FORMALIZED MATHEMATICS
Volume 6, Number 4, 1997
University of Białystok

The Ordering of Points on a Curve. Part II

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Summary. The proof of the Jordan Curve Theorem according to [14] is continued. The notions of the first and last point of a oriented arc are introduced as well as ordering of points on a curve in $\mathcal{E}^2_T$.

MML Identifier: JORDAN5C.

The papers [15], [18], [10], [1], [13], [20], [2], [3], [4], [8], [17], [11], [9], [12], [6], [5], [16], [7], and [19] provide the terminology and notation for this paper.

1. First and last point of a curve

One can prove the following proposition

(1) Let P, Q be subsets of the carrier of $\mathcal{E}^2_{\rm T}$, p_1, p_2, q_1 be points of $\mathcal{E}^2_{\rm T}$, f be a map from $\mathbb{I}$ into $(\mathcal{E}^2_{\rm T})\restriction P$, and s_1 be a real number. Suppose that

(i) P is an arc from p_1 to p_2,
(ii) $q_1 \in P$,
(iii) $q_1 \in Q$,
(iv) $f(s_1) = q_1$,
(v) f is a homeomorphism,
(vi) $f(0) = p_1$,
(vii) $f(1) = p_2$,
(viii) $0 \leqslant s_1$,
(ix) $s_1 \leqslant 1$, and

[1]This paper was written while the author visited the Shinshu University in the winter of 1997.

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ISSN 1426–2630

(x) for every real number t such that $0 \leqslant t$ and $t < s_1$ holds $f(t) \notin Q$.
Let g be a map from $\mathbb{I}$ into $(\mathcal{E}^2_{\rm T})\restriction P$ and s_2 be a real number. Suppose g is a homeomorphism and $g(0) = p_1$ and $g(1) = p_2$ and $g(s_2) = q_1$ and $0 \leqslant s_2$ and $s_2 \leqslant 1$. Let t be a real number. If $0 \leqslant t$ and $t < s_2$, then $g(t) \notin Q$.

Let P, Q be subsets of the carrier of $\mathcal{E}^2_{\rm T}$ and let p_1, p_2 be points of $\mathcal{E}^2_{\rm T}$. Let us assume that P meets Q and $P \cap Q$ is closed and P is an arc from p_1 to p_2. The functor $\mathrm{FPoint}(P, p_1, p_2, Q)$ yielding a point of $\mathcal{E}^2_{\rm T}$ is defined by the conditions (Def. 1).

(Def. 1)(i) $\mathrm{FPoint}(P, p_1, p_2, Q) \in P \cap Q$, and

(ii) for every map g from $\mathbb{I}$ into $(\mathcal{E}^2_{\rm T})\restriction P$ and for every real number s_2 such that g is a homeomorphism and $g(0) = p_1$ and $g(1) = p_2$ and $g(s_2) = \mathrm{FPoint}(P, p_1, p_2, Q)$ and $0 \leqslant s_2$ and $s_2 \leqslant 1$ and for every real number t such that $0 \leqslant t$ and $t < s_2$ holds $g(t) \notin Q$.

One can prove the following three propositions:

(2) Let P, Q be subsets of the carrier of $\mathcal{E}^2_{\rm T}$ and p, p_1, p_2 be points of $\mathcal{E}^2_{\rm T}$. If $p \in P$ and P is an arc from p_1 to p_2 and $Q = \{p\}$, then $\mathrm{FPoint}(P, p_1, p_2, Q) = p$.

(3) Let P be a non empty subset of the carrier of $\mathcal{E}^2_{\rm T}$, Q be a subset of the carrier of $\mathcal{E}^2_{\rm T}$, and p_1, p_2 be points of $\mathcal{E}^2_{\rm T}$. If $p_1 \in Q$ and $P \cap Q$ is closed and P is an arc from p_1 to p_2, then $\mathrm{FPoint}(P, p_1, p_2, Q) = p_1$.

