FORMALIZED MATHEMATICS
Volume 6, Number 4, 1997
University of Białystok

On the Rectangular Finite Sequences of the Points of the Plane

Andrzej Trybulec
University of Białystok

Yatsuka Nakamura
Shinshu University
Nagano

Summary. The article deals with a rather technical concept – rectangular sequences of the points of the plane. We mean by that a finite sequence consisting of five elements, that is circular, i.e. the first element and the fifth one of it are equal, and such that the polygon determined by it is a non degenerated rectangle, with sides parallel to axes. The main result is that for the rectangle determined by such a sequence the left and the right component of the complement of it are different and disjoint.

MML Identifier: SPRECT_1.

The terminology and notation used in this paper are introduced in the following papers: [29], [35], [34], [28], [7], [36], [13], [2], [25], [1], [27], [32], [5], [6], [3], [33], [31], [17], [16], [14], [15], [4], [26], [24], [37], [10], [23], [11], [12], [21], [18], [19], [22], [30], [20], [8], and [9].

1. General preliminaries

One can prove the following proposition

(1) For every trivial set A and for every set B such that $B \subseteq A$ holds B is trivial.

One can verify that every function which is non constant is also non trivial.

Let us observe that every function which is trivial is also constant.

One can prove the following proposition

(2) For every function f such that $\operatorname{rng} f$ is trivial holds f is constant.

ISSN 1426–2630

Let f be a constant function. One can verify that $\operatorname{rng} f$ is trivial.

Let us observe that there exists a finite sequence which is non empty and constant.

We now state three propositions:

(3) For all finite sequences f, g such that $f \frown g$ is constant holds f is constant and g is constant.

(4) For all sets x, y such that $\langle x, y \rangle$ is constant holds $x = y$.

(5) For all sets x, y, z such that $\langle x, y, z \rangle$ is constant holds $x = y$ and $y = z$ and $z = x$.

2. Preliminaries (general topology)

One can prove the following four propositions:

(6) Let G_1 be a non empty topological space, A be a subset of the carrier of G_1, and B be a non empty subset of the carrier of G_1. If A is a component of B, then $A \neq \emptyset$.

(7) Let G_1 be a non empty topological space, A be a subset of the carrier of G_1, and B be a non empty subset of the carrier of G_1. If A is a component of B, then $A \subseteq B$.

(8) Let T be a non empty topological space, A be a non empty subset of the carrier of T, and B_1, B_2, C be subsets of the carrier of T. Suppose B_1 is a component of A and B_2 is a component of A and C is a component of A and $B_1 \cup B_2 = A$. Then $C = B_1$ or $C = B_2$.

(9) Let T be a non empty topological space, A be a non empty subset of the carrier of T, and B_1, B_2, C_1, C_2 be subsets of the carrier of T. Suppose B_1 is a component of A and B_2 is a component of A and C_1 is a component of A and C_2 is a component of A and $B_1 \cup B_2 = A$ and $C_1 \cup C_2 = A$. Then $\{B_1, B_2\} = \{C_1, C_2\}$.

3. Preliminaries (the topology of the plane)

We follow the rules: C, C_1, C_2 are non empty compact subsets of $\mathcal{E}^2_{\mathrm{T}}$ and p, q are points of $\mathcal{E}^2_{\mathrm{T}}$.

Next we state the proposition

(10) For all points p, q, r of $\mathcal{E}^2_{\mathrm{T}}$ holds $\widetilde{\mathcal{L}}(\langle p, q, r \rangle) = \mathcal{L}(p, q) \cup \mathcal{L}(q, r)$.

Let n be a natural number and let f be a non trivial finite sequence of elements of $\mathcal{E}^n_{\mathrm{T}}$. Observe that $\widetilde{\mathcal{L}}(f)$ is non empty.

Let f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. Note that $\widetilde{\mathcal{L}}(f)$ is compact.

We now state two propositions:

(11) For all subsets A, B of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ such that $A \subseteq B$ and B is horizontal holds A is horizontal.

