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Predicate Calculus for Boolean Valued Functions. Part II

Shunichi Kobayashi
Shinshu University
Nagano

Yatsuka Nakamura
Shinshu University
Nagano

Summary. In this paper, we have proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of usual predicate logic.

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The terminology and notation used in this paper are introduced in the following articles: [8], [10], [11], [2], [3], [7], [6], [9], [1], [4], and [5].

1. Preliminaries

In this paper Y denotes a non empty set.

Next we state a number of propositions:

(1) For all elements a, b, c of $\mathrm{BVF}(Y)$ such that $a \Subset b \Rightarrow c$ holds $a \wedge b \Subset c$.

(2) For all elements a, b, c of $\mathrm{BVF}(Y)$ such that $a \wedge b \Subset c$ holds $a \Subset b \Rightarrow c$.

(3) For all elements a, b of $\mathrm{BVF}(Y)$ holds $a \vee a \wedge b = a$.

(4) For all elements a, b of $\mathrm{BVF}(Y)$ holds $a \wedge (a \vee b) = a$.

(5) For every element a of $\mathrm{BVF}(Y)$ holds $a \wedge \neg a = \mathit{false}(Y)$.

(6) For every element a of $\mathrm{BVF}(Y)$ holds $a \vee \neg a = \mathit{true}(Y)$.

(7) For all elements a, b of $\mathrm{BVF}(Y)$ holds $a \Leftrightarrow b = (a \Rightarrow b) \wedge (b \Rightarrow a)$.

(8) For all elements a, b of $\mathrm{BVF}(Y)$ holds $a \Rightarrow b = \neg a \vee b$.

(9) For all elements a, b of $\mathrm{BVF}(Y)$ holds $a \oplus b = \neg a \wedge b \vee a \wedge \neg b$.

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(10) For all elements a, b of BVF(Y) holds $a \Leftrightarrow b = \mathit{true}(Y)$ iff $a \Rightarrow b = \mathit{true}(Y)$ and $b \Rightarrow a = \mathit{true}(Y)$.

(11) For all elements a, b, c of BVF(Y) such that $a \Leftrightarrow b = \mathit{true}(Y)$ and $b \Leftrightarrow c = \mathit{true}(Y)$ holds $a \Leftrightarrow c = \mathit{true}(Y)$.

(12) For all elements a, b of BVF(Y) such that $a \Leftrightarrow b = \mathit{true}(Y)$ holds $\neg a \Leftrightarrow \neg b = \mathit{true}(Y)$.

(13) For all elements a, b, c, d of BVF(Y) such that $a \Leftrightarrow b = \mathit{true}(Y)$ and $c \Leftrightarrow d = \mathit{true}(Y)$ holds $a \wedge c \Leftrightarrow b \wedge d = \mathit{true}(Y)$.

(14) For all elements a, b, c, d of BVF(Y) such that $a \Leftrightarrow b = \mathit{true}(Y)$ and $c \Leftrightarrow d = \mathit{true}(Y)$ holds $a \Rightarrow c \Leftrightarrow b \Rightarrow d = \mathit{true}(Y)$.

(15) For all elements a, b, c, d of BVF(Y) such that $a \Leftrightarrow b = \mathit{true}(Y)$ and $c \Leftrightarrow d = \mathit{true}(Y)$ holds $a \vee c \Leftrightarrow b \vee d = \mathit{true}(Y)$.

(16) For all elements a, b, c, d of BVF(Y) such that $a \Leftrightarrow b = \mathit{true}(Y)$ and $c \Leftrightarrow d = \mathit{true}(Y)$ holds $a \Leftrightarrow c \Leftrightarrow b \Leftrightarrow d = \mathit{true}(Y)$.

2. Predicate Calculus

Next we state a number of propositions:

(17) Let a, b be elements of BVF(Y), G be a subset of PARTITIONS(Y), and P_1 be a partition of Y. If G is a coordinate and $P_1 \in G$, then $\forall_{a \Leftrightarrow b, P_1} G = \forall_{a \Rightarrow b, P_1} G \wedge \forall_{b \Rightarrow a, P_1} G$.

(18) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and P_1, P_2 be partitions of Y. Suppose G is a coordinate and $P_1 \in G$ and $P_2 \in G$. Then $\forall_{a, P_1} G \Subset \exists_{a, P_1} G$ and $\forall_{a, P_1} G \Subset \exists_{a, P_2} G$.

(19) Let a, u be elements of BVF(Y), G be a subset of PARTITIONS(Y), and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$ and u is independent of P_1, G. If $a \Rightarrow u = \mathit{true}(Y)$, then $\forall_{a, P_1} G \Rightarrow u = \mathit{true}(Y)$.

(20) Let u be an element of BVF(Y), G be a subset of PARTITIONS(Y), and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$ and u is independent of P_1, G. Then $\exists_{u, P_1} G \Subset u$.

(21) Let u be an element of BVF(Y), G be a subset of PARTITIONS(Y), and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$ and u is independent of P_1, G. Then $u \Subset \forall_{u, P_1} G$.

(22) Let u be an element of BVF(Y), G be a subset of PARTITIONS(Y), and P_1, P_2 be partitions of Y. Suppose G is a coordinate and $P_1 \in G$ and $P_2 \in G$ and u is independent of P_2, G. Then $\forall_{u, P_1} G \Subset \forall_{u, P_2} G$.

(23) Let u be an element of BVF(Y), G be a subset of PARTITIONS(Y), and P_1, P_2 be partitions of Y. Suppose G is a coordinate and $P_1 \in G$ and $P_2 \in G$ and u is independent of P_1, G. Then $\exists_{u, P_1} G \Subset \exists_{u, P_2} G$.

(24) Let a, b be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. If G is a coordinate and $P_1 \in G$, then $\forall_{a \Leftrightarrow b, P_1} G \Subset \forall_{a,P_1} G \Leftrightarrow \forall_{b,P_1} G$.

(25) Let a, b be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. If G is a coordinate and $P_1 \in G$, then $\forall_{a \wedge b, P_1} G \Subset a \wedge \forall_{b,P_1} G$.

(26) Let a, u be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$ and u is independent of P_1, G. Then $\forall_{a,P_1} G \Rightarrow u \Subset \exists_{a \Rightarrow u, P_1} G$.

(27) Let a, b be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$. If $a \Leftrightarrow b = \mathit{true}(Y)$, then $\forall_{a,P_1} G \Leftrightarrow \forall_{b,P_1} G = \mathit{true}(Y)$.

(28) Let a, b be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$. If $a \Leftrightarrow b = \mathit{true}(Y)$, then $\exists_{a,P_1} G \Leftrightarrow \exists_{b,P_1} G = \mathit{true}(Y)$.

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