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Convergence and the Limit of Complex Sequences. Serieses

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The papers [5], [4], [8], [6], [2], [7], [10], [12], [3], [1], [9], [13], and [11] provide the terminology and notation for this paper.

1. Preliminaries

For simplicity, we adopt the following convention: r_1, r_2, r_3 are sequences of real numbers, s_1, s_2, s_3 are complex sequences, k, n, m are natural numbers, and p, r are elements of $\mathbb{R}$.

The following propositions are true:

(1) $(n+1)+0i \neq 0_{\mathbb{C}}$ and $0+(n+1)i \neq 0_{\mathbb{C}}$.

(2) If for every n holds $r_1(n) = 0$, then for every m holds $(\sum_{\alpha=0}^{\kappa} |r_1|(\alpha))_{\kappa \in \mathbb{N}}(m) = 0$.

(3) If for every n holds $r_1(n) = 0$, then r_1 is absolutely summable.

Let us note that there exists a sequence of real numbers which is absolutely summable.

One can check that every sequence of real numbers which is summable is also convergent.

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Next we state several propositions:

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(4) Suppose r_1 is convergent. Let given p. Suppose $0 < p$. Then there exists n such that for all natural numbers m, l such that $n \leqslant m$ and $n \leqslant l$ holds $|r_1(m) - r_1(l)| < p$.

(5) If for every n holds $r_1(n) \leqslant p$, then for all natural numbers n, l holds $(\sum_{\alpha=0}^{\kappa}(r_1)(\alpha))_{\kappa\in\mathbb{N}}(n+l) - (\sum_{\alpha=0}^{\kappa}(r_1)(\alpha))_{\kappa\in\mathbb{N}}(n) \leqslant p \cdot l$.

(6) If for every n holds $r_1(n) \leqslant p$, then for every n holds $(\sum_{\alpha=0}^{\kappa}(r_1)(\alpha))_{\kappa\in\mathbb{N}}(n) \leqslant p \cdot (n+1)$.

(7) If for every n such that $n \leqslant m$ holds $r_2(n) \leqslant p \cdot r_3(n)$, then $(\sum_{\alpha=0}^{\kappa}(r_2)(\alpha))_{\kappa\in\mathbb{N}}(m) \leqslant p \cdot (\sum_{\alpha=0}^{\kappa}(r_3)(\alpha))_{\kappa\in\mathbb{N}}(m)$.

(8) Suppose that for every n such that $n \leqslant m$ holds $r_2(n) \leqslant p \cdot r_3(n)$. Let given n. Suppose $n \leqslant m$. Let l be a natural number. If $n + l \leqslant m$, then $(\sum_{\alpha=0}^{\kappa}(r_2)(\alpha))_{\kappa\in\mathbb{N}}(n+l) - (\sum_{\alpha=0}^{\kappa}(r_2)(\alpha))_{\kappa\in\mathbb{N}}(n) \leqslant p \cdot ((\sum_{\alpha=0}^{\kappa}(r_3)(\alpha))_{\kappa\in\mathbb{N}}(n+l) - (\sum_{\alpha=0}^{\kappa}(r_3)(\alpha))_{\kappa\in\mathbb{N}}(n))$.

(9) If for every n holds $0 \leqslant r_1(n)$, then for all n, m such that $n \leqslant m$ holds $|(\sum_{\alpha=0}^{\kappa}(r_1)(\alpha))_{\kappa\in\mathbb{N}}(m) - (\sum_{\alpha=0}^{\kappa}(r_1)(\alpha))_{\kappa\in\mathbb{N}}(n)| = (\sum_{\alpha=0}^{\kappa}(r_1)(\alpha))_{\kappa\in\mathbb{N}}(m) - (\sum_{\alpha=0}^{\kappa}(r_1)(\alpha))_{\kappa\in\mathbb{N}}(n)$ and for every n holds $|(\sum_{\alpha=0}^{\kappa}(r_1)(\alpha))_{\kappa\in\mathbb{N}}(n)| = (\sum_{\alpha=0}^{\kappa}(r_1)(\alpha))_{\kappa\in\mathbb{N}}(n)$.

(10) If s_2 is convergent and s_3 is convergent and $\lim(s_2 - s_3) = 0_{\mathbb{C}}$, then $\lim s_2 = \lim s_3$.