(4) Let P, Q be subsets of the carrier of $\mathcal{E}^2_{\rm T}$, p_1, p_2, q_1 be points of $\mathcal{E}^2_{\rm T}$, f be a map from $\mathbb{I}$ into $(\mathcal{E}^2_{\rm T})\restriction P$, and s_1 be a real number. Suppose that

(i) P is an arc from p_1 to p_2,
(ii) $q_1 \in P$,
(iii) $q_1 \in Q$,
(iv) $f(s_1) = q_1$,
(v) f is a homeomorphism,
(vi) $f(0) = p_1$,
(vii) $f(1) = p_2$,
(viii) $0 \leqslant s_1$,
(ix) $s_1 \leqslant 1$, and
(x) for every real number t such that $1 \geqslant t$ and $t > s_1$ holds $f(t) \notin Q$.

Let g be a map from $\mathbb{I}$ into $(\mathcal{E}^2_{\rm T})\restriction P$ and s_2 be a real number. Suppose g is a homeomorphism and $g(0) = p_1$ and $g(1) = p_2$ and $g(s_2) = q_1$ and $0 \leqslant s_2$ and $s_2 \leqslant 1$. Let t be a real number. If $1 \geqslant t$ and $t > s_2$, then $g(t) \notin Q$.

Let P, Q be subsets of the carrier of $\mathcal{E}^2_{\rm T}$ and let p_1, p_2 be points of $\mathcal{E}^2_{\rm T}$. Let us assume that P meets Q and $P \cap Q$ is closed and P is an arc from p_1 to p_2. The functor $\mathrm{LPoint}(P, p_1, p_2, Q)$ yielding a point of $\mathcal{E}^2_{\rm T}$ is defined by the conditions (Def. 2).

(Def. 2)(i) $\mathrm{LPoint}(P, p_1, p_2, Q) \in P \cap Q$, and

(ii) for every map g from $\mathbb{I}$ into $(\mathcal{E}^2_{\rm T})\restriction P$ and for every real number s_2 such that g is a homeomorphism and $g(0) = p_1$ and $g(1) = p_2$ and $g(s_2) =$

LPoint(P, p_1, p_2, Q) and $0 \leqslant s_2$ and $s_2 \leqslant 1$ and for every real number t such that $1 \geqslant t$ and $t > s_2$ holds $g(t) \notin Q$.

One can prove the following propositions:

(5) Let P, Q be subsets of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ and p, p_1, p_2 be points of $\mathcal{E}^2_{\mathrm{T}}$. If $p \in P$ and P is an arc from p_1 to p_2 and $Q = \{p\}$, then LPoint$(P, p_1, p_2, Q) = p$.

(6) Let P be a non empty subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$, Q be a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$, and p_1, p_2 be points of $\mathcal{E}^2_{\mathrm{T}}$. If $p_2 \in Q$ and $P \cap Q$ is closed and P is an arc from p_1 to p_2, then LPoint$(P, p_1, p_2, Q) = p_2$.

(7) Let P be a non empty subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$, Q be a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$, and p_1, p_2 be points of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $P \subseteq Q$ and P is closed and an arc from p_1 to p_2. Then FPoint$(P, p_1, p_2, Q) = p_1$ and LPoint$(P, p_1, p_2, Q) = p_2$.

2. The ordering of points on a curve

Let P be a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ and let p_1, p_2, q_1, q_2 be points of $\mathcal{E}^2_{\mathrm{T}}$. We say that LE q_1, q_2, P, p_1, p_2 if and only if the conditions (Def. 3) are satisfied.

(Def. 3)(i) $q_1 \in P$,

(ii) $q_2 \in P$, and

(iii) for every map g from $\mathbb{I}$ into $(\mathcal{E}^2_{\mathrm{T}}) \restriction P$ and for all real numbers s_1, s_2 such that g is a homeomorphism and $g(0) = p_1$ and $g(1) = p_2$ and $g(s_1) = q_1$ and $0 \leqslant s_1$ and $s_1 \leqslant 1$ and $g(s_2) = q_2$ and $0 \leqslant s_2$ and $s_2 \leqslant 1$ holds $s_1 \leqslant s_2$.