(12) For all subsets A, B of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ such that $A \subseteq B$ and B is vertical holds A is vertical.

Let us observe that $\square_{\mathcal{E}^2}$ is special polygonal.

One can check that $\square_{\mathcal{E}^2}$ is non horizontal and non vertical.

One can check that there exists a subset of $\mathcal{E}^2_{\mathrm{T}}$ which is non vertical, non horizontal, non empty, and compact.

4. Special points of a compact non empty subset of the plane

The following propositions are true:

(13) N-min $C \in C$ and N-max $C \in C$.

(14) S-min $C \in C$ and S-max $C \in C$.

(15) W-min $C \in C$ and W-max $C \in C$.

(16) E-min $C \in C$ and E-max $C \in C$.

(17) C is vertical iff W-bound $C =$ E-bound C.

(18) C is horizontal iff S-bound $C =$ N-bound C.

(19) For every C such that NW-corner $C =$ NE-corner C holds C is vertical.

(20) For every C such that SW-corner $C =$ SE-corner C holds C is vertical.

(21) For every C such that NW-corner $C =$ SW-corner C holds C is horizontal.

(22) For every C such that NE-corner $C =$ SE-corner C holds C is horizontal.

In the sequel t, r_1, r_2, s_1, s_2 are real numbers.

The following propositions are true:

(23) W-bound $C \leqslant$ E-bound C.

(24) S-bound $C \leqslant$ N-bound C.

(25) $\mathcal{L}(\text{SE-corner}\, C, \text{NE-corner}\, C) = \{p : p_{\mathbf{1}} = \text{E-bound}\, C \wedge p_{\mathbf{2}} \leqslant \text{N-bound}\, C \wedge p_{\mathbf{2}} \geqslant \text{S-bound}\, C\}$.

(26) $\mathcal{L}(\text{SW-corner}\, C, \text{SE-corner}\, C) = \{p : p_{\mathbf{1}} \leqslant \text{E-bound}\, C \wedge p_{\mathbf{1}} \geqslant \text{W-bound}\, C \wedge p_{\mathbf{2}} = \text{S-bound}\, C\}$.

(27) $\mathcal{L}(\text{NW-corner}\, C, \text{NE-corner}\, C) = \{p : p_{\mathbf{1}} \leqslant \text{E-bound}\, C \wedge p_{\mathbf{1}} \geqslant \text{W-bound}\, C \wedge p_{\mathbf{2}} = \text{N-bound}\, C\}$.

(28) $\mathcal{L}(\text{SW-corner}\, C, \text{NW-corner}\, C) = \{p : p_{\mathbf{1}} = \text{W-bound}\, C \wedge p_{\mathbf{2}} \leqslant \text{N-bound}\, C \wedge p_{\mathbf{2}} \geqslant \text{S-bound}\, C\}$.

(29) $\mathcal{L}(\text{SW-corner}\,C, \text{NW-corner}\,C) \cap \mathcal{L}(\text{NW-corner}\,C, \text{NE-corner}\,C) = \{\text{NW-corner}\,C\}$.

(30) $\mathcal{L}(\text{NW-corner}\,C, \text{NE-corner}\,C) \cap \mathcal{L}(\text{NE-corner}\,C, \text{SE-corner}\,C) = \{\text{NE-corner}\,C\}$.

(31) $\mathcal{L}(\text{SE-corner}\,C, \text{NE-corner}\,C) \cap \mathcal{L}(\text{SW-corner}\,C, \text{SE-corner}\,C) = \{\text{SE-corner}\,C\}$.

(32) $\mathcal{L}(\text{NW-corner}\,C, \text{SW-corner}\,C) \cap \mathcal{L}(\text{SW-corner}\,C, \text{SE-corner}\,C) = \{\text{SW-corner}\,C\}$.

5. Subsets of the plane that are neither vertical nor horizontal

In the sequel D is a non vertical non horizontal non empty compact subset of $\mathcal{E}^2_{\mathrm{T}}$.

The following propositions are true:

(33) $\text{W-bound}\,D < \text{E-bound}\,D$.

