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Propositional Calculus for Boolean Valued Functions. Part I

Shunichi Kobayashi
Shinshu University
Nagano

Yatsuka Nakamura
Shinshu University
Nagano

Summary. In this paper, we have proved some elementary propositional calculus formulae for Boolean valued functions.

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The terminology and notation used in this paper have been introduced in the following articles: [6], [8], [9], [2], [3], [5], [1], [7], and [4].

In this paper Y is a non empty set.

Next we state a number of propositions:

(1) For all elements a, b of $\mathrm{BVF}(Y)$ holds $a = \mathit{true}(Y)$ and $b = \mathit{true}(Y)$ iff $a \wedge b = \mathit{true}(Y)$.

(2) For all elements a, b of $\mathrm{BVF}(Y)$ such that $a = \mathit{true}(Y)$ and $a \Rightarrow b = \mathit{true}(Y)$ holds $b = \mathit{true}(Y)$.

(3) For all elements a, b of $\mathrm{BVF}(Y)$ such that $a = \mathit{true}(Y)$ holds $a \vee b = \mathit{true}(Y)$.

(5)[1] For all elements a, b of $\mathrm{BVF}(Y)$ such that $b = \mathit{true}(Y)$ holds $a \Rightarrow b = \mathit{true}(Y)$.

(6) For all elements a, b of $\mathrm{BVF}(Y)$ such that $\neg a = \mathit{true}(Y)$ holds $a \Rightarrow b = \mathit{true}(Y)$.

(7) For every element a of $\mathrm{BVF}(Y)$ holds $\neg(a \wedge \neg a) = \mathit{true}(Y)$.

(8) For every element a of $\mathrm{BVF}(Y)$ holds $a \Rightarrow a = \mathit{true}(Y)$.

(9) For all elements a, b of $\mathrm{BVF}(Y)$ holds $a \Rightarrow b = \mathit{true}(Y)$ iff $\neg b \Rightarrow \neg a = \mathit{true}(Y)$.

[1]The proposition (4) has been removed.

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(10) For all elements a, b, c of BVF(Y) such that $a \Rightarrow b = \mathit{true}(Y)$ and $b \Rightarrow c = \mathit{true}(Y)$ holds $a \Rightarrow c = \mathit{true}(Y)$.

(11) For all elements a, b of BVF(Y) such that $a \Rightarrow b = \mathit{true}(Y)$ and $a \Rightarrow \neg b = \mathit{true}(Y)$ holds $\neg a = \mathit{true}(Y)$.

(12) For every element a of BVF(Y) holds $\neg a \Rightarrow a \Rightarrow a = \mathit{true}(Y)$.

(13) For all elements a, b, c of BVF(Y) holds $a \Rightarrow b \Rightarrow \neg(b \wedge c) \Rightarrow \neg(a \wedge c) = \mathit{true}(Y)$.

(14) For all elements a, b, c of BVF(Y) holds $a \Rightarrow b \Rightarrow b \Rightarrow c \Rightarrow a \Rightarrow c = \mathit{true}(Y)$.

(15) For all elements a, b, c of BVF(Y) such that $a \Rightarrow b = \mathit{true}(Y)$ holds $b \Rightarrow c \Rightarrow a \Rightarrow c = \mathit{true}(Y)$.

(16) For all elements a, b of BVF(Y) holds $b \Rightarrow a \Rightarrow b = \mathit{true}(Y)$.

(17) For all elements a, b, c of BVF(Y) holds $a \Rightarrow b \Rightarrow c \Rightarrow b \Rightarrow c = \mathit{true}(Y)$.

(18) For all elements a, b of BVF(Y) holds $b \Rightarrow b \Rightarrow a \Rightarrow a = \mathit{true}(Y)$.

(19) For all elements a, b, c of BVF(Y) holds $c \Rightarrow b \Rightarrow a \Rightarrow b \Rightarrow c \Rightarrow a = \mathit{true}(Y)$.

(20) For all elements a, b, c of BVF(Y) holds $b \Rightarrow c \Rightarrow a \Rightarrow b \Rightarrow a \Rightarrow c = \mathit{true}(Y)$.

(21) For all elements a, b, c of BVF(Y) holds $b \Rightarrow b \Rightarrow c \Rightarrow b \Rightarrow c = \mathit{true}(Y)$.

