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On the Components of the Complement of a Special Polygonal Curve

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Summary. By the special polygonal curve we meana simple closed curve, that is a polygone and moreover has edges parallel to axes. We continue the formalization of the Takeuti-Nakamura proof [21] of the Jordan curve theorem. In the paper we prove that the complement of the special polygonal curve consists of at least two components. With the theorem which has at most two components we completed the theorem that a special polygonal curve cuts the plane into exactly two components.

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The articles [22], [29], [1], [11], [3], [2], [27], [28], [19], [12], [20], [30], [7], [8], [9], [16], [4], [24], [13], [14], [15], [5], [18], [23], [17], [6], [10], [26], and [25] provide the terminology and notation for this paper.

In this paper j denotes a natural number.

One can prove the following propositions:

(1) Let f be a S-sequence in $\mathbb{R}^2$ and Q be a non empty compact subset of $\mathcal{E}^2_{\rm T}$. If $\widetilde{\mathcal{L}}(f)$ meets Q and $\pi_1 f \notin Q$, then $\widetilde{\mathcal{L}}(\downharpoonleft f, \mathrm{FPoint}(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\mathrm{len}\, f} f, Q)) \cap Q = \{\mathrm{FPoint}(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\mathrm{len}\, f} f, Q)\}$.

(2) Let f be a finite sequence of elements of $\mathcal{E}^2_{\rm T}$ and p be a point of $\mathcal{E}^2_{\rm T}$. If f is a special sequence and $p = \pi_{\mathrm{len}\, f} f$, then $\widetilde{\mathcal{L}}(\downharpoonleft p, f) = \{p\}$.

(3) Let f be a finite sequence of elements of $\mathcal{E}^2_{\rm T}$ and p be a point of $\mathcal{E}^2_{\rm T}$. If f is a special sequence and $p \in \widetilde{\mathcal{L}}(f)$, then $\widetilde{\mathcal{L}}(\downharpoonleft p, f) \subseteq \widetilde{\mathcal{L}}(f)$.

(4) Let f be a S-sequence in $\mathbb{R}^2$, p be a point of $\mathcal{E}^2_{\rm T}$, and given j. If $1 \leqslant j$ and $j < \mathrm{len}\, f$ and $p \in \widetilde{\mathcal{L}}(\mathrm{mid}(f, j, \mathrm{len}\, f))$, then LE $\pi_j f$, p, $\widetilde{\mathcal{L}}(f)$, $\pi_1 f$, $\pi_{\mathrm{len}\, f} f$.

[1]The work had been done when the first author visited Nagano in fall of 1998.

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(5) Let f be a S-sequence in $\mathbb{R}^2$, p, q be points of $\mathcal{E}^2_{\mathrm{T}}$, and given j. If $1 \leqslant j$ and $j < \operatorname{len} f$ and $p \in \mathcal{L}(f, j)$ and $q \in \mathcal{L}(p, \pi_{j+1} f)$, then LE p, q, $\widetilde{\mathcal{L}}(f)$, $\pi_1 f$, $\pi_{\operatorname{len} f} f$.

(6) Let f be a S-sequence in $\mathbb{R}^2$ and Q be a non empty compact subset of $\mathcal{E}^2_{\mathrm{T}}$. If $\widetilde{\mathcal{L}}(f)$ meets Q and $\pi_{\operatorname{len} f} f \notin Q$, then $\widetilde{\mathcal{L}}(\downharpoonleft \mathrm{LPoint}(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\operatorname{len} f} f, Q), f) \cap Q = \{\mathrm{LPoint}(\widetilde{\mathcal{L}}(f), \pi_1 f, \pi_{\operatorname{len} f} f, Q)\}$.

(7) For every non constant standard special circular sequence f holds $\mathrm{LeftComp}(f) \neq \mathrm{RightComp}(f)$.

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