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The Construction of SCM over Ring

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The terminology and notation used in this paper have been introduced in the following articles: [6], [11], [2], [3], [9], [4], [5], [7], [1], [10], and [8].

For simplicity, we follow the rules: i, k are natural numbers, I is an element of $\mathbb{Z}_8$, i_1 is an element of Instr-Loc$_{\rm SCM}$, d_1 is an element of Data-Loc$_{\rm SCM}$, and S is a non empty 1-sorted structure.

Let us observe that every non empty loop structure which is trivial is also Abelian, add-associative, right zeroed, and right complementable and every non empty double loop structure which is trivial is also right unital and right-distributive.

Let us note that every element of Data-Loc$_{\rm SCM}$ is natural.

One can check the following observations:

* Data-Loc$_{\rm SCM}$ is non trivial,
* Instr$_{\rm SCM}$ is non trivial, and
* Instr-Loc$_{\rm SCM}$ is non trivial.

Let S be a non empty 1-sorted structure. The functor Instr$_{\rm SCM}(S)$ yields a subset of $[\!: \mathbb{Z}_8, (\bigcup\{\text{the carrier of } S\} \cup \mathbb{N})^* :\!]$ and is defined by the condition (Def. 1).

(Def. 1) Instr$_{\rm SCM}(S) = \{\langle 0, \varepsilon\rangle\} \cup \{\langle I, \langle a, b\rangle\rangle; I$ ranges over elements of $\mathbb{Z}_8$, a ranges over elements of Data-Loc$_{\rm SCM}$, b ranges over elements of Data-Loc$_{\rm SCM}$: $I \in \{1, 2, 3, 4\}\} \cup \{\langle 6, \langle i\rangle\rangle : i$ ranges over elements of Instr-Loc$_{\rm SCM}\} \cup \{\langle 7, \langle i, a\rangle\rangle : i$ ranges over elements of Instr-Loc$_{\rm SCM}$, a ranges over elements of Data-Loc$_{\rm SCM}\} \cup \{\langle 5, \langle a, r\rangle\rangle : a$ ranges over elements of Data-Loc$_{\rm SCM}$, r ranges over elements of the carrier of $S\}$.

Let S be a non empty 1-sorted structure. Note that Instr$_{\rm SCM}(S)$ is non trivial.

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Let S be a non empty 1-sorted structure. We say that S is good if and only if:

(Def. 2) The carrier of $S \neq \text{Instr-Loc}_{\text{SCM}}$ and the carrier of $S \neq \text{Instr}_{\text{SCM}}(S)$.

One can verify that every non empty 1-sorted structure which is trivial is also good.

Let us observe that there exists a 1-sorted structure which is strict, trivial, and non empty.

Let us observe that there exists a double loop structure which is strict, trivial, and non empty.

One can check that there exists a ring which is strict and trivial.

In the sequel G denotes a good non empty 1-sorted structure.

Let S be a non empty 1-sorted structure. The functor $\text{OK}_{\text{SCM}}(S)$ yielding a function from $\mathbb{N}$ into $\{\text{the carrier of } S\} \cup \{\text{Instr}_{\text{SCM}}(S), \text{Instr-Loc}_{\text{SCM}}\}$ is defined as follows:

(Def. 3) $(\text{OK}_{\text{SCM}}(S))(0) = \text{Instr-Loc}_{\text{SCM}}$ and for every natural number k holds $(\text{OK}_{\text{SCM}}(S))(2 \cdot k + 1) =$ the carrier of S and $(\text{OK}_{\text{SCM}}(S))(2 \cdot k + 2) = \text{Instr}_{\text{SCM}}(S)$.

Let S be a non empty 1-sorted structure. An **SCM**-state over S is an element of $\prod \text{OK}_{\text{SCM}}(S)$.

Next we state several propositions:

(1) $\text{Instr-Loc}_{\text{SCM}} \neq \text{Instr}_{\text{SCM}}(S)$.

(2) $(\text{OK}_{\text{SCM}}(G))(i) = \text{Instr-Loc}_{\text{SCM}}$ iff $i = 0$.

(3) $(\text{OK}_{\text{SCM}}(G))(i) =$ the carrier of G iff there exists k such that $i = 2 \cdot k + 1$.

(4) $(\text{OK}_{\text{SCM}}(G))(i) = \text{Instr}_{\text{SCM}}(G)$ iff there exists k such that $i = 2 \cdot k + 2$.

(5) $(\text{OK}_{\text{SCM}}(G))(d_1) =$ the carrier of G.

(6) $(\text{OK}_{\text{SCM}}(G))(i_1) = \text{Instr}_{\text{SCM}}(G)$.