(22) For all elements a, b, c of BVF(Y) holds $a \Rightarrow b \Rightarrow c \Rightarrow a \Rightarrow b \Rightarrow a \Rightarrow c = \mathit{true}(Y)$.

(23) For all elements a, b of BVF(Y) such that $a = \mathit{true}(Y)$ holds $a \Rightarrow b \Rightarrow b = \mathit{true}(Y)$.

(24) For all elements a, b, c of BVF(Y) such that $c \Rightarrow b \Rightarrow a = \mathit{true}(Y)$ holds $b \Rightarrow c \Rightarrow a = \mathit{true}(Y)$.

(25) For all elements a, b, c of BVF(Y) such that $c \Rightarrow b \Rightarrow a = \mathit{true}(Y)$ and $b = \mathit{true}(Y)$ holds $c \Rightarrow a = \mathit{true}(Y)$.

(26) For all elements a, b, c of BVF(Y) such that $c \Rightarrow b \Rightarrow a = \mathit{true}(Y)$ and $b = \mathit{true}(Y)$ and $c = \mathit{true}(Y)$ holds $a = \mathit{true}(Y)$.

(27) For all elements b, c of BVF(Y) such that $b \Rightarrow b \Rightarrow c = \mathit{true}(Y)$ holds $b \Rightarrow c = \mathit{true}(Y)$.

(28) For all elements a, b, c of BVF(Y) such that $a \Rightarrow b \Rightarrow c = \mathit{true}(Y)$ holds $a \Rightarrow b \Rightarrow a \Rightarrow c = \mathit{true}(Y)$.

(29) For all elements a, b, c of BVF(Y) such that $a \Rightarrow b \Rightarrow c = \mathit{true}(Y)$ and $a \Rightarrow b = \mathit{true}(Y)$ holds $a \Rightarrow c = \mathit{true}(Y)$.

(30) For all elements a, b, c of BVF(Y) such that $a \Rightarrow b \Rightarrow c = \mathit{true}(Y)$ and $a \Rightarrow b = \mathit{true}(Y)$ and $a = \mathit{true}(Y)$ holds $c = \mathit{true}(Y)$.

(31) For all elements a, b, c, d of BVF(Y) such that $a \Rightarrow b \Rightarrow c = \mathit{true}(Y)$ and $a \Rightarrow c \Rightarrow d = \mathit{true}(Y)$ holds $a \Rightarrow b \Rightarrow d = \mathit{true}(Y)$.

(32) For all elements a, b of $\mathrm{BVF}(Y)$ holds $\neg a \Rightarrow \neg b \Rightarrow b \Rightarrow a = \mathit{true}(Y)$.

(33) For all elements a, b of $\mathrm{BVF}(Y)$ holds $a \Rightarrow b \Rightarrow \neg b \Rightarrow \neg a = \mathit{true}(Y)$.

(34) For all elements a, b of $\mathrm{BVF}(Y)$ holds $a \Rightarrow \neg b \Rightarrow b \Rightarrow \neg a = \mathit{true}(Y)$.

(35) For all elements a, b of $\mathrm{BVF}(Y)$ holds $\neg a \Rightarrow b \Rightarrow \neg b \Rightarrow a = \mathit{true}(Y)$.

(36) For every element a of $\mathrm{BVF}(Y)$ holds $a \Rightarrow \neg a \Rightarrow \neg a = \mathit{true}(Y)$.

(37) For all elements a, b of $\mathrm{BVF}(Y)$ holds $\neg a \Rightarrow a \Rightarrow b = \mathit{true}(Y)$.

(38) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $\neg(a \wedge b \wedge c) = \neg a \vee \neg b \vee \neg c$.

(39) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $\neg(a \vee b \vee c) = \neg a \wedge \neg b \wedge \neg c$.

(40) For all elements a, b, c, d of $\mathrm{BVF}(Y)$ holds $a \vee b \wedge c \wedge d = (a \vee b) \wedge (a \vee c) \wedge (a \vee d)$.

(41) For all elements a, b, c, d of $\mathrm{BVF}(Y)$ holds $a \wedge (b \vee c \vee d) = a \wedge b \vee a \wedge c \vee a \wedge d$.

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