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Some Properties of Cells on Go-Board

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The terminology and notation used in this paper have been introduced in the following articles: [23], [9], [13], [3], [20], [22], [25], [26], [7], [8], [2], [1], [5], [6], [24], [10], [19], [4], [15], [14], [21], [11], [12], [16], [17], and [18].

We use the following convention: i, i_1, i_2, j, j_1, j_2, k, n are natural numbers, D is a non empty set, and f is a finite sequence of elements of D.

Let E be a non empty set, let S be a non empty set of finite sequences of the carrier of $\mathcal{E}^2_{\mathrm{T}}$, let F be a function from E into S, and let e be an element of E. Then $F(e)$ is a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$.

Let F be a function. The functor $\mathrm{Values}\, F$ yielding a set is defined by:

(Def. 1) $\mathrm{Values}\, F = \mathrm{Union}(\mathrm{rng}_\kappa\, F(\kappa))$.

We now state three propositions:

(1) Let M be a finite sequence of elements of D^*. If $i \in \mathrm{dom}\, M$, then $M(i)$ is a finite sequence of elements of D.

(2) For every finite sequence M of elements of D^* holds $\mathrm{dom}(\mathrm{rng}_\kappa\, M(\kappa)) = \mathrm{dom}\, M$.

(3) For every finite sequence M of elements of D^* holds $\mathrm{Values}\, M = \bigcup\{\mathrm{rng}\, f; f$ ranges over elements of D^*: $f \in \mathrm{rng}\, M\}$.

Let D be a non empty set and let M be a finite sequence of elements of D^*. Note that $\mathrm{Values}\, M$ is finite.

The following propositions are true:

(4) For every matrix M over D such that $i \in \mathrm{dom}\, M$ and $M(i) = f$ holds $\mathrm{len}\, f = \mathrm{width}\, M$.

(5) For every matrix M over D such that $i \in \mathrm{dom}\, M$ and $M(i) = f$ and $j \in \mathrm{dom}\, f$ holds $\langle i, j \rangle \in$ the indices of M.

(6) For every matrix M over D such that $\langle i, j \rangle \in$ the indices of M and $M(i) = f$ holds $\mathrm{len}\, f = \mathrm{width}\, M$ and $j \in \mathrm{dom}\, f$.

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(7) For every matrix M over D holds Values $M = \{M_{i,j} : \langle i, j\rangle \in$ the indices of $M\}$.

(8) For every non empty set D and for every matrix M over D holds card Values $M \leqslant \operatorname{len} M \cdot \operatorname{width} M$.

In the sequel f, f_1, f_2 are finite sequences of elements of $\mathcal{E}^2_{\mathrm{T}}$ and G is a Go-board.

Next we state a number of propositions:

(9) If f is a sequence which elements belong to G, then $\operatorname{rng} f \subseteq$ Values G.

(10) For all Go-boards G_1, G_2 such that Values $G_1 \subseteq$ Values G_2 and $\langle i_1, j_1\rangle \in$ the indices of G_1 and $1 \leqslant j_2$ and $j_2 \leqslant \operatorname{width} G_2$ and $(G_1)_{i_1,j_1} = (G_2)_{1,j_2}$ holds $i_1 = 1$.

(11) For all Go-boards G_1, G_2 such that Values $G_1 \subseteq$ Values G_2 and $\langle i_1, j_1\rangle \in$ the indices of G_1 and $1 \leqslant j_2$ and $j_2 \leqslant \operatorname{width} G_2$ and $(G_1)_{i_1,j_1} = (G_2)_{\operatorname{len} G_2,j_2}$ holds $i_1 = \operatorname{len} G_1$.

(12) For all Go-boards G_1, G_2 such that Values $G_1 \subseteq$ Values G_2 and $\langle i_1, j_1\rangle \in$ the indices of G_1 and $1 \leqslant i_2$ and $i_2 \leqslant \operatorname{len} G_2$ and $(G_1)_{i_1,j_1} = (G_2)_{i_2,1}$ holds $j_1 = 1$.