2. The Operations on Complex Sequences

In the sequel z denotes an element of $\mathbb{C}$ and N_1 denotes an increasing sequence of naturals.

Let z be an element of $\mathbb{C}$. The functor $(z^{\kappa})_{\kappa\in\mathbb{N}}$ yielding a complex sequence is defined as follows:

(Def. 1) $(z^{\kappa})_{\kappa\in\mathbb{N}}(0) = 1_{\mathbb{C}}$ and for every n holds $(z^{\kappa})_{\kappa\in\mathbb{N}}(n+1) = (z^{\kappa})_{\kappa\in\mathbb{N}}(n) \cdot z$.

Let z be an element of $\mathbb{C}$ and let n be a natural number. The functor $z_{\mathbb{N}}^{n}$ yielding an element of $\mathbb{C}$ is defined by:

(Def. 2) $z_{\mathbb{N}}^{n} = (z^{\kappa})_{\kappa\in\mathbb{N}}(n)$.

The following proposition is true

(11) $z_{\mathbb{N}}^{0} = 1_{\mathbb{C}}$.

Let c be a complex sequence. The functor $\Re(c)$ yields a sequence of real numbers and is defined as follows:

(Def. 3) For every n holds $\Re(c)(n) = \Re(c(n))$.

Let c be a complex sequence. The functor $\Im(c)$ yielding a sequence of real numbers is defined as follows:

(Def. 4) For every n holds $\Im(c)(n) = \Im(c(n))$.

We now state a number of propositions:

(12) $|z| \leqslant |\Re(z)| + |\Im(z)|$.

(13) $|\Re(z)| \leqslant |z|$ and $|\Im(z)| \leqslant |z|$.

(14) $\Re(s_2) = \Re(s_3)$ and $\Im(s_2) = \Im(s_3)$ iff $s_2 = s_3$.

(15) $\Re(s_2) + \Re(s_3) = \Re(s_2 + s_3)$ and $\Im(s_2) + \Im(s_3) = \Im(s_2 + s_3)$.

(16) $-\Re(s_1) = \Re(-s_1)$ and $-\Im(s_1) = \Im(-s_1)$.

(17) $r \cdot \Re(z) = \Re((r + 0i) \cdot z)$ and $r \cdot \Im(z) = \Im((r + 0i) \cdot z)$.

(18) $\Re(s_2) - \Re(s_3) = \Re(s_2 - s_3)$ and $\Im(s_2) - \Im(s_3) = \Im(s_2 - s_3)$.

(19) $r\,\Re(s_1) = \Re((r + 0i)\, s_1)$ and $r\,\Im(s_1) = \Im((r + 0i)\, s_1)$.

(20) $\Re(z\, s_1) = \Re(z)\,\Re(s_1) - \Im(z)\,\Im(s_1)$ and $\Im(z\, s_1) = \Re(z)\,\Im(s_1) + \Im(z)\,\Re(s_1)$.

(21) $\Re(s_2\, s_3) = \Re(s_2)\,\Re(s_3) - \Im(s_2)\,\Im(s_3)$ and $\Im(s_2\, s_3) = \Re(s_2)\,\Im(s_3) + \Im(s_2)\,\Re(s_3)$.

Let s_1 be a complex sequence and let N_1 be an increasing sequence of naturals. The functor $s_1\, N_1$ yielding a complex sequence is defined by:

(Def. 5) For every n holds $(s_1\, N_1)(n) = s_1(N_1(n))$.

Next we state the proposition

(22) $\Re(s_1\, N_1) = \Re(s_1) \cdot N_1$ and $\Im(s_1\, N_1) = \Im(s_1) \cdot N_1$.

Let s_1 be a complex sequence and let k be a natural number. The functor $s_1 \uparrow k$ yields a complex sequence and is defined by:

(Def. 6) For every n holds $(s_1 \uparrow k)(n) = s_1(n + k)$.

The following proposition is true

(23) $\Re(s_1) \uparrow k = \Re(s_1 \uparrow k)$ and $\Im(s_1) \uparrow k = \Im(s_1 \uparrow k)$.