The following propositions are true:

(8) Let P be a non empty subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$, p_1, p_2, q_1, q_2 be points of $\mathcal{E}^2_{\mathrm{T}}$, g be a map from $\mathbb{I}$ into $(\mathcal{E}^2_{\mathrm{T}}) \restriction P$, and s_1, s_2 be real numbers. Suppose that

(i) P is an arc from p_1 to p_2,

(ii) g is a homeomorphism,

(iii) $g(0) = p_1$,

(iv) $g(1) = p_2$,

(v) $g(s_1) = q_1$,

(vi) $0 \leqslant s_1$,

(vii) $s_1 \leqslant 1$,

(viii) $g(s_2) = q_2$,

(ix) $0 \leqslant s_2$,

(x) $s_2 \leqslant 1$, and

(xi) $s_1 \leqslant s_2$.

Then LE q_1, q_2, P, p_1, p_2.

(9) Let P be a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ and p_1, p_2, q_1 be points of $\mathcal{E}^2_{\mathrm{T}}$. If P is an arc from p_1 to p_2 and $q_1 \in P$, then LE q_1, q_1, P, p_1, p_2.

(10) Let P be a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ and p_1, p_2, q_1 be points of $\mathcal{E}^2_{\mathrm{T}}$. Suppose P is an arc from p_1 to p_2 and $q_1 \in P$. Then LE p_1, q_1, P, p_1, p_2 and LE q_1, p_2, P, p_1, p_2.

(11) Let P be a non empty subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ and p_1, p_2 be points of $\mathcal{E}^2_{\mathrm{T}}$. If P is an arc from p_1 to p_2, then LE p_1, p_2, P, p_1, p_2.

(12) Let P be a non empty subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ and p_1, p_2, q_1, q_2 be points of $\mathcal{E}^2_{\mathrm{T}}$. Suppose P is an arc from p_1 to p_2 and LE q_1, q_2, P, p_1, p_2 and LE q_2, q_1, P, p_1, p_2. Then $q_1 = q_2$.

(13) Let P be a non empty subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ and p_1, p_2, q_1, q_2, q_3 be points of $\mathcal{E}^2_{\mathrm{T}}$. Suppose P is an arc from p_1 to p_2 and LE q_1, q_2, P, p_1, p_2 and LE q_2, q_3, P, p_1, p_2. Then LE q_1, q_3, P, p_1, p_2.

(14) Let P be a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ and p_1, p_2, q_1, q_2 be points of $\mathcal{E}^2_{\mathrm{T}}$. Suppose P is an arc from p_1 to p_2 and $q_1 \in P$ and $q_2 \in P$ and $q_1 \neq q_2$. Then LE q_1, q_2, P, p_1, p_2 and not LE q_2, q_1, P, p_1, p_2 or LE q_2, q_1, P, p_1, p_2 and not LE q_1, q_2, P, p_1, p_2.

3. Some properties of the ordering of points on a curve

We now state a number of propositions:

(15) Let f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$, Q be a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$, and q be a point of $\mathcal{E}^2_{\mathrm{T}}$. Suppose f is a special sequence and $\widetilde{\mathcal{L}}(f) \cap Q$ is closed and $q \in \widetilde{\mathcal{L}}(f)$ and $q \in Q$. Then LE FPoint$(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\mathrm{len}\, f} f, Q)$, q, $\widetilde{\mathcal{L}}(f)$, $\pi_1 f$, $\pi_{\mathrm{len}\, f} f$.

(16) Let f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$, Q be a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$, and q be a point of $\mathcal{E}^2_{\mathrm{T}}$. Suppose f is a special sequence and $\widetilde{\mathcal{L}}(f) \cap Q$ is closed and $q \in \widetilde{\mathcal{L}}(f)$ and $q \in Q$. Then LE q, LPoint$(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\mathrm{len}\, f} f, Q)$, $\widetilde{\mathcal{L}}(f)$, $\pi_1 f$, $\pi_{\mathrm{len}\, f} f$.