(13) For all Go-boards G_1, G_2 such that Values $G_1 \subseteq$ Values G_2 and $\langle i_1, j_1\rangle \in$ the indices of G_1 and $1 \leqslant i_2$ and $i_2 \leqslant \operatorname{len} G_2$ and $(G_1)_{i_1,j_1} = (G_2)_{i_2,\operatorname{width} G_2}$ holds $j_1 = \operatorname{width} G_1$.

(14) Let G_1, G_2 be Go-boards. Suppose Values $G_1 \subseteq$ Values G_2 and $1 \leqslant i_1$ and $i_1 < \operatorname{len} G_1$ and $1 \leqslant j_1$ and $j_1 \leqslant \operatorname{width} G_1$ and $1 \leqslant i_2$ and $i_2 < \operatorname{len} G_2$ and $1 \leqslant j_2$ and $j_2 \leqslant \operatorname{width} G_2$ and $(G_1)_{i_1,j_1} = (G_2)_{i_2,j_2}$. Then $((G_2)_{i_2+1,j_2})_{\mathbf{1}} \leqslant ((G_1)_{i_1+1,j_1})_{\mathbf{1}}$.

(15) Let G_1, G_2 be Go-boards. Suppose Values $G_1 \subseteq$ Values G_2 and $1 < i_1$ and $i_1 \leqslant \operatorname{len} G_1$ and $1 \leqslant j_1$ and $j_1 \leqslant \operatorname{width} G_1$ and $1 < i_2$ and $i_2 \leqslant \operatorname{len} G_2$ and $1 \leqslant j_2$ and $j_2 \leqslant \operatorname{width} G_2$ and $(G_1)_{i_1,j_1} = (G_2)_{i_2,j_2}$. Then $((G_1)_{i_1-'1,j_1})_{\mathbf{1}} \leqslant ((G_2)_{i_2-'1,j_2})_{\mathbf{1}}$.

(16) Let G_1, G_2 be Go-boards. Suppose Values $G_1 \subseteq$ Values G_2 and $1 \leqslant i_1$ and $i_1 \leqslant \operatorname{len} G_1$ and $1 \leqslant j_1$ and $j_1 < \operatorname{width} G_1$ and $1 \leqslant i_2$ and $i_2 \leqslant \operatorname{len} G_2$ and $1 \leqslant j_2$ and $j_2 < \operatorname{width} G_2$ and $(G_1)_{i_1,j_1} = (G_2)_{i_2,j_2}$. Then $((G_2)_{i_2,j_2+1})_{\mathbf{2}} \leqslant ((G_1)_{i_1,j_1+1})_{\mathbf{2}}$.

(17) Let G_1, G_2 be Go-boards. Suppose Values $G_1 \subseteq$ Values G_2 and $1 \leqslant i_1$ and $i_1 \leqslant \operatorname{len} G_1$ and $1 < j_1$ and $j_1 \leqslant \operatorname{width} G_1$ and $1 \leqslant i_2$ and $i_2 \leqslant \operatorname{len} G_2$ and $1 < j_2$ and $j_2 \leqslant \operatorname{width} G_2$ and $(G_1)_{i_1,j_1} = (G_2)_{i_2,j_2}$. Then $((G_1)_{i_1,j_1-'1})_{\mathbf{2}} \leqslant ((G_2)_{i_2,j_2-'1})_{\mathbf{2}}$.

(18) Let G_1, G_2 be Go-boards. Suppose Values $G_1 \subseteq$ Values G_2 and $\langle i_1, j_1\rangle \in$ the indices of G_1 and $\langle i_2, j_2\rangle \in$ the indices of G_2 and $(G_1)_{i_1,j_1} = (G_2)_{i_2,j_2}$. Then $\operatorname{cell}(G_2, i_2, j_2) \subseteq \operatorname{cell}(G_1, i_1, j_1)$.

