *Formalized Mathematics*, 2(**5**):719–725, 1991.
[6] Grzegorz Bancerek. Bounds in posets and relational substructures. *Formalized Mathematics*, 6(**1**):81–91, 1997.
[7] Grzegorz Bancerek. Directed sets, nets, ideals, filters, and maps. *Formalized Mathematics*, 6(**1**):93–107, 1997.
[8] Grzegorz Bancerek. The "way-below" relation. *Formalized Mathematics*, 6(**1**):169–176, 1997.
[9] Józef Białas. Group and field definitions. *Formalized Mathematics*, 1(**3**):433–439, 1990.
[10] Czesław Byliński. Binary operations. *Formalized Mathematics*, 1(**1**):175–180, 1990.
[11] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.
[12] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.
[13] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(**1**):47–53, 1990.
[14] Czesław Byliński and Andrzej Trybulec. Complex spaces. *Formalized Mathematics*, 2(**1**):151–158, 1991.
[15] Agata Darmochwał. Compact spaces. *Formalized Mathematics*, 1(**2**):383–386, 1990.
[16] G. Gierz, K.H. Hofmann, K. Keimel, J.D. Lawson, M. Mislove, and D.S. Scott. *A Compendium of Continuous Lattices*. Springer-Verlag, Berlin, Heidelberg, New York, 1980.
[17] Adam Grabowski and Robert Milewski. Boolean posets, posets under inclusion and products of relational structures. *Formalized Mathematics*, 6(**1**):117–121, 1997.
[18] Krzysztof Hryniewiecki. Relations of tolerance. *Formalized Mathematics*, 2(**1**):105–109, 1991.
[19] Artur Korniłowicz. Cartesian products of relations and relational structures. *Formalized Mathematics*, 6(**1**):145–152, 1997.
[20] Jarosław Kotowicz. Monotone real sequences. Subsequences. *Formalized Mathematics*, 1(**3**):471–475, 1990.
[21] Beata Madras. Product of family of universal algebras. *Formalized Mathematics*, 4(**1**):103–108, 1993.
[22] Michał Muzalewski. Categories of groups. *Formalized Mathematics*, 2(**4**):563–571, 1991.
[23] Yatsuka Nakamura, Piotr Rudnicki, Andrzej Trybulec, and Pauline N. Kawamoto. Preliminaries to circuits, I. *Formalized Mathematics*, 5(**2**):167–172, 1996.
[24] Andrzej Nędzusiak. σ-fields and probability. *Formalized Mathematics*, 1(**2**):401–407, 1990.
[25] Bogdan Nowak and Grzegorz Bancerek. Universal classes. *Formalized Mathematics*, 1(**3**):595–600, 1990.
[26] Beata Padlewska. Families of sets. *Formalized Mathematics*, 1(**1**):147–152, 1990.
[27] Beata Padlewska. Locally connected spaces. *Formalized Mathematics*, 2(**1**):93–96, 1991.
[28] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Formalized Mathematics*, 1(**1**):223–230, 1990.
[29] Andrzej Trybulec. Function domains and Frænkel operator. *Formalized Mathematics*, 1(**3**):495–500, 1990.
[30] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.

[31] Andrzej Trybulec. Tuples, projections and Cartesian products. *Formalized Mathematics*, 1(**1**):97–105, 1990.
[32] Andrzej Trybulec. Many-sorted sets. *Formalized Mathematics*, 4(**1**):15–22, 1993.
[33] Andrzej Trybulec. Many sorted algebras. *Formalized Mathematics*, 5(**1**):37–42, 1996.
[34] Wojciech A. Trybulec. Partially ordered sets. *Formalized Mathematics*, 1(**2**):313–319, 1990.
[35] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[36] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.
[37] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.
[38] Edmund Woronowicz. Relations defined on sets. *Formalized Mathematics*, 1(**1**):181–186, 1990.
[39] Mirosław Wysocki and Agata Darmochwał. Subsets of topological spaces. *Formalized Mathematics*, 1(**1**):231–237, 1990.

Received November 12, 1996