(7) $\pi_0 \prod \text{OK}_{\text{SCM}}(S) = \text{Instr-Loc}_{\text{SCM}}$.

(8) $\pi_{d_1} \prod \text{OK}_{\text{SCM}}(G) =$ the carrier of G.

(9) $\pi_{i_1} \prod \text{OK}_{\text{SCM}}(G) = \text{Instr}_{\text{SCM}}(G)$.

Let S be a non empty 1-sorted structure and let s be an **SCM**-state over S. The functor $\mathbf{IC}_s$ yielding an element of $\text{Instr-Loc}_{\text{SCM}}$ is defined by:

(Def. 4) $\mathbf{IC}_s = s(0)$.

Let R be a good non empty 1-sorted structure, let s be an **SCM**-state over R, and let u be an element of $\text{Instr-Loc}_{\text{SCM}}$. The functor $\text{Chg}_{\text{SCM}}(s, u)$ yielding an **SCM**-state over R is defined by:

(Def. 5) $\text{Chg}_{\text{SCM}}(s, u) = s {+\cdot} (0 \longmapsto u)$.

The following three propositions are true:

(10) For every **SCM**-state s over G and for every element u of $\text{Instr-Loc}_{\text{SCM}}$ holds $(\text{Chg}_{\text{SCM}}(s, u))(0) = u$.

(11) For every **SCM**-state s over G and for every element u of Instr-Loc$_{\mathrm{SCM}}$ and for every element m_1 of Data-Loc$_{\mathrm{SCM}}$ holds $(\mathrm{Chg}_{\mathrm{SCM}}(s,u))(m_1) = s(m_1)$.

(12) For every **SCM**-state s over G and for all elements u, v of Instr-Loc$_{\mathrm{SCM}}$ holds $(\mathrm{Chg}_{\mathrm{SCM}}(s,u))(v) = s(v)$.

Let R be a good non empty 1-sorted structure, let s be an **SCM**-state over R, let t be an element of Data-Loc$_{\mathrm{SCM}}$, and let u be an element of the carrier of R. The functor $\mathrm{Chg}_{\mathrm{SCM}}(s,t,u)$ yielding an **SCM**-state over R is defined as follows:

(Def. 6) $\mathrm{Chg}_{\mathrm{SCM}}(s,t,u) = s {+}\cdot (t \,\dot{\longmapsto}\, u)$.

One can prove the following propositions:

(13) Let s be an **SCM**-state over G, t be an element of Data-Loc$_{\mathrm{SCM}}$, and u be an element of the carrier of G. Then $(\mathrm{Chg}_{\mathrm{SCM}}(s,t,u))(0) = s(0)$.

(14) Let s be an **SCM**-state over G, t be an element of Data-Loc$_{\mathrm{SCM}}$, and u be an element of the carrier of G. Then $(\mathrm{Chg}_{\mathrm{SCM}}(s,t,u))(t) = u$.

(15) Let s be an **SCM**-state over G, t be an element of Data-Loc$_{\mathrm{SCM}}$, u be an element of the carrier of G, and m_1 be an element of Data-Loc$_{\mathrm{SCM}}$. If $m_1 \neq t$, then $(\mathrm{Chg}_{\mathrm{SCM}}(s,t,u))(m_1) = s(m_1)$.

(16) Let s be an **SCM**-state over G, t be an element of Data-Loc$_{\mathrm{SCM}}$, u be an element of the carrier of G, and v be an element of Instr-Loc$_{\mathrm{SCM}}$. Then $(\mathrm{Chg}_{\mathrm{SCM}}(s,t,u))(v) = s(v)$.

Let R be a good non empty 1-sorted structure, let s be an **SCM**-state over R, and let a be an element of Data-Loc$_{\mathrm{SCM}}$. Then $s(a)$ is an element of R.

Let S be a non empty 1-sorted structure and let x be an element of $\mathrm{Instr}_{\mathrm{SCM}}(S)$. Let us assume that there exist elements m_1, m_2 of Data-Loc$_{\mathrm{SCM}}$ and I such that $x = \langle I, \langle m_1, m_2 \rangle \rangle$. The functor $x\,\mathrm{address}_1$ yielding an element of Data-Loc$_{\mathrm{SCM}}$ is defined by:

(Def. 7) There exists a finite sequence f of elements of Data-Loc$_{\mathrm{SCM}}$ such that $f = x_{\mathbf{2}}$ and $x\,\mathrm{address}_1 = \pi_1 f$.

The functor $x\,\mathrm{address}_2$ yields an element of Data-Loc$_{\mathrm{SCM}}$ and is defined by:

(Def. 8) There exists a finite sequence f of elements of Data-Loc$_{\mathrm{SCM}}$ such that $f = x_{\mathbf{2}}$ and $x\,\mathrm{address}_2 = \pi_2 f$.