(19) Let G_1, G_2 be Go-boards. Suppose Values $G_1 \subseteq$ Values G_2 and $\langle i_1, j_1\rangle \in$

the indices of G_1 and $\langle i_2, j_2\rangle \in$ the indices of G_2 and $(G_1)_{i_1,j_1} = (G_2)_{i_2,j_2}$. Then $\mathrm{cell}(G_2, i_2 -' 1, j_2) \subseteq \mathrm{cell}(G_1, i_1 -' 1, j_1)$.

(20) Let G_1, G_2 be Go-boards. Suppose $\mathrm{Values}\, G_1 \subseteq \mathrm{Values}\, G_2$ and $\langle i_1, j_1\rangle \in$ the indices of G_1 and $\langle i_2, j_2\rangle \in$ the indices of G_2 and $(G_1)_{i_1,j_1} = (G_2)_{i_2,j_2}$. Then $\mathrm{cell}(G_2, i_2, j_2 -' 1) \subseteq \mathrm{cell}(G_1, i_1, j_1 -' 1)$.

(21) Let f be a standard special circular sequence. Suppose f is a sequence which elements belong to G. Then Values the Go-board of $f \subseteq \mathrm{Values}\, G$.

Let us consider f, G, k. Let us assume that $1 \leqslant k$ and $k + 1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G. The functor right_cell(f, k, G) yields a subset of $\mathcal{E}^2_{\mathrm{T}}$ and is defined by the condition (Def. 2).

(Def. 2) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $\langle i_1, j_1\rangle \in$ the indices of G and $\langle i_2, j_2\rangle \in$ the indices of G and $\pi_k f = G_{i_1,j_1}$ and $\pi_{k+1} f = G_{i_2,j_2}$. Then
- (i) $i_1 = i_2$ and $j_1 + 1 = j_2$ and right_cell$(f, k, G) = \mathrm{cell}(G, i_1, j_1)$, or
- (ii) $i_1 + 1 = i_2$ and $j_1 = j_2$ and right_cell$(f, k, G) = \mathrm{cell}(G, i_1, j_1 -' 1)$, or
- (iii) $i_1 = i_2 + 1$ and $j_1 = j_2$ and right_cell$(f, k, G) = \mathrm{cell}(G, i_2, j_2)$, or
- (iv) $i_1 = i_2$ and $j_1 = j_2 + 1$ and right_cell$(f, k, G) = \mathrm{cell}(G, i_1 -' 1, j_2)$.

The functor left_cell(f, k, G) yields a subset of $\mathcal{E}^2_{\mathrm{T}}$ and is defined by the condition (Def. 3).

(Def. 3) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $\langle i_1, j_1\rangle \in$ the indices of G and $\langle i_2, j_2\rangle \in$ the indices of G and $\pi_k f = G_{i_1,j_1}$ and $\pi_{k+1} f = G_{i_2,j_2}$. Then
- (i) $i_1 = i_2$ and $j_1 + 1 = j_2$ and left_cell$(f, k, G) = \mathrm{cell}(G, i_1 -' 1, j_1)$, or
- (ii) $i_1 + 1 = i_2$ and $j_1 = j_2$ and left_cell$(f, k, G) = \mathrm{cell}(G, i_1, j_1)$, or
- (iii) $i_1 = i_2 + 1$ and $j_1 = j_2$ and left_cell$(f, k, G) = \mathrm{cell}(G, i_2, j_2 -' 1)$, or
- (iv) $i_1 = i_2$ and $j_1 = j_2 + 1$ and left_cell$(f, k, G) = \mathrm{cell}(G, i_1, j_2)$.

We now state a number of propositions:

(22) Suppose that
$1 \leqslant k$ and $k + 1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G and $\langle i, j\rangle \in$ the indices of G and $\langle i, j + 1\rangle \in$ the indices of G and $\pi_k f = G_{i,j}$ and $\pi_{k+1} f = G_{i,j+1}$. Then left_cell$(f, k, G) = \mathrm{cell}(G, i -' 1, j)$.