(34) $\text{S-bound}\,D < \text{N-bound}\,D$.

(35) $\mathcal{L}(\text{SW-corner}\,D, \text{NW-corner}\,D) \cap \mathcal{L}(\text{SE-corner}\,D, \text{NE-corner}\,D) = \emptyset$.

(36) $\mathcal{L}(\text{SW-corner}\,D, \text{SE-corner}\,D) \cap \mathcal{L}(\text{NW-corner}\,D, \text{NE-corner}\,D) = \emptyset$.

6. A special sequence related to a compact non empty subset of the plane

Let us consider C. The functor $\text{SpStSeq}\,C$ yielding a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ is defined as follows:

(Def. 1) $\text{SpStSeq}\,C = \langle \text{NW-corner}\,C, \text{NE-corner}\,C, \text{SE-corner}\,C \rangle \frown \langle \text{SW-corner}\,C, \text{NW-corner}\,C \rangle$.

The following propositions are true:

(37) $\pi_1\,\text{SpStSeq}\,C = \text{NW-corner}\,C$.

(38) $\pi_2\,\text{SpStSeq}\,C = \text{NE-corner}\,C$.

(39) $\pi_3\,\text{SpStSeq}\,C = \text{SE-corner}\,C$.

(40) $\pi_4\,\text{SpStSeq}\,C = \text{SW-corner}\,C$.

(41) $\pi_5\,\text{SpStSeq}\,C = \text{NW-corner}\,C$.

(42) $\text{len}\,\text{SpStSeq}\,C = 5$.

(43) $\widetilde{\mathcal{L}}(\text{SpStSeq}\,C) = \mathcal{L}(\text{NW-corner}\,C, \text{NE-corner}\,C) \cup \mathcal{L}(\text{NE-corner}\,C, \text{SE-corner}\,C) \cup (\mathcal{L}(\text{SE-corner}\,C, \text{SW-corner}\,C) \cup \mathcal{L}(\text{SW-corner}\,C, \text{NW-corner}\,C))$.

Let D be a non vertical non empty compact subset of $\mathcal{E}^2_{\mathrm{T}}$. Note that SpStSeq D is non constant.

Let D be a non horizontal non empty compact subset of $\mathcal{E}^2_{\mathrm{T}}$. Note that SpStSeq D is non constant.

Let us consider D. One can check that SpStSeq D is special unfolded circular s.c.c. and standard.

Next we state four propositions:

(44) $\widetilde{\mathcal{L}}(\mathrm{SpStSeq}\,D) = [.\,\text{W-bound}\,D, \text{E-bound}\,D, \text{S-bound}\,D, \text{N-bound}\,D.]$.

(45) Let T be a non empty topological space, X be a non empty subset of T, and f be a real map of T. Then $\mathrm{rng}(f \restriction X) = f^{\circ}X$.

(46) Let T be a non empty topological space, X be a non empty compact subset of T, and f be a continuous real map of T. Then $f^{\circ}X$ is lower bounded.

(47) Let T be a non empty topological space, X be a non empty compact subset of T, and f be a continuous real map of T. Then $f^{\circ}X$ is upper bounded.

Let us observe that there exists a subset of $\mathbb{R}$ which is non empty, upper bounded, and lower bounded.

We now state a number of propositions:

(48) $\text{W-bound}\,C = \inf((\mathrm{proj1})^{\circ}C)$.

(49) $\text{S-bound}\,C = \inf((\mathrm{proj2})^{\circ}C)$.

(50) $\text{N-bound}\,C = \sup((\mathrm{proj2})^{\circ}C)$.

(51) $\text{E-bound}\,C = \sup((\mathrm{proj1})^{\circ}C)$.

(52) For all non empty lower bounded subsets A, B of $\mathbb{R}$ holds $\inf(A \cup B) = \min(\inf A, \inf B)$.

(53) For all non empty upper bounded subsets A, B of $\mathbb{R}$ holds $\sup(A \cup B) = \max(\sup A, \sup B)$.