One can prove the following proposition

(17) For every element x of $\mathrm{Instr}_{\mathrm{SCM}}(S)$ and for all elements m_1, m_2 of Data-Loc$_{\mathrm{SCM}}$ such that $x = \langle I, \langle m_1, m_2 \rangle \rangle$ holds $x\,\mathrm{address}_1 = m_1$ and $x\,\mathrm{address}_2 = m_2$.

Let R be a non empty 1-sorted structure and let x be an element of $\mathrm{Instr}_{\mathrm{SCM}}(R)$. Let us assume that there exist an element m_1 of Instr-Loc$_{\mathrm{SCM}}$ and I such that $x = \langle I, \langle m_1 \rangle \rangle$. The functor $x\,\mathrm{address}_{\mathrm{j}}$ yielding an element of Instr-Loc$_{\mathrm{SCM}}$ is defined as follows:

(Def. 9) There exists a finite sequence f of elements of Instr-Loc$_{\mathrm{SCM}}$ such that $f = x_{\mathbf{2}}$ and $x\,\mathrm{address_j} = \pi_1 f$.

Next we state the proposition

(18) For every element x of Instr$_{\mathrm{SCM}}(S)$ and for every element m_1 of Instr-Loc$_{\mathrm{SCM}}$ such that $x = \langle I, \langle m_1 \rangle \rangle$ holds $x\,\mathrm{address_j} = m_1$.

Let S be a non empty 1-sorted structure and let x be an element of Instr$_{\mathrm{SCM}}(S)$. Let us assume that there exist an element m_1 of Instr-Loc$_{\mathrm{SCM}}$, an element m_2 of Data-Loc$_{\mathrm{SCM}}$, and I such that $x = \langle I, \langle m_1, m_2 \rangle \rangle$. The functor $x\,\mathrm{address_j}$ yields an element of Instr-Loc$_{\mathrm{SCM}}$ and is defined as follows:

(Def. 10) There exists an element m_1 of Instr-Loc$_{\mathrm{SCM}}$ and there exists an element m_2 of Data-Loc$_{\mathrm{SCM}}$ such that $\langle m_1, m_2 \rangle = x_{\mathbf{2}}$ and $x\,\mathrm{address_j} = \pi_1 \langle m_1, m_2 \rangle$.

The functor $x\,\mathrm{address_c}$ yields an element of Data-Loc$_{\mathrm{SCM}}$ and is defined as follows:

(Def. 11) There exists an element m_1 of Instr-Loc$_{\mathrm{SCM}}$ and there exists an element m_2 of Data-Loc$_{\mathrm{SCM}}$ such that $\langle m_1, m_2 \rangle = x_{\mathbf{2}}$ and $x\,\mathrm{address_c} = \pi_2 \langle m_1, m_2 \rangle$.

We now state the proposition

(19) Let x be an element of Instr$_{\mathrm{SCM}}(S)$, m_1 be an element of Instr-Loc$_{\mathrm{SCM}}$, and m_2 be an element of Data-Loc$_{\mathrm{SCM}}$. If $x = \langle I, \langle m_1, m_2 \rangle \rangle$, then $x\,\mathrm{address_j} = m_1$ and $x\,\mathrm{address_c} = m_2$.

Let S be a non empty 1-sorted structure, let d be an element of Data-Loc$_{\mathrm{SCM}}$, and let s be an element of the carrier of S. Then $\langle d, s \rangle$ is a finite sequence of elements of Data-Loc$_{\mathrm{SCM}} \cup$ the carrier of S.

Let S be a non empty 1-sorted structure and let x be an element of Instr$_{\mathrm{SCM}}(S)$. Let us assume that there exist an element m_1 of Data-Loc$_{\mathrm{SCM}}$, an element r of the carrier of S, and I such that $x = \langle I, \langle m_1, r \rangle \rangle$. The functor $x\,\mathrm{const\_address}$ yields an element of Data-Loc$_{\mathrm{SCM}}$ and is defined as follows:

(Def. 12) There exists a finite sequence f of elements of Data-Loc$_{\mathrm{SCM}} \cup$ the carrier of S such that $f = x_{\mathbf{2}}$ and $x\,\mathrm{const\_address} = \pi_1 f$.

The functor $x\,\mathrm{const\_value}$ yields an element of the carrier of S and is defined by:

(Def. 13) There exists a finite sequence f of elements of Data-Loc$_{\mathrm{SCM}} \cup$ the carrier of S such that $f = x_{\mathbf{2}}$ and $x\,\mathrm{const\_value} = \pi_2 f$.