(23) Suppose that
$1 \leqslant k$ and $k + 1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G and $\langle i, j\rangle \in$ the indices of G and $\langle i, j + 1\rangle \in$ the indices of G and $\pi_k f = G_{i,j}$ and $\pi_{k+1} f = G_{i,j+1}$. Then right_cell$(f, k, G) = \mathrm{cell}(G, i, j)$.

(24) Suppose that
$1 \leqslant k$ and $k + 1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G and $\langle i, j\rangle \in$ the indices of G and $\langle i + 1, j\rangle \in$ the indices of G and $\pi_k f = G_{i,j}$ and $\pi_{k+1} f = G_{i+1,j}$. Then left_cell$(f, k, G) = \mathrm{cell}(G, i, j)$.

(25) Suppose that
$1 \leqslant k$ and $k+1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G and $\langle i, j\rangle \in$ the indices of G and $\langle i + 1, j\rangle \in$ the indices of G and $\pi_k f = G_{i,j}$

and $\pi_{k+1}f = G_{i+1,j}$. Then right_cell$(f,k,G) = \mathrm{cell}(G,i,j -' 1)$.

(26) Suppose that
$1 \leqslant k$ and $k+1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G and $\langle i,\, j\rangle \in$ the indices of G and $\langle i+1,\, j\rangle \in$ the indices of G and $\pi_k f = G_{i+1,j}$ and $\pi_{k+1}f = G_{i,j}$. Then left_cell$(f,k,G) = \mathrm{cell}(G,i,j -' 1)$.

(27) Suppose that
$1 \leqslant k$ and $k+1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G and $\langle i,\, j\rangle \in$ the indices of G and $\langle i+1,\, j\rangle \in$ the indices of G and $\pi_k f = G_{i+1,j}$ and $\pi_{k+1}f = G_{i,j}$. Then right_cell$(f,k,G) = \mathrm{cell}(G,i,j)$.

(28) Suppose that
$1 \leqslant k$ and $k+1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G and $\langle i,\, j+1\rangle \in$ the indices of G and $\langle i,\, j\rangle \in$ the indices of G and $\pi_k f = G_{i,j+1}$ and $\pi_{k+1}f = G_{i,j}$. Then left_cell$(f,k,G) = \mathrm{cell}(G,i,j)$.

(29) Suppose that
$1 \leqslant k$ and $k+1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G and $\langle i,\, j+1\rangle \in$ the indices of G and $\langle i,\, j\rangle \in$ the indices of G and $\pi_k f = G_{i,j+1}$ and $\pi_{k+1}f = G_{i,j}$. Then right_cell$(f,k,G) = \mathrm{cell}(G,i -' 1,j)$.

(30) If $1 \leqslant k$ and $k+1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G, then left_cell$(f,k,G) \cap$ right_cell$(f,k,G) = \mathcal{L}(f,k)$.

(31) If $1 \leqslant k$ and $k+1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G, then right_cell(f,k,G) is closed.

(32) Suppose $1 \leqslant k$ and $k+1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G and $k+1 \leqslant n$. Then left_cell$(f,k,G) =$ left_cell$(f{\restriction}n,k,G)$ and right_cell$(f,k,G) =$ right_cell$(f{\restriction}n,k,G)$.

(33) Suppose $1 \leqslant k$ and $k+1 \leqslant \mathrm{len}(f_{\lfloor n})$ and $n \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G. Then left_cell$(f,k+n,G) =$ left_cell$(f_{\lfloor n},k,G)$ and right_cell$(f,k+n,G) =$ right_cell$(f_{\lfloor n},k,G)$.

(34) Let G be a Go-board and f be a standard special circular sequence. Suppose $1 \leqslant n$ and $n+1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G. Then left_cell$(f,n,G) \subseteq \mathrm{leftcell}(f,n)$ and right_cell$(f,n,G) \subseteq \mathrm{rightcell}(f,n)$.

Let us consider f, G, k. Let us assume that $1 \leqslant k$ and $k+1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G. The functor front_right_cell(f,k,G) yielding a subset of $\mathcal{E}^2_{\mathrm{T}}$ is defined by the condition (Def. 4).