(54) If $C = C_1 \cup C_2$, then $\text{W-bound}\,C = \min(\text{W-bound}\,C_1, \text{W-bound}\,C_2)$.

(55) If $C = C_1 \cup C_2$, then $\text{S-bound}\,C = \min(\text{S-bound}\,C_1, \text{S-bound}\,C_2)$.

(56) If $C = C_1 \cup C_2$, then $\text{N-bound}\,C = \max(\text{N-bound}\,C_1, \text{N-bound}\,C_2)$.

(57) If $C = C_1 \cup C_2$, then $\text{E-bound}\,C = \max(\text{E-bound}\,C_1, \text{E-bound}\,C_2)$.

Let us consider p, q. One can check that $\mathcal{L}(p, q)$ is compact.

One can verify that $\emptyset_{\mathbb{R}}$ is bounded.

Next we state the proposition

(58) $s_1 \in [r_1, r_2]$ iff $r_1 \leqslant s_1$ and $s_1 \leqslant r_2$.

Let us consider r_1, r_2. One can check that $[r_1, r_2]$ is bounded.

Let us observe that every subset of $\mathbb{R}$ which is bounded is also lower bounded and upper bounded and every subset of $\mathbb{R}$ which is lower bounded and upper bounded is also bounded.

The following propositions are true:

(59) If $r_1 \leqslant r_2$, then $t \in [r_1, r_2]$ iff there exists s_1 such that $0 \leqslant s_1$ and $s_1 \leqslant 1$ and $t = s_1 \cdot r_1 + (1 - s_1) \cdot r_2$.

(60) If $p_{\mathbf{1}} \leqslant q_{\mathbf{1}}$, then $(\text{proj1})^\circ\mathcal{L}(p, q) = [p_{\mathbf{1}}, q_{\mathbf{1}}]$.

(61) If $p_{\mathbf{2}} \leqslant q_{\mathbf{2}}$, then $(\text{proj2})^\circ\mathcal{L}(p, q) = [p_{\mathbf{2}}, q_{\mathbf{2}}]$.

(62) If $p_{\mathbf{1}} \leqslant q_{\mathbf{1}}$, then $\text{W-bound}\,\mathcal{L}(p, q) = p_{\mathbf{1}}$.

(63) If $p_{\mathbf{2}} \leqslant q_{\mathbf{2}}$, then $\text{S-bound}\,\mathcal{L}(p, q) = p_{\mathbf{2}}$.

(64) If $p_{\mathbf{2}} \leqslant q_{\mathbf{2}}$, then $\text{N-bound}\,\mathcal{L}(p, q) = q_{\mathbf{2}}$.

(65) If $p_{\mathbf{1}} \leqslant q_{\mathbf{1}}$, then $\text{E-bound}\,\mathcal{L}(p, q) = q_{\mathbf{1}}$.

(66) $\text{W-bound}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \text{W-bound}\,D$.

(67) $\text{S-bound}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \text{S-bound}\,D$.

(68) $\text{N-bound}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \text{N-bound}\,D$.

(69) $\text{E-bound}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \text{E-bound}\,D$.

(70) $\text{NW-corner}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \text{NW-corner}\,D$.

(71) $\text{NE-corner}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \text{NE-corner}\,D$.

(72) $\text{SW-corner}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \text{SW-corner}\,D$.

(73) $\text{SE-corner}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \text{SE-corner}\,D$.

(74) $\text{W-most}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \mathcal{L}(\text{SW-corner}\,D, \text{NW-corner}\,D)$.

(75) $\text{N-most}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \mathcal{L}(\text{NW-corner}\,D, \text{NE-corner}\,D)$.

(76) $\text{S-most}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \mathcal{L}(\text{SW-corner}\,D, \text{SE-corner}\,D)$.

(77) $\text{E-most}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \mathcal{L}(\text{SE-corner}\,D, \text{NE-corner}\,D)$.

(78) $(\text{proj2})^\circ\mathcal{L}(\text{SW-corner}\,D, \text{NW-corner}\,D) = [\text{S-bound}\,D, \text{N-bound}\,D]$.