Let s_1 be a complex sequence. The functor $(\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa \in \mathbb{N}}$ yields a complex sequence and is defined as follows:

(Def. 7) $(\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa \in \mathbb{N}}(0) = s_1(0)$ and for every n holds $(\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa \in \mathbb{N}}(n+1) = (\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa \in \mathbb{N}}(n) + s_1(n + 1)$.

Let s_1 be a complex sequence. The functor $\sum s_1$ yields an element of $\mathbb{C}$ and is defined as follows:

(Def. 8) $\sum s_1 = \lim((\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa \in \mathbb{N}})$.

Next we state a number of propositions:

(24) If for every n holds $s_1(n) = 0_{\mathbb{C}}$, then for every m holds $(\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa \in \mathbb{N}}(m) = 0_{\mathbb{C}}$.

(25) If for every n holds $s_1(n) = 0_{\mathbb{C}}$, then for every m holds $(\sum_{\alpha=0}^{\kappa}|s_1|(\alpha))_{\kappa \in \mathbb{N}}(m) = 0$.

(26) $(\sum_{\alpha=0}^{\kappa}\Re(s_1)(\alpha))_{\kappa \in \mathbb{N}} = \Re((\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa \in \mathbb{N}})$ and $(\sum_{\alpha=0}^{\kappa}\Im(s_1)(\alpha))_{\kappa \in \mathbb{N}} = \Im((\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa \in \mathbb{N}})$.

(27) $(\sum_{\alpha=0}^{\kappa}(s_2)(\alpha))_{\kappa \in \mathbb{N}} + (\sum_{\alpha=0}^{\kappa}(s_3)(\alpha))_{\kappa \in \mathbb{N}} = (\sum_{\alpha=0}^{\kappa}(s_2 + s_3)(\alpha))_{\kappa \in \mathbb{N}}$.

(28) $(\sum_{\alpha=0}^{\kappa}(s_2)(\alpha))_{\kappa \in \mathbb{N}} - (\sum_{\alpha=0}^{\kappa}(s_3)(\alpha))_{\kappa \in \mathbb{N}} = (\sum_{\alpha=0}^{\kappa}(s_2 - s_3)(\alpha))_{\kappa \in \mathbb{N}}$.

(29) $(\sum_{\alpha=0}^{\kappa}(z\, s_1)(\alpha))_{\kappa\in\mathbb{N}} = z\,(\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa\in\mathbb{N}}$.

(30) $|(\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa\in\mathbb{N}}(k)| \leqslant (\sum_{\alpha=0}^{\kappa}|s_1|(\alpha))_{\kappa\in\mathbb{N}}(k)$.

(31) $|(\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa\in\mathbb{N}}(m) - (\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa\in\mathbb{N}}(n)| \leqslant |(\sum_{\alpha=0}^{\kappa}|s_1|(\alpha))_{\kappa\in\mathbb{N}}(m) - (\sum_{\alpha=0}^{\kappa}|s_1|(\alpha))_{\kappa\in\mathbb{N}}(n)|$.

(32) $(\sum_{\alpha=0}^{\kappa}\Re(s_1)(\alpha))_{\kappa\in\mathbb{N}} \uparrow k = \Re((\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa\in\mathbb{N}} \uparrow k)$ and $(\sum_{\alpha=0}^{\kappa}\Im(s_1)(\alpha))_{\kappa\in\mathbb{N}} \uparrow k = \Im((\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa\in\mathbb{N}} \uparrow k)$.

(33) If for every n holds $s_2(n) = s_1(0)$, then $(\sum_{\alpha=0}^{\kappa}(s_1 \uparrow 1)(\alpha))_{\kappa\in\mathbb{N}} = (\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa\in\mathbb{N}} \uparrow 1 - s_2$.

(34) $(\sum_{\alpha=0}^{\kappa}|s_1|(\alpha))_{\kappa\in\mathbb{N}}$ is non-decreasing.

Let s_1 be a complex sequence. Note that $(\sum_{\alpha=0}^{\kappa}|s_1|(\alpha))_{\kappa\in\mathbb{N}}$ is non-decreasing.

Next we state three propositions:

(35) If for every n such that $n \leqslant m$ holds $s_2(n) = s_3(n)$, then $(\sum_{\alpha=0}^{\kappa}(s_2)(\alpha))_{\kappa\in\mathbb{N}}(m) = (\sum_{\alpha=0}^{\kappa}(s_3)(\alpha))_{\kappa\in\mathbb{N}}(m)$.