(17) For all points q_1, q_2, p_1, p_2 of $\mathcal{E}^2_{\mathrm{T}}$ such that $p_1 \neq p_2$ holds if LE q_1, q_2, $\mathcal{L}(p_1, p_2)$, p_1, p_2, then LE(q_1, q_2, p_1, p_2).

(18) Let P, Q be subsets of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ and p_1, p_2 be points of $\mathcal{E}^2_{\mathrm{T}}$. Suppose P is an arc from p_1 to p_2 and $P \cap Q \neq \emptyset$ and $P \cap Q$ is closed. Then FPoint(P, p_1, p_2, Q) = LPoint(P, p_2, p_1, Q) and LPoint(P, p_1, p_2, Q) = FPoint(P, p_2, p_1, Q).

(19) Let f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$, Q be a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$, and i be a natural number. Suppose $\widetilde{\mathcal{L}}(f)$ meets Q and Q is closed and f is a special sequence and $1 \leqslant i$

and $i + 1 \leqslant \operatorname{len} f$ and $\mathrm{FPoint}(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\operatorname{len} f} f, Q) \in \mathcal{L}(f, i)$. Then $\mathrm{FPoint}(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\operatorname{len} f} f, Q) = \mathrm{FPoint}(\mathcal{L}(f, i), \pi_i f, \pi_{i+1} f, Q)$.

(20) Let f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$, Q be a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$, and i be a natural number. Suppose $\widetilde{\mathcal{L}}(f)$ meets Q and Q is closed and f is a special sequence and $1 \leqslant i$ and $i + 1 \leqslant \operatorname{len} f$ and $\mathrm{LPoint}(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\operatorname{len} f} f, Q) \in \mathcal{L}(f, i)$. Then $\mathrm{LPoint}(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\operatorname{len} f} f, Q) = \mathrm{LPoint}(\mathcal{L}(f, i), \pi_i f, \pi_{i+1} f, Q)$.

(21) Let f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ and i be a natural number. Suppose $1 \leqslant i$ and $i + 1 \leqslant \operatorname{len} f$ and f is a special sequence and $\mathrm{FPoint}(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\operatorname{len} f} f, \mathcal{L}(f, i)) \in \mathcal{L}(f, i)$. Then $\mathrm{FPoint}(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\operatorname{len} f} f, \mathcal{L}(f, i)) = \pi_i f$.

(22) Let f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ and i be a natural number. Suppose $1 \leqslant i$ and $i + 1 \leqslant \operatorname{len} f$ and f is a special sequence and $\mathrm{LPoint}(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\operatorname{len} f} f, \mathcal{L}(f, i)) \in \mathcal{L}(f, i)$. Then $\mathrm{LPoint}(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\operatorname{len} f} f, \mathcal{L}(f, i)) = \pi_{i+1} f$.

(23) Let f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ and i be a natural number. Suppose f is a special sequence and $1 \leqslant i$ and $i + 1 \leqslant \operatorname{len} f$. Then LE $\pi_i f$, $\pi_{i+1} f$, $\widetilde{\mathcal{L}}(f)$, $\pi_1 f$, $\pi_{\operatorname{len} f} f$.

(24) Let f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ and i, k be natural numbers. Suppose f is a special sequence and $1 \leqslant i$ and $i + k + 1 \leqslant \operatorname{len} f$. Then LE $\pi_i f$, $\pi_{i+k} f$, $\widetilde{\mathcal{L}}(f)$, $\pi_1 f$, $\pi_{\operatorname{len} f} f$.

(25) Let f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$, q be a point of $\mathcal{E}^2_{\mathrm{T}}$, and i be a natural number. Suppose f is a special sequence and $1 \leqslant i$ and $i + 1 \leqslant \operatorname{len} f$ and $q \in \mathcal{L}(f, i)$. Then LE $\pi_i f$, q, $\widetilde{\mathcal{L}}(f)$, $\pi_1 f$, $\pi_{\operatorname{len} f} f$.