We now state the proposition

(20) Let x be an element of Instr$_{\mathrm{SCM}}(S)$, m_1 be an element of Data-Loc$_{\mathrm{SCM}}$, and r be an element of the carrier of S. If $x = \langle I, \langle m_1, r \rangle \rangle$, then $x\,\mathrm{const\_address} = m_1$ and $x\,\mathrm{const\_value} = r$.

Let R be a good ring, let x be an element of Instr$_{\mathrm{SCM}}(R)$, and let s be an **SCM**-state over R. The functor Exec-Res$_{\mathrm{SCM}}(x, s)$ yields an **SCM**-state over

R and is defined by:

(Def. 14) $\text{Exec-Res}_{\text{SCM}}(x, s) =$

$$\begin{cases} \text{Chg}_{\text{SCM}}(\text{Chg}_{\text{SCM}}(s, x\,\text{address}_1, s(x\,\text{address}_2)), \text{Next}(\mathbf{IC}_s)), \text{ if there exist elements } m_1, m_2 \text{ of Data-Loc}_{\text{SCM}} \text{ such that } x = \langle 1, \langle m_1, m_2\rangle\rangle, \\ \text{Chg}_{\text{SCM}}(\text{Chg}_{\text{SCM}}(s, x\,\text{address}_1, s(x\,\text{address}_1) + s(x\,\text{address}_2)), \text{Next}(\mathbf{IC}_s)), \text{ if there exist elements } m_1, m_2 \text{ of Data-Loc}_{\text{SCM}} \text{ such that } x = \langle 2, \langle m_1, m_2\rangle\rangle, \\ \text{Chg}_{\text{SCM}}(\text{Chg}_{\text{SCM}}(s, x\,\text{address}_1, s(x\,\text{address}_1) - s(x\,\text{address}_2)), \text{Next}(\mathbf{IC}_s)), \text{ if there exist elements } m_1, m_2 \text{ of Data-Loc}_{\text{SCM}} \text{ such that } x = \langle 3, \langle m_1, m_2\rangle\rangle, \\ \text{Chg}_{\text{SCM}}(\text{Chg}_{\text{SCM}}(s, x\,\text{address}_1, s(x\,\text{address}_1) \cdot s(x\,\text{address}_2)), \text{Next}(\mathbf{IC}_s)), \text{ if there exist elements } m_1, m_2 \text{ of Data-Loc}_{\text{SCM}} \text{ such that } x = \langle 4, \langle m_1, m_2\rangle\rangle, \\ \text{Chg}_{\text{SCM}}(s, x\,\text{address}_{\text{j}}), \text{ if there exists an element } m_1 \text{ of Instr-Loc}_{\text{SCM}} \text{ such that } x = \langle 6, \langle m_1\rangle\rangle, \\ \text{Chg}_{\text{SCM}}(s, (s(x\,\text{address}_{\text{c}}) = 0_R \rightarrow x\,\text{address}_{\text{j}}, \text{Next}(\mathbf{IC}_s))), \text{ if there exists an element } m_1 \text{ of Instr-Loc}_{\text{SCM}} \text{ and there exists an element } m_2 \text{ of Data-Loc}_{\text{SCM}} \text{ such that } x = \langle 7, \langle m_1, m_2\rangle\rangle, \\ \text{Chg}_{\text{SCM}}(\text{Chg}_{\text{SCM}}(s, x\,\text{const\_address}, x\,\text{const\_value}), \text{Next}(\mathbf{IC}_s)), \text{ if there exists an element } m_1 \text{ of Data-Loc}_{\text{SCM}} \text{ and there exists an element } r \text{ of the carrier of } R \text{ such that } x = \langle 5, \langle m_1, r\rangle\rangle, \\ s, \text{ otherwise.} \end{cases}$$

Let S be a non empty 1-sorted structure, let f be a function from $\text{Instr}_{\text{SCM}}(S)$ into $(\prod \text{OK}_{\text{SCM}}(S))^{\prod \text{OK}_{\text{SCM}}(S)}$, and let x be an element of $\text{Instr}_{\text{SCM}}(S)$. One can check that $f(x)$ is function-like and relation-like.

Let R be a good ring. The functor $\text{Exec}_{\text{SCM}}(R)$ yielding a function from $\text{Instr}_{\text{SCM}}(R)$ into $(\prod \text{OK}_{\text{SCM}}(R))^{\prod \text{OK}_{\text{SCM}}(R)}$ is defined as follows:

(Def. 15) For every element x of $\text{Instr}_{\text{SCM}}(R)$ and for every **SCM**-state y over R holds $(\text{Exec}_{\text{SCM}}(R))(x)(y) = \text{Exec-Res}_{\text{SCM}}(x, y)$.

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