(36) If $1_{\mathbb{C}} \neq z$, then for every n holds $(\sum_{\alpha=0}^{\kappa}((z^{\kappa})_{\kappa\in\mathbb{N}})(\alpha))_{\kappa\in\mathbb{N}}(n) = \frac{1_{\mathbb{C}} - z_{\mathbb{N}}^{n+1}}{1_{\mathbb{C}} - z}$.

(37) If $z \neq 1_{\mathbb{C}}$ and for every n holds $s_1(n+1) = z \cdot s_1(n)$, then for every n holds $(\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa\in\mathbb{N}}(n) = s_1(0) \cdot \frac{1_{\mathbb{C}} - z_{\mathbb{N}}^{n+1}}{1_{\mathbb{C}} - z}$.

3. Convergence of Complex Sequences

Next we state four propositions:

(38) Let a, b be sequences of real numbers and c be a complex sequence. Suppose that for every n holds $\Re(c(n)) = a(n)$ and $\Im(c(n)) = b(n)$. Then a is convergent and b is convergent if and only if c is convergent.

(39) Let a, b be convergent sequences of real numbers and c be a complex sequence. Suppose that for every n holds $\Re(c(n)) = a(n)$ and $\Im(c(n)) = b(n)$. Then c is convergent and $\lim c = \lim a + \lim b i$.

(40) Let a, b be sequences of real numbers and c be a convergent complex sequence. Suppose that for every n holds $\Re(c(n)) = a(n)$ and $\Im(c(n)) = b(n)$. Then a is convergent and b is convergent and $\lim a = \Re(\lim c)$ and $\lim b = \Im(\lim c)$.

(41) For every convergent complex sequence c holds $\Re(c)$ is convergent and $\Im(c)$ is convergent and $\lim \Re(c) = \Re(\lim c)$ and $\lim \Im(c) = \Im(\lim c)$.

Let c be a convergent complex sequence. Observe that $\Re(c)$ is convergent and $\Im(c)$ is convergent.

The following propositions are true:

(42) Let c be a complex sequence. Suppose $\Re(c)$ is convergent and $\Im(c)$ is convergent. Then c is convergent and $\Re(\lim c) = \lim \Re(c)$ and $\Im(\lim c) = \lim \Im(c)$.

(43) If $0 < |z|$ and $|z| < 1$ and $s_1(0) = z$ and for every n holds $s_1(n+1) = s_1(n) \cdot z$, then s_1 is convergent and $\lim s_1 = 0_{\mathbb{C}}$.

(44) If $|z| < 1$ and for every n holds $s_1(n) = z_{\mathbb{N}}^{n+1}$, then s_1 is convergent and $\lim s_1 = 0_{\mathbb{C}}$.

(45) If $r > 0$ and there exists m such that for every n such that $n \geqslant m$ holds $|s_1(n)| \geqslant r$, then $|s_1|$ is not convergent or $\lim |s_1| \neq 0$.

(46) s_1 is convergent iff for every p such that $0 < p$ there exists n such that for every m such that $n \leqslant m$ holds $|s_1(m) - s_1(n)| < p$.

(47) Suppose s_1 is convergent. Let given p. Suppose $0 < p$. Then there exists n such that for all natural numbers m, l such that $n \leqslant m$ and $n \leqslant l$ holds $|s_1(m) - s_1(l)| < p$.

(48) If for every n holds $|s_1(n)| \leqslant r_1(n)$ and r_1 is convergent and $\lim r_1 = 0$, then s_1 is convergent and $\lim s_1 = 0_{\mathbb{C}}$.

4. Summable and Absolutely Summable Complex Sequences

Let s_1, s_2 be complex sequences. We say that s_1 is a subsequence of s_2 if and only if:

(Def. 9) There exists N_1 such that $s_1 = s_2\, N_1$.

Next we state three propositions:

(49) If s_1 is a subsequence of s_2, then $\Re(s_1)$ is a subsequence of $\Re(s_2)$ and $\Im(s_1)$ is a subsequence of $\Im(s_2)$.

(50) If s_1 is a subsequence of s_2 and s_2 is a subsequence of s_3, then s_1 is a subsequence of s_3.