(79) $(\text{proj1})^\circ\mathcal{L}(\text{NW-corner}\,D, \text{NE-corner}\,D) = [\text{W-bound}\,D, \text{E-bound}\,D]$.

(80) $(\text{proj2})^\circ\mathcal{L}(\text{NE-corner}\,D, \text{SE-corner}\,D) = [\text{S-bound}\,D, \text{N-bound}\,D]$.

(81) $(\text{proj1})^\circ\mathcal{L}(\text{SE-corner}\,D, \text{SW-corner}\,D) = [\text{W-bound}\,D, \text{E-bound}\,D]$.

(82) $\text{W-min}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \text{SW-corner}\,D$.

(83) $\text{W-max}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \text{NW-corner}\,D$.

(84) $\text{N-min}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \text{NW-corner}\,D$.

(85) $\text{N-max}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \text{NE-corner}\,D$.

(86) $\text{E-min}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \text{SE-corner}\,D$.

(87) $\text{E-max}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \text{NE-corner}\,D$.

(88) $\text{S-min}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \text{SW-corner}\,D$.

(89) $\text{S-max}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,D) = \text{SE-corner}\,D$.

7. Rectangular finite suequences of the points of the plane

Let f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. We say that f is rectangular if and only if:

(Def. 2) There exists D such that $f = \mathrm{SpStSeq}\,D$.

Let us consider D. Note that $\mathrm{SpStSeq}\,D$ is rectangular.

Let us mention that there exists a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ which is rectangular.

In the sequel s denotes a rectangular finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$.

The following proposition is true

(90) $\mathrm{len}\, s = 5$.

Let us note that every finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ which is rectangular is also non constant.

One can verify that every non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ which is rectangular is also standard, special, unfolded, circular, and s.c.c..

In the sequel s is a rectangular finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$.

Next we state four propositions:

(91) $\pi_1 s = \text{N-min}\,\widetilde{\mathcal{L}}(s)$ and $\pi_1 s = \text{W-max}\,\widetilde{\mathcal{L}}(s)$.

(92) $\pi_2 s = \text{N-max}\,\widetilde{\mathcal{L}}(s)$ and $\pi_2 s = \text{E-max}\,\widetilde{\mathcal{L}}(s)$.

(93) $\pi_3 s = \text{S-max}\,\widetilde{\mathcal{L}}(s)$ and $\pi_3 s = \text{E-min}\,\widetilde{\mathcal{L}}(s)$.

(94) $\pi_4 s = \text{S-min}\,\widetilde{\mathcal{L}}(s)$ and $\pi_4 s = \text{W-min}\,\widetilde{\mathcal{L}}(s)$.

8. Jordan property

One can prove the following proposition

(95) If $r_1 < r_2$ and $s_1 < s_2$, then $[.r_1, r_2, s_1, s_2.]$ is Jordan.

Let f be a rectangular finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. Observe that $\widetilde{\mathcal{L}}(f)$ is Jordan.

Let S be a subset of the carrier of $\mathcal{E}^2_{\mathrm{T}}$. Let us observe that S is Jordan if and only if the conditions (Def. 3) are satisfied.

(Def. 3)(i) $S^{\mathrm{c}} \neq \emptyset$, and

(ii) there exist subsets A_1, A_2 of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ such that $S^{\mathrm{c}} = A_1 \cup A_2$ and A_1 misses A_2 and $\overline{A_1} \setminus A_1 = \overline{A_2} \setminus A_2$ and A_1 is a component of S^{c} and A_2 is a component of S^{c}.

Next we state the proposition

(96) For every rectangular finite sequence f of elements of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{LeftComp}(f)$ misses $\mathrm{RightComp}(f)$.

Let f be a non constant standard special circular sequence. One can verify that LeftComp(f) is non empty and RightComp(f) is non empty.

The following proposition is true

(97) For every rectangular finite sequence f of elements of $\mathcal{E}^2_{\rm T}$ holds LeftComp(f) $\neq$ RightComp(f).