(Def. 4) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $\langle i_1,\, j_1\rangle \in$ the indices of G and $\langle i_2,\, j_2\rangle \in$ the indices of G and $\pi_k f = G_{i_1,j_1}$ and $\pi_{k+1}f = G_{i_2,j_2}$. Then

(i) $i_1 = i_2$ and $j_1 + 1 = j_2$ and front_right_cell$(f,k,G) = \mathrm{cell}(G,i_2,j_2)$, or

(ii) $i_1 + 1 = i_2$ and $j_1 = j_2$ and front_right_cell$(f,k,G) = \mathrm{cell}(G,i_2,j_2 -' 1)$, or

(iii) $i_1 = i_2 + 1$ and $j_1 = j_2$ and front_right_cell$(f, k, G) = \mathrm{cell}(G, i_2 -' 1, j_2)$, or

(iv) $i_1 = i_2$ and $j_1 = j_2 + 1$ and front_right_cell$(f, k, G) = \mathrm{cell}(G, i_2 -' 1, j_2 -' 1)$.

The functor front_left_cell(f, k, G) yields a subset of $\mathcal{E}^2_{\mathrm{T}}$ and is defined by the condition (Def. 5).

(Def. 5) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $\langle i_1, j_1 \rangle \in$ the indices of G and $\langle i_2, j_2 \rangle \in$ the indices of G and $\pi_k f = G_{i_1,j_1}$ and $\pi_{k+1} f = G_{i_2,j_2}$. Then

(i) $i_1 = i_2$ and $j_1 + 1 = j_2$ and front_left_cell$(f, k, G) = \mathrm{cell}(G, i_2 -' 1, j_2)$, or

(ii) $i_1 + 1 = i_2$ and $j_1 = j_2$ and front_left_cell$(f, k, G) = \mathrm{cell}(G, i_2, j_2)$, or

(iii) $i_1 = i_2 + 1$ and $j_1 = j_2$ and front_left_cell$(f, k, G) = \mathrm{cell}(G, i_2 -' 1, j_2 -' 1)$, or

(iv) $i_1 = i_2$ and $j_1 = j_2 + 1$ and front_left_cell$(f, k, G) = \mathrm{cell}(G, i_2, j_2 -' 1)$.

Next we state several propositions:

(35) Suppose that
$1 \leqslant k$ and $k+1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G and $\langle i, j \rangle \in$ the indices of G and $\langle i, j+1 \rangle \in$ the indices of G and $\pi_k f = G_{i,j}$ and $\pi_{k+1} f = G_{i,j+1}$. Then front_left_cell$(f, k, G) = \mathrm{cell}(G, i -' 1, j + 1)$.

(36) Suppose that
$1 \leqslant k$ and $k+1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G and $\langle i, j \rangle \in$ the indices of G and $\langle i, j+1 \rangle \in$ the indices of G and $\pi_k f = G_{i,j}$ and $\pi_{k+1} f = G_{i,j+1}$. Then front_right_cell$(f, k, G) = \mathrm{cell}(G, i, j + 1)$.

(37) Suppose that
$1 \leqslant k$ and $k+1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G and $\langle i, j \rangle \in$ the indices of G and $\langle i+1, j \rangle \in$ the indices of G and $\pi_k f = G_{i,j}$ and $\pi_{k+1} f = G_{i+1,j}$. Then front_left_cell$(f, k, G) = \mathrm{cell}(G, i + 1, j)$.

(38) Suppose that
$1 \leqslant k$ and $k+1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G and $\langle i, j \rangle \in$ the indices of G and $\langle i+1, j \rangle \in$ the indices of G and $\pi_k f = G_{i,j}$ and $\pi_{k+1} f = G_{i+1,j}$. Then front_right_cell$(f, k, G) = \mathrm{cell}(G, i + 1, j -' 1)$.