(51) If s_1 is bounded, then there exists s_2 which is a subsequence of s_1 and convergent.

Let s_1 be a complex sequence. We say that s_1 is summable if and only if:

(Def. 10) $(\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa \in \mathbb{N}}$ is convergent.

Let us observe that there exists a complex sequence which is summable.

Let s_1 be a summable complex sequence. Observe that $(\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa \in \mathbb{N}}$ is convergent.

Let us consider s_1. We say that s_1 is absolutely summable if and only if:

(Def. 11) $|s_1|$ is summable.

One can prove the following proposition

(52) If for every n holds $s_1(n) = 0_{\mathbb{C}}$, then s_1 is absolutely summable.

Let us observe that there exists a complex sequence which is absolutely summable.

Let s_1 be an absolutely summable complex sequence. Observe that $|s_1|$ is summable.

The following proposition is true

(53) If s_1 is summable, then s_1 is convergent and $\lim s_1 = 0_{\mathbb{C}}$.

One can verify that every complex sequence which is summable is also convergent.

We now state the proposition

(54) If s_1 is summable, then $\Re(s_1)$ is summable and $\Im(s_1)$ is summable and $\sum s_1 = \sum \Re(s_1) + \sum \Im(s_1) i$.

Let s_1 be a summable complex sequence. One can verify that $\Re(s_1)$ is summable and $\Im(s_1)$ is summable.

We now state two propositions:

(55) If s_2 is summable and s_3 is summable, then $s_2 + s_3$ is summable and $\sum(s_2 + s_3) = \sum s_2 + \sum s_3$.

(56) If s_2 is summable and s_3 is summable, then $s_2 - s_3$ is summable and $\sum(s_2 - s_3) = \sum s_2 - \sum s_3$.

Let s_2, s_3 be summable complex sequences. One can check that $s_2 + s_3$ is summable and $s_2 - s_3$ is summable.

The following proposition is true

(57) If s_1 is summable, then $z\, s_1$ is summable and $\sum(z\, s_1) = z \cdot \sum s_1$.

Let z be an element of $\mathbb{C}$ and let s_1 be a summable complex sequence. One can check that $z\, s_1$ is summable.

The following two propositions are true:

(58) If $\Re(s_1)$ is summable and $\Im(s_1)$ is summable, then s_1 is summable and $\sum s_1 = \sum \Re(s_1) + \sum \Im(s_1) i$.

(59) If s_1 is summable, then for every n holds $s_1 \uparrow n$ is summable.

Let s_1 be a summable complex sequence and let n be a natural number. Note that $s_1 \uparrow n$ is summable.

One can prove the following propositions:

(60) If there exists n such that $s_1 \uparrow n$ is summable, then s_1 is summable.

(61) If s_1 is summable, then for every n holds $\sum s_1 = (\sum_{\alpha=0}^{\kappa} (s_1)(\alpha))_{\kappa \in \mathbb{N}}(n) + \sum(s_1 \uparrow (n+1))$.

(62) $(\sum_{\alpha=0}^{\kappa} |s_1|(\alpha))_{\kappa \in \mathbb{N}}$ is upper bounded iff s_1 is absolutely summable.

Let s_1 be an absolutely summable complex sequence. One can check that $(\sum_{\alpha=0}^{\kappa} |s_1|(\alpha))_{\kappa \in \mathbb{N}}$ is upper bounded.

One can prove the following two propositions:

(63) s_1 is summable iff for every p such that $0 < p$ there exists n such that for every m such that $n \leqslant m$ holds $|(\sum_{\alpha=0}^{\kappa} (s_1)(\alpha))_{\kappa \in \mathbb{N}}(m) - (\sum_{\alpha=0}^{\kappa} (s_1)(\alpha))_{\kappa \in \mathbb{N}}(n)| < p$.

(64) If s_1 is absolutely summable, then s_1 is summable.

One can check that every complex sequence which is absolutely summable is also summable.

Let us note that there exists a complex sequence which is absolutely summable.

The following propositions are true:

(65) If $|z| < 1$, then $(z^\kappa)_{\kappa \in \mathbb{N}}$ is summable and $\sum((z^\kappa)_{\kappa \in \mathbb{N}}) = \frac{1_\mathbb{C}}{1_\mathbb{C} - z}$.

