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Propositional Calculus for Boolean Valued Functions. Part III

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Summary. In this paper, we have proved some elementary propositional calculus formulae for Boolean valued functions.

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The articles [6], [8], [9], [2], [3], [5], [1], [7], and [4] provide the terminology and notation for this paper.

In this paper Y is a non empty set.

Next we state a number of propositions:

(1) For all elements a, b of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (\neg a \Rightarrow b) = b$.

(2) For all elements a, b of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (a \Rightarrow \neg b) = \neg a$.

(3) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \Rightarrow b \vee c = (a \Rightarrow b) \vee (a \Rightarrow c)$.

(4) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \Rightarrow b \wedge c = (a \Rightarrow b) \wedge (a \Rightarrow c)$.

(5) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \vee b \Rightarrow c = (a \Rightarrow c) \wedge (b \Rightarrow c)$.

(6) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \wedge b \Rightarrow c = (a \Rightarrow c) \vee (b \Rightarrow c)$.

(7) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \wedge b \Rightarrow c = a \Rightarrow b \Rightarrow c$.

(8) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \wedge b \Rightarrow c = a \Rightarrow \neg b \vee c$.

(9) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \Rightarrow b \vee c = a \wedge \neg b \Rightarrow c$.

(10) For all elements a, b of $\mathrm{BVF}(Y)$ holds $a \wedge (a \Rightarrow b) = a \wedge b$.

(11) For all elements a, b of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge \neg b = \neg a \wedge \neg b$.

(12) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) = (a \Rightarrow b) \wedge (b \Rightarrow c) \wedge (a \Rightarrow c)$.

(13) For every element a of $\mathrm{BVF}(Y)$ holds $\mathit{true}(Y) \Rightarrow a = a$.

(14) For every element a of $\mathrm{BVF}(Y)$ holds $a \Rightarrow \mathit{false}(Y) = \neg a$.

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(15) For every element a of $\mathrm{BVF}(Y)$ holds $\mathit{false}(Y) \Rightarrow a = \mathit{true}(Y)$.

(16) For every element a of $\mathrm{BVF}(Y)$ holds $a \Rightarrow \mathit{true}(Y) = \mathit{true}(Y)$.

(17) For every element a of $\mathrm{BVF}(Y)$ holds $a \Rightarrow \neg a = \neg a$.

(18) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \Rightarrow b \Subset c \Rightarrow a \Rightarrow c \Rightarrow b$.

(19) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \Leftrightarrow b \Subset a \Leftrightarrow c \Leftrightarrow b \Leftrightarrow c$.

(20) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \Leftrightarrow b \Subset a \Rightarrow c \Leftrightarrow b \Rightarrow c$.

(21) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \Leftrightarrow b \Subset c \Rightarrow a \Leftrightarrow c \Rightarrow b$.

(22) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \Leftrightarrow b \Subset a \wedge c \Leftrightarrow b \wedge c$.

(23) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \Leftrightarrow b \Subset a \vee c \Leftrightarrow b \vee c$.

(24) For all elements a, b of $\mathrm{BVF}(Y)$ holds $a \Subset a \Leftrightarrow b \Leftrightarrow b \Leftrightarrow a \Leftrightarrow a$.

(25) For all elements a, b of $\mathrm{BVF}(Y)$ holds $a \Subset a \Rightarrow b \Leftrightarrow b$.

(26) For all elements a, b of $\mathrm{BVF}(Y)$ holds $a \Subset b \Rightarrow a \Leftrightarrow a$.

(27) For all elements a, b of $\mathrm{BVF}(Y)$ holds $a \Subset a \wedge b \Leftrightarrow b \wedge a \Leftrightarrow a$.

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