References

[1] Grzegorz Bancerek. Cardinal numbers. *Formalized Mathematics*, 1(**2**):377–382, 1990.

[2] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.

[3] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.

[4] Józef Białas. Group and field definitions. *Formalized Mathematics*, 1(**3**):433–439, 1990.

[5] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[6] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.

[7] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(**1**):47–53, 1990.

[8] Czesław Byliński and Yatsuka Nakamura. Special polygons. *Formalized Mathematics*, 5(**2**):247–252, 1996.

[9] Czesław Byliński and Piotr Rudnicki. Bounding boxes for compact sets in $\mathcal{E}^2$. *Formalized Mathematics*, 6(**3**):427–440, 1997.

[10] Agata Darmochwał. Compact spaces. *Formalized Mathematics*, 1(**2**):383–386, 1990.

[11] Agata Darmochwał. The Euclidean space. *Formalized Mathematics*, 2(**4**):599–603, 1991.

[12] Agata Darmochwał and Yatsuka Nakamura. The topological space $\mathcal{E}^2_{\rm T}$. Arcs, line segments and special polygonal arcs. *Formalized Mathematics*, 2(**5**):617–621, 1991.

[13] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.

[14] Katarzyna Jankowska. Matrices. Abelian group of matrices. *Formalized Mathematics*, 2(**4**):475–480, 1991.

[15] Katarzyna Jankowska. Transpose matrices and groups of permutations. *Formalized Mathematics*, 2(**5**):711–717, 1991.

[16] Jarosław Kotowicz. Convergent real sequences. Upper and lower bound of sets of real numbers. *Formalized Mathematics*, 1(**3**):477–481, 1990.

[17] Jarosław Kotowicz. Monotone real sequences. Subsequences. *Formalized Mathematics*, 1(**3**):471–475, 1990.

[18] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-board - part I. *Formalized Mathematics*, 3(**1**):107–115, 1992.

[19] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-board - part II. *Formalized Mathematics*, 3(**1**):117–121, 1992.

[20] Yatsuka Nakamura and Czesław Byliński. Extremal properties of vertices on special polygons, part I. *Formalized Mathematics*, 5(**1**):97–102, 1996.

[21] Yatsuka Nakamura and Jarosław Kotowicz. The Jordan's property for certain subsets of the plane. *Formalized Mathematics*, 3(**2**):137–142, 1992.

[22] Yatsuka Nakamura and Andrzej Trybulec. Decomposing a Go-board into cells. *Formalized Mathematics*, 5(**3**):323–328, 1996.

[23] Beata Padlewska. Connected spaces. *Formalized Mathematics*, 1(**1**):239–244, 1990.

[24] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Formalized Mathematics*, 1(**1**):223–230, 1990.

[25] Jan Popiołek. Some properties of functions modul and signum. *Formalized Mathematics*, 1(**2**):263–264, 1990.

[26] Konrad Raczkowski and Paweł Sadowski. Topological properties of subsets in real numbers. *Formalized Mathematics*, 1(**4**):777–780, 1990.

[27] Andrzej Trybulec. Binary operations applied to functions. *Formalized Mathematics*, 1(**2**):329–334, 1990.
[28] Andrzej Trybulec. Enumerated sets. *Formalized Mathematics*, 1(**1**):25–34, 1990.
[29] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[30] Andrzej Trybulec. Left and right component of the complement of a special closed curve. *Formalized Mathematics*, 5(**4**):465–468, 1996.
[31] Andrzej Trybulec. On the decomposition of finite sequences. *Formalized Mathematics*, 5(**3**):317–322, 1996.
[32] Andrzej Trybulec and Czesław Byliński. Some properties of real numbers. *Formalized Mathematics*, 1(**3**):445–449, 1990.
[33] Wojciech A. Trybulec. Pigeon hole principle. *Formalized Mathematics*, 1(**3**):575–579, 1990.
[34] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[35] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.
[36] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.
[37] Mirosław Wysocki and Agata Darmochwał. Subsets of topological spaces. *Formalized Mathematics*, 1(**1**):231–237, 1990.

Received November 30, 1997