(39) Suppose that
$1 \leqslant k$ and $k+1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G and $\langle i, j \rangle \in$ the indices of G and $\langle i+1, j \rangle \in$ the indices of G and $\pi_k f = G_{i+1,j}$ and $\pi_{k+1} f = G_{i,j}$. Then front_left_cell$(f, k, G) = \mathrm{cell}(G, i -' 1, j -' 1)$.

(40) Suppose that
$1 \leqslant k$ and $k+1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G and $\langle i, j \rangle \in$ the indices of G and $\langle i+1, j \rangle \in$ the indices of G and $\pi_k f = G_{i+1,j}$ and $\pi_{k+1} f = G_{i,j}$. Then front_right_cell$(f, k, G) = \mathrm{cell}(G, i -' 1, j)$.

(41) Suppose that

$1 \leqslant k$ and $k+1 \leqslant \operatorname{len} f$ and f is a sequence which elements belong to G and $\langle i, j+1\rangle \in$ the indices of G and $\langle i, j\rangle \in$ the indices of G and $\pi_k f = G_{i,j+1}$ and $\pi_{k+1} f = G_{i,j}$. Then front_left_cell$(f, k, G) =$ cell$(G, i, j -' 1)$.

(42) Suppose that
$1 \leqslant k$ and $k+1 \leqslant \operatorname{len} f$ and f is a sequence which elements belong to G and $\langle i, j+1\rangle \in$ the indices of G and $\langle i, j\rangle \in$ the indices of G and $\pi_k f = G_{i,j+1}$ and $\pi_{k+1} f = G_{i,j}$. Then front_right_cell$(f, k, G) =$ cell$(G, i -' 1, j -' 1)$.

(43) Suppose $1 \leqslant k$ and $k + 1 \leqslant \operatorname{len} f$ and f is a sequence which elements belong to G and $k + 1 \leqslant n$. Then front_left_cell$(f, k, G) =$ front_left_cell$(f{\restriction}n, k, G)$ and front_right_cell$(f, k, G) =$ front_right_cell$(f{\restriction}n, k, G)$.

Let us consider f, G, k. We say that f turns right k, G if and only if the condition (Def. 6) is satisfied.

(Def. 6) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $\langle i_1, j_1\rangle \in$ the indices of G and $\langle i_2, j_2\rangle \in$ the indices of G and $\pi_k f = G_{i_1,j_1}$ and $\pi_{k+1} f = G_{i_2,j_2}$. Then

(i) $i_1 = i_2$ and $j_1 + 1 = j_2$ and $\langle i_2 + 1, j_2\rangle \in$ the indices of G and $\pi_{k+2} f = G_{i_2+1,j_2}$, or

(ii) $i_1 + 1 = i_2$ and $j_1 = j_2$ and $\langle i_2, j_2 -' 1\rangle \in$ the indices of G and $\pi_{k+2} f = G_{i_2,j_2-'1}$, or

(iii) $i_1 = i_2 + 1$ and $j_1 = j_2$ and $\langle i_2, j_2 + 1\rangle \in$ the indices of G and $\pi_{k+2} f = G_{i_2,j_2+1}$, or

(iv) $i_1 = i_2$ and $j_1 = j_2 + 1$ and $\langle i_2 -' 1, j_2\rangle \in$ the indices of G and $\pi_{k+2} f = G_{i_2-'1,j_2}$.

We say that f turns left k, G if and only if the condition (Def. 7) is satisfied.

(Def. 7) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $\langle i_1, j_1\rangle \in$ the indices of G and $\langle i_2, j_2\rangle \in$ the indices of G and $\pi_k f = G_{i_1,j_1}$ and $\pi_{k+1} f = G_{i_2,j_2}$. Then

(i) $i_1 = i_2$ and $j_1 + 1 = j_2$ and $\langle i_2 -' 1, j_2\rangle \in$ the indices of G and $\pi_{k+2} f = G_{i_2-'1,j_2}$, or

(ii) $i_1 + 1 = i_2$ and $j_1 = j_2$ and $\langle i_2, j_2 + 1\rangle \in$ the indices of G and $\pi_{k+2} f = G_{i_2,j_2+1}$, or

(iii) $i_1 = i_2 + 1$ and $j_1 = j_2$ and $\langle i_2, j_2 -' 1\rangle \in$ the indices of G and $\pi_{k+2} f = G_{i_2,j_2-'1}$, or

(iv) $i_1 = i_2$ and $j_1 = j_2 + 1$ and $\langle i_2 + 1, j_2\rangle \in$ the indices of G and $\pi_{k+2} f = G_{i_2+1,j_2}$.