(26) Let f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$, q be a point of $\mathcal{E}^2_{\mathrm{T}}$, and i be a natural number. Suppose f is a special sequence and $1 \leqslant i$ and $i + 1 \leqslant \operatorname{len} f$ and $q \in \mathcal{L}(f, i)$. Then LE q, $\pi_{i+1} f$, $\widetilde{\mathcal{L}}(f)$, $\pi_1 f$, $\pi_{\operatorname{len} f} f$.

(27) Let f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$, Q be a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$, q be a point of $\mathcal{E}^2_{\mathrm{T}}$, and i, j be natural numbers. Suppose that

(i) $\widetilde{\mathcal{L}}(f)$ meets Q,
(ii) f is a special sequence,
(iii) Q is closed,
(iv) $\mathrm{FPoint}(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\operatorname{len} f} f, Q) \in \mathcal{L}(f, i)$,
(v) $1 \leqslant i$,
(vi) $i + 1 \leqslant \operatorname{len} f$,
(vii) $q \in \mathcal{L}(f, j)$,
(viii) $1 \leqslant j$,
(ix) $j + 1 \leqslant \operatorname{len} f$,
(x) $q \in Q$, and
(xi) $\mathrm{FPoint}(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\operatorname{len} f} f, Q) \neq q$.

Then $i \leqslant j$ and if $i = j$, then $\mathrm{LE}(\mathrm{FPoint}(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\mathrm{len}\, f} f, Q), q, \pi_i f, \pi_{i+1} f)$.

(28) Let f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$, Q be a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$, q be a point of $\mathcal{E}^2_{\mathrm{T}}$, and i, j be natural numbers. Suppose that

- (i) $\widetilde{\mathcal{L}}(f)$ meets Q,
- (ii) f is a special sequence,
- (iii) Q is closed,
- (iv) $\mathrm{LPoint}(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\mathrm{len}\, f} f, Q) \in \mathcal{L}(f, i)$,
- (v) $1 \leqslant i$,
- (vi) $i + 1 \leqslant \mathrm{len}\, f$,
- (vii) $q \in \mathcal{L}(f, j)$,
- (viii) $1 \leqslant j$,
- (ix) $j + 1 \leqslant \mathrm{len}\, f$,
- (x) $q \in Q$, and
- (xi) $\mathrm{LPoint}(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\mathrm{len}\, f} f, Q) \neq q$.

Then $i \geqslant j$ and if $i = j$, then $\mathrm{LE}(q, \mathrm{LPoint}(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\mathrm{len}\, f} f, Q), \pi_i f, \pi_{i+1} f)$.

(29) Let f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$, q_1, q_2 be points of $\mathcal{E}^2_{\mathrm{T}}$, and i be a natural number. Suppose $q_1 \in \mathcal{L}(f, i)$ and $q_2 \in \mathcal{L}(f, i)$ and f is a special sequence and $1 \leqslant i$ and $i + 1 \leqslant \mathrm{len}\, f$. If LE q_1, q_2, $\widetilde{\mathcal{L}}(f)$, $\pi_1 f$, $\pi_{\mathrm{len}\, f} f$, then LE q_1, q_2, $\mathcal{L}(f, i)$, $\pi_i f$, $\pi_{i+1} f$.

(30) Let f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ and q_1, q_2 be points of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $q_1 \in \widetilde{\mathcal{L}}(f)$ and $q_2 \in \widetilde{\mathcal{L}}(f)$ and f is a special sequence and $q_1 \neq q_2$. Then LE q_1, q_2, $\widetilde{\mathcal{L}}(f)$, $\pi_1 f$, $\pi_{\mathrm{len}\, f} f$ if and only if for all natural numbers i, j such that $q_1 \in \mathcal{L}(f, i)$ and $q_2 \in \mathcal{L}(f, j)$ and $1 \leqslant i$ and $i + 1 \leqslant \mathrm{len}\, f$ and $1 \leqslant j$ and $j + 1 \leqslant \mathrm{len}\, f$ holds $i \leqslant j$ and if $i = j$, then $\mathrm{LE}(q_1, q_2, \pi_i f, \pi_{i+1} f)$.

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Received September 10, 1997