(66) If $|z| < 1$ and for every n holds $s_1(n+1) = z \cdot s_1(n)$, then s_1 is summable and $\sum s_1 = \frac{s_1(0)}{1_\mathbb{C} - z}$.

(67) If r_2 is summable and there exists m such that for every n such that $m \leqslant n$ holds $|s_3(n)| \leqslant r_2(n)$, then s_3 is absolutely summable.

(68) Suppose for every n holds $0 \leqslant |s_2|(n)$ and $|s_2|(n) \leqslant |s_3|(n)$ and s_3 is absolutely summable. Then s_2 is absolutely summable and $\sum |s_2| \leqslant \sum |s_3|$.

(69) If for every n holds $|s_1|(n) > 0$ and there exists m such that for every n such that $n \geqslant m$ holds $\frac{|s_1|(n+1)}{|s_1|(n)} \geqslant 1$, then s_1 is not absolutely summable.

(70) If for every n holds $r_2(n) = \sqrt[n]{|s_1|(n)}$ and r_2 is convergent and $\lim r_2 < 1$, then s_1 is absolutely summable.

(71) If for every n holds $r_2(n) = \sqrt[n]{|s_1|(n)}$ and there exists m such that for every n such that $m \leqslant n$ holds $r_2(n) \geqslant 1$, then $|s_1|$ is not summable.

(72) If for every n holds $r_2(n) = \sqrt[n]{|s_1|(n)}$ and r_2 is convergent and $\lim r_2 > 1$, then s_1 is not absolutely summable.

(73) Suppose $|s_1|$ is non-increasing and for every n holds $r_2(n) = 2^n \cdot |s_1|$(the n-th power of 2). Then s_1 is absolutely summable if and only if r_2 is summable.

(74) If $p > 1$ and for every n such that $n \geqslant 1$ holds $|s_1|(n) = \frac{1}{n^p}$, then s_1 is absolutely summable.

(75) If $p \leqslant 1$ and for every n such that $n \geqslant 1$ holds $|s_1|(n) = \frac{1}{n^p}$, then s_1 is not absolutely summable.

(76) If for every n holds $s_1(n) \neq 0_\mathbb{C}$ and $r_2(n) = \frac{|s_1|(n+1)}{|s_1|(n)}$ and r_2 is convergent and $\lim r_2 < 1$, then s_1 is absolutely summable.

(77) If for every n holds $s_1(n) \neq 0_\mathbb{C}$ and there exists m such that for every n such that $n \geqslant m$ holds $\frac{|s_1|(n+1)}{|s_1|(n)} \geqslant 1$, then s_1 is not absolutely summable.

References

[1] Agnieszka Banachowicz and Anna Winnicka. Complex sequences. *Formalized Mathematics*, 4(**1**):121–124, 1993.

[2] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.

[3] Czesław Byliński. The complex numbers. *Formalized Mathematics*, 1(**3**):507–513, 1990.

[4] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.
[5] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.
[6] Jarosław Kotowicz. Convergent sequences and the limit of sequences. *Formalized Mathematics*, 1(**2**):273–275, 1990.
[7] Jarosław Kotowicz. Monotone real sequences. Subsequences. *Formalized Mathematics*, 1(**3**):471–475, 1990.
[8] Jarosław Kotowicz. Real sequences and basic operations on them. *Formalized Mathematics*, 1(**2**):269–272, 1990.
[9] Adam Naumowicz. Conjugate sequences, bounded complex sequences and convergent complex sequences. *Formalized Mathematics*, 6(**2**):265–268, 1997.
[10] Jan Popiołek. Some properties of functions modul and signum. *Formalized Mathematics*, 1(**2**):263–264, 1990.
[11] Konrad Raczkowski and Andrzej Nędzusiak. Real exponents and logarithms. *Formalized Mathematics*, 2(**2**):213–216, 1991.
[12] Konrad Raczkowski and Andrzej Nędzusiak. Serieses. *Formalized Mathematics*, 2(**4**):449–452, 1991.
[13] Andrzej Trybulec and Czesław Byliński. Some properties of real numbers. *Formalized Mathematics*, 1(**3**):445–449, 1990.

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