We say that f goes straight k, G if and only if the condition (Def. 8) is satisfied.

(Def. 8) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $\langle i_1, j_1\rangle \in$ the indices of G and $\langle i_2, j_2\rangle \in$ the indices of G and $\pi_k f = G_{i_1,j_1}$ and $\pi_{k+1} f = G_{i_2,j_2}$. Then

(i) $i_1 = i_2$ and $j_1 + 1 = j_2$ and $\langle i_2, j_2 + 1\rangle \in$ the indices of G and $\pi_{k+2} f = G_{i_2,j_2+1}$, or

(ii) $i_1 + 1 = i_2$ and $j_1 = j_2$ and $\langle i_2 + 1, j_2\rangle \in$ the indices of G and $\pi_{k+2} f = G_{i_2+1,j_2}$, or

(iii) $i_1 = i_2 + 1$ and $j_1 = j_2$ and $\langle i_2 -' 1, j_2 \rangle \in$ the indices of G and $\pi_{k+2} f = G_{i_2 -' 1, j_2}$, or

(iv) $i_1 = i_2$ and $j_1 = j_2 + 1$ and $\langle i_2, j_2 -' 1 \rangle \in$ the indices of G and $\pi_{k+2} f = G_{i_2, j_2 -' 1}$.

One can prove the following propositions:

(44) Suppose $1 \leqslant k$ and $k + 2 \leqslant \operatorname{len} f$ and f is a sequence which elements belong to G and $k + 2 \leqslant n$ and $f \restriction n$ turns right k, G. Then f turns right k, G.

(45) Suppose $1 \leqslant k$ and $k + 2 \leqslant \operatorname{len} f$ and f is a sequence which elements belong to G and $k + 2 \leqslant n$ and $f \restriction n$ turns left k, G. Then f turns left k, G.

(46) Suppose $1 \leqslant k$ and $k + 2 \leqslant \operatorname{len} f$ and f is a sequence which elements belong to G and $k+2 \leqslant n$ and $f \restriction n$ goes straight k, G. Then f goes straight k, G.

(47) Suppose that
$1 < k$ and $k + 1 \leqslant \operatorname{len} f_1$ and $k + 1 \leqslant \operatorname{len} f_2$ and f_1 is a sequence which elements belong to G and f_2 is a sequence which elements belong to G and $f_1 \restriction k = f_2 \restriction k$ and f_1 turns right $k -' 1$, G and f_2 turns right $k -' 1$, G. Then $f_1 \restriction (k + 1) = f_2 \restriction (k + 1)$.

(48) Suppose that
$1 < k$ and $k + 1 \leqslant \operatorname{len} f_1$ and $k + 1 \leqslant \operatorname{len} f_2$ and f_1 is a sequence which elements belong to G and f_2 is a sequence which elements belong to G and $f_1 \restriction k = f_2 \restriction k$ and f_1 turns left $k -' 1$, G and f_2 turns left $k -' 1$, G. Then $f_1 \restriction (k + 1) = f_2 \restriction (k + 1)$.

(49) Suppose that
$1 < k$ and $k + 1 \leqslant \operatorname{len} f_1$ and $k + 1 \leqslant \operatorname{len} f_2$ and f_1 is a sequence which elements belong to G and f_2 is a sequence which elements belong to G and $f_1 \restriction k = f_2 \restriction k$ and f_1 goes straight $k -' 1$, G and f_2 goes straight $k -' 1$, G. Then $f_1 \restriction (k + 1) = f_2 \restriction (k + 